Generating Li–Yorke chaos in a stable continuous-time T–S fuzzy model via time-delay feedback control*

Sun Qiu-Ye(孙秋野)[a)†], Zhang Hua-Guang(张化光)[a)b)], and Zhao Yan(赵 琰)[c)]

[a)] *College of Information Science and Engineering, Northeastern University, Shenyang* 110004, *China*

[b)] *Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang* 110004, *China*

[c)] *Department of Automatic Control Engineering, Shenyang Institute of Engineering, Shenyang* 110136, *China*

(Received 21 December 2009; revised manuscript received 21 January 2010)

This paper investigates the chaotification problem of a stable continuous-time T–S fuzzy system. A simple nonlinear state time-delay feedback controller is designed by parallel distributed compensation technique. Then, the asymptotically approximate relationship between the controlled continuous-time T–S fuzzy system with time-delay and a discrete-time T–S fuzzy system is established. Based on the discrete-time T–S fuzzy system, it proves that the chaos in the discrete-time T–S fuzzy system satisfies the Li–Yorke definition by choosing appropriate controller parameters via the revised Marotto theorem. Finally, the effectiveness of the proposed chaotic anticontrol method is verified by a practical example.

Keywords: chaotification, T–S fuzzy model, time-delay feedback, Li–Yorke chaos

PACC: 0545

1. Introduction

Recently, chaotic systems and chaotic phenomena have been paid more and more attention due to its potential applications in many practical application fields.[1−4] As a reverse process of suppressing or eliminating chaotic behaviours in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, chaotification is to generate some chaotic behaviours from a given system, which is non-chaotic or even stable originally. It is well-known that chaotification of dynamical systems has many potential applications in electronic, mechanical, biological and medical systems, etc. Ito and Narikiyo found that the grinding efficiency was higher when the rotating speed of the grinding machine was under chaotic state.[5] Chau *et al.* found that when the stirrer was working under low-Reynolds-number mixing condition, the stirring effect of the stirrer is better with the chaotic rotating speed of the stirring paddle than that with the constant stirring speed.[6] The relevant researches indicated that about 0.1 to 1 billion dollars were wasted due to the inefficiency of the stirring devices per year.[7] Thus, it is necessary to develop proper method to make an originally stable permanent magnet synchronous motor (PMSM) system generate chaotic behaviour. There are several chaotification methods proposed in recent years.[8−25] However, in fact, there is not a unified definition of chaos in a continuous-time nonlinear system currently. Several kinds of acknowledged definitions of chaos, such as Li–Yorke definition, Deveney definition and Melnikov definition, are all set up based on the discrete-time nonlinear system. At present, some chaotification approaches[26,27] for continuous-time systems are generating chaos by tracking a known chaotic system. Strictly speaking, this kind of 'chaos' is not chaos in true sense because the controlled system is not extremely sensitive to initial conditions. Others[28,29] only generate chaos in phenomena by simulation, but lack strict theoretical proof.

In this paper, the PMSM system is considered as the practical continuous-time dynamical system to be chaotified. In many relevant literatures, the PMSM systems are assumed to be smooth-air-gap for simplicity. However, all the PMSM systems are non-smooth-air-gap in real world. But the non-smooth-air-gap PMSM systems are scarcely studied.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60904101, 60972164 and 60904046), the Fundamental Research Funds for the Central Universities (Grant No. N090404009) and the Research Foundation of Education Bureau of Liaoning Province, China (Grant No. 2009A544).

†Corresponding author. E-mail: sunqiuye@mail.neu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

Inspired by the above ideas, this paper is devoted to investigating the chaotification problem of a continuous-time T–S fuzzy model which is originally stable. Moreover, we consider the non-smooth-air-gap PMSM system as the practical dynamical system to be chaotified. First, the exact T–S fuzzy representation of the non-smooth-air-gap PMSM is presented. Based on the continuous-time T–S fuzzy model, a fuzzy nonlinear time-delay feedback controller is designed by parallel distributed compensation (PDC) technique. Then, the asymptotically approximate relationship between the controlled continuous-time T–S fuzzy non-smooth-air-gap PMSM system with time-delay and a discrete-time T–S fuzzy system is established. Finally, based on the discrete-time T–S fuzzy system theory, it proves that the chaos in the discrete-time T–S fuzzy system satisfies the Li–Yorke definition by choosing appropriate controller parameters.

The rest of this paper is organized as follows. In Section 2, the T–S fuzzy model is reviewed and the non-smooth-air-gap PMSM system is introduced briefly. The exact continuous-time T–S fuzzy representation for the non-smooth-air-gap PMSM system is given in Section 3. In Section 4, at first, the approximate relationship between the continuos-time T–S fuzzy delayed system and a discrete-time T–S fuzzy model is established. Then, the corresponding chaotification controller is designed based on the PDC technique. At last, the revised Marotto theorem is applied to prove that the chaos generated in the controlled non-smooth-air-gap PMSM system satisfies the Li–Yorke definition. Simulation study is carried out in Section 5. Some conclusions are drawn in Section 6.

2. Preliminaries and problem formulation

2.1. T–S fuzzy model

Consider the continuous-time T–S fuzzy rule base described as follows:[30]

R^i: IF $p_1(t)$ is M_{i1}, ..., and $p_q(t)$ is M_{iq}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t), \tag{1}$$

where $i = 1, 2, \ldots, r$ (r is the number of fuzzy rules), $\boldsymbol{x}(t) \in R^n$ and $\boldsymbol{u}(t) \in R^m$ denote the state vector and the input vector, respectively, $\boldsymbol{A}_i \in R^{n\times n}$ and $\boldsymbol{B}_i \in R^{n\times m}$ are known system matrix and input matrix with appropriate dimensions, respectively, $p_1(t)$, $p_2(t)$, ..., $p_q(t)$ are premise variables, M_{ij} is a fuzzy set ($j = 1, 2, \ldots, q$). By taking a standard fuzzy inference strategy, i.e., using a singleton fuzzifier, product fuzzy inference and centre average defuzzifier, we infer the final continuous-time fuzzy T–S system is as follows:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{r} \boldsymbol{\omega}_i(\boldsymbol{p}(t)) \left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right)}{\sum_{i=1}^{r} \boldsymbol{\omega}_i(\boldsymbol{p}(t))}, \tag{2}$$

where

$$\boldsymbol{\omega}_i(\boldsymbol{p}(t)) = \prod_{j=1}^{q} M_{ij}(p_j(t)),$$

in which $M_{ij}(p_j(t))$ is the degree of membership of $p_j(t)$ in M_{ij}, with

$$\begin{cases} \sum\limits_{i=1}^{r} \boldsymbol{\omega}_i(\boldsymbol{p}(t)) > 0, \\ \boldsymbol{\omega}_i(\boldsymbol{p}(t)) \geq 0, \end{cases} \quad i = 1, 2, \ldots, r.$$

By using $\boldsymbol{\mu}_i(\boldsymbol{p}(t))(= \boldsymbol{\omega}_i(\boldsymbol{p}(t))/\sum_{i=1}^{r} \boldsymbol{\omega}_i(\boldsymbol{p}(t)))$ instead of $\boldsymbol{\omega}_i(\boldsymbol{p}(t))$, we rewrite Eq. (2) as:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(t)) \left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right). \tag{3}$$

Note that

$$\begin{cases} \sum\limits_{i=1}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(t)) = 1, \\ \boldsymbol{\mu}_i(\boldsymbol{p}(t)) \geq 0, \end{cases} \quad i = 1, 2, \ldots, r,$$

in which $\boldsymbol{\mu}_i(\boldsymbol{p}(t))$ can be regarded as the firing strength of the ith IF–THEN rules.

2.2. Non-smooth-air-gap PMSM plant

Based on d–q axis, the non-smooth-air gap PMSM plant can be modeled as follows:[31]

$$\begin{cases} \dot{i}_q(t) = \dfrac{u_q(t) - R_1 i_q(t) + \omega(t) L_d i_d(t) - \omega(t)\psi_r}{L_q}, \\ \dot{i}_d(t) = \dfrac{u_d(t) - R_1 i_d(t) + \omega(t) L_q i_q(t)}{L_d}, \\ \dot{\omega}(t) = \dfrac{n_p\psi_r i_q(t) + n_p\left(L_d - L_q\right) i_d(t) i_q(t) - T_L(t) - \beta\omega(t)}{J}, \end{cases} \tag{4}$$

where $i_q(t)$, $i_d(t)$ and $\omega(t)$ are state variables, which represent currents and motor angular frequency, respectively, $u_q(t)$ and $u_d(t)$ are the quadrature- and direct-axis stator voltage components, respectively, J is the polar moment of inertia, $T_{\rm L}(t)$ is the external load torque, β is the viscous damping coefficient, R_1 is the stator winding resistance, L_q and L_d are the quadrature- and direct-axis stator inductors, respectively, ψ_r is the permanent magnet flux, and n_p is the number of pole-pairs.

Applying an affine transformation and a time-scaling transformation, we can transform Eq. (4) into

$$\begin{cases} \dot{\tilde{i}}_q(t) = -\tilde{i}_q(t) - \tilde{\omega}(t)\tilde{i}_d(t) + \gamma\tilde{\omega}(t) + \tilde{u}_q(t), \\ \dot{\tilde{i}}_d(t) = -\tilde{i}_d(t) + \tilde{\omega}(t)\tilde{i}_q(t) + \tilde{u}_d(t), \\ \dot{\tilde{\omega}}(t) = \sigma\left(\tilde{i}_q(t) - \tilde{\omega}(t)\right) + \varepsilon\tilde{i}_d(t)\tilde{i}_q(t) - \tilde{T}_{\rm L}(t), \end{cases} \tag{5}$$

where γ, σ and ε are system's parameters.

As is well-known, the external load torque $T_{\rm L}(t)$ is not arbitrarily controllable. Therefore, it is difficult to realize the chaotification of the non-smooth-air gap PMSM system via an external control signal in practice. Thus, we can choose the quadrature-axis stator voltage $\tilde{u}_q(t)$ and the direct-axis stator voltage $\tilde{u}_d(t)$ as the feedback signals.

Our goal is to design an appropriate controller to chaotify the non-smooth-air-gap PMSM system. To realize this target, we will employ the T–S fuzzy model and the corresponding fuzzy control idea, i.e., first, we propose the exact T–S fuzzy representation of the non-smooth-air-gap PMSM system, then, a fuzzy delayed state feedback controller is designed. Moreover, the asymptotically approximate relationship between the controlled continuous-time T–S fuzzy delayed non-smooth-air-gap PMSM system and a discrete-time T–S fuzzy model is established. Finally, based on the discrete-time T–S fuzzy model, we prove that the Li–Yorke chaos will be generated in the controlled T–S fuzzy delayed non-smooth-air-gap PMSM system when the controller parameters are chosen appropriately.

3. Fuzzy modeling of the non-smooth-air-gap PMSM plant

In this section, the following problem is investigated. i.e., how to choose appropriate membership functions and how to set up appropriate fuzzy rules, such that an exact T–S fuzzy model of the non-smooth-air-gap PMSM system can be derived. The following results indicate that a simple T–S fuzzy system with few fuzzy sets and fuzzy rules can represent the non-smooth-air-gap PMSM system exactly.

According to the bounded property of the state variables of the PMSM systems, we can get $\tilde{i}_q(t) \in [c_1 - d_1, c_1 + d_1]$, $\tilde{i}_d(t) \in [c_2 - d_2, c_2 + d_2]$ in which the values of c_1, c_2, d_1 and d_2 can be obtained from simulation. The exact T–S fuzzy model of the above non-smooth-air-gap PMSM system Eq. (5) is given in the following theorem.

Theorem 1 For the non-smooth-air-gap PMSM system Eq. (5) with the input signal $\tilde{u}_q(t)$, $\tilde{u}_d(t)$, the following T–S fuzzy model can represent it accurately.

R^1: IF $x_1(t)$ is M_{11} and $x_2(t)$ is M_{12}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{u}(t),$$

R^2: IF $x_1(t)$ is M_{21} and $x_2(t)$ is M_{22}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_2\boldsymbol{x}(t) + \boldsymbol{B}_2\boldsymbol{u}(t),$$

R^3: IF $x_1(t)$ is M_{31} and $x_2(t)$ is M_{32}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_3\boldsymbol{x}(t) + \boldsymbol{B}_3\boldsymbol{u}(t),$$

R^4: IF $x_1(t)$ is M_{41} and $x_2(t)$ is M_{42}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_4\boldsymbol{x}(t) + \boldsymbol{B}_4\boldsymbol{u}(t),$$

where $x(t) = [x_1(t), x_2(t), x_3(t)]^{\rm T} = [\tilde{i}_q, \tilde{i}_d, \tilde{\omega}]^{\rm T}$,

$$\boldsymbol{A}_1 = \begin{pmatrix} -1 & 0 & \gamma \\ 0 & -1 & 2(c_1 + d_1) \\ \sigma & 2\varepsilon(c_1 + d_1) & -\sigma \end{pmatrix},$$

$$\boldsymbol{A}_2 = \begin{pmatrix} -1 & 0 & \gamma \\ 0 & -1 & 2(c_1 - d_1) \\ \sigma & 2\varepsilon(c_1 - d_1) & -\sigma \end{pmatrix},$$

$$\boldsymbol{A}_3 = \begin{pmatrix} -1 & 0 & \gamma - 2(c_2 + d_2) \\ 0 & -1 & 0 \\ \sigma & 0 & -\sigma \end{pmatrix},$$

$$\boldsymbol{A}_4 = \begin{pmatrix} -1 & 0 & \gamma - 2(c_2 - d_2) \\ 0 & -1 & 0 \\ \sigma & 0 & -\sigma \end{pmatrix},$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}_3 = \boldsymbol{B}_4 = B = [1, 1, 0]^{\rm T}.$$

The membership functions are chosen as

$$M_{11}(x_1(t)) = 0.5\left(1 - \frac{c_1 - x_1(t)}{d_1}\right),$$
$$M_{21}(x_1(t)) = 0.5\left(1 + \frac{c_1 - x_1(t)}{d_1}\right),$$
$$M_{32}(x_2(t)) = 0.5\left(1 - \frac{c_2 - x_2(t)}{d_2}\right),$$
$$M_{42}(x_2(t)) = 0.5\left(1 + \frac{c_2 - x_2(t)}{d_2}\right),$$
$$M_{12}(x_2(t)) = M_{22}(x_2(t))$$
$$= M_{31}(x_1(t)) = M_{41}(x_1(t)) = 1.$$

Proof The choice of the structure of the fuzzy IF–THEN rules, the form of the fuzzy membership functions and the number of the fuzzy IF–THEN rules are mainly determined by the nonlinear terms of the non-smooth-air-gap PMSM system. Unlike the smooth-air-gap PMSM system, there are three nonlinear terms in the equations of the non-smooth-air-gap PMSM system. Moreover, there is not a common factor in these nonlinear terms. Therefore, we have to choose two state variables occurring in these nonlinear terms as the premise variables. Consequently, there are four fuzzy IF–THEN rules in the T–S fuzzy model of the non-smooth-air-gap PMSM system as given in Theorem 1. Then, for each $x_k(t)$ ($k = 1, 2, 3$), applying the product-inference rule, singleton fuzzifier, and the centre of gravity defuzzifier to the above fuzzy rule base, we can infer the overall fuzzy model as

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\}, \qquad (6)$$

where $r = 4$,

$$\boldsymbol{\mu}_i(\boldsymbol{p}(t)) = \prod_{j=1}^{2} M_{ij}(p_j(t)) \Big/ \sum_{i=1}^{4}\prod_{j=1}^{2} M_{ij}(p_j(t)).$$

Substituting $\boldsymbol{A}_i$, $\boldsymbol{B}$ and $M_{ij}(p_j(t))$ as defined above into Eq. (6), we easily see that Eq. (6) is absolutely equivalent to Eq. (5). That is, the original non-smooth-air-gap PMSM system is described by the above T–S fuzzy model. The proof is completed.

Remark 1 Note that there is not any approximation error between the T–S fuzzy model (6) and the original non-smooth-air-gap PMSM system (5). In other words, the established T–S fuzzy model (6) is absolutely equivalent to the original non-smooth-air-gap PMSM system (5).

4. Chaotification controller design and verification of Li–Yorke chaos in the closed-loop fuzzy system

Based on the PDC technique, a fuzzy controller is designed as follows:

R^i: IF $x_1(t)$ is M_{i1} and $x_2(t)$ is M_{i2}, THEN

$$\boldsymbol{u}(t) = \alpha \sin\left(\beta\left(\boldsymbol{T}_i^{-1}\boldsymbol{x}\right)_1(t-\tau)\right), \quad i = 1, 2, 3, 4,$$

where α and β are constant parameters to be determined, $\sin(\cdot)$ denotes the sinusoid function, τ is the time delay, $\left(\boldsymbol{T}_i^{-1}\boldsymbol{x}\right)_1(t-\tau)$ is the first element of the vector $\left(\boldsymbol{T}_i^{-1}\boldsymbol{x}\right)(t-\tau)$ and the transformation matrix $\boldsymbol{T}_i$ is defined as $\boldsymbol{T}_i = \boldsymbol{C}_i\boldsymbol{W}_i$, in which $\boldsymbol{C}_i$ is the controllability matrix $\boldsymbol{C}_i = \left[\begin{array}{ccc}\boldsymbol{B}_i & \boldsymbol{A}_i\boldsymbol{B}_i & \boldsymbol{A}_i^2\boldsymbol{B}_i\end{array}\right]$ and

$$\boldsymbol{W}_i = \begin{pmatrix} a_{i2} & a_{i1} & 1 \\ a_{i1} & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

where a_{ij} are the coefficients of the following characteristic polynomial:

$$\det(\lambda\boldsymbol{I} - \boldsymbol{A}_i) = \lambda^3 + a_{i1}\lambda^2 + a_{i2}\lambda + a_{i3}.$$

Then, the overall fuzzy controller is constructed by the parallel connection, i.e.,

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(t))\left\{\alpha \sin\left(\beta\left(\boldsymbol{T}_i^{-1}\boldsymbol{x}\right)_1(t-\tau)\right)\right\}. \qquad (7)$$

Substituting Eq. (7) into Eq. (6), we can derive the controlled close-loop T–S fuzzy PMSM system as follows:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(t))\mu_j(\boldsymbol{p}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + B\alpha\sin(\beta(T_j^{-1}x)_1(t-\tau))\}. \qquad (8)$$

The following lemma establishes the asymptotically approximate relationship between the continuous-time T–S fuzzy delayed system Eq. (8) and a discrete-time T–S fuzzy model.

Lemma 1[32] There is asymptotically approximate relationship between the continuous-time T–S fuzzy system with time-delay Eq. (8) and the following discrete-time T–S fuzzy system if the time delay τ is sufficiently large and all elements of $(\boldsymbol{T}_i)_1$ are nonzero.

R^{ii}: IF $x_1(k)$ is M_{i1} and $x_2(k)$ is M_{i2}, THEN

$$\boldsymbol{x}(k+1) \approx \begin{pmatrix} \dfrac{\alpha\,(T_i)_{11}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{11}} x_1(k)\right) \\ \dfrac{\alpha\,(T_i)_{21}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{21}} x_2(k)\right) \\ \dfrac{\alpha\,(T_i)_{31}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{31}} x_3(k)\right) \end{pmatrix},$$

R^{ij}: IF $x_1(t)$ is (M_{i1} and M_{j1}) and $x_2(t)$ is (M_{i2} and M_{j2}), THEN

$$\boldsymbol{x}(k+1) \approx \begin{pmatrix} \dfrac{\alpha\,(T_i)_{11}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{11}} x_1(k)\right) \\ \dfrac{\alpha\,(T_i)_{21}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{21}} x_2(k)\right) \\ \dfrac{\alpha\,(T_i)_{31}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{31}} x_3(k)\right) \end{pmatrix},$$

where a'_{i0} are determined in the following matrices

$$\boldsymbol{T}_i^{-1}\boldsymbol{A}_i\boldsymbol{T}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a'_{i0} & -a'_{i1} & -a'_{i2} \end{pmatrix}.$$

Remark 2 In Lemma 1, the state matrices $\boldsymbol{A}_i$ in the continuous-time T–S fuzzy model Eq. (8) should be Hurwitz stable. Since the PMSM system to be chaotified is assumed to be stable at the origin, the above condition can be satisfied naturally. If the uncontrolled PMSM system is unstable at the origin, a simple fuzzy state feedback controller can be designed by applying PDC technique. The controller gain matrices $\boldsymbol{K}_i$ can be obtained by solving the following LMIs:

$$\begin{cases} \boldsymbol{A}_i\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{B}\boldsymbol{M}_i + \boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} < 0, \quad i = 1, 2, \ldots, r, \\ \boldsymbol{Q} > 0, \end{cases}$$

where $\boldsymbol{K}_i = \boldsymbol{M}_i\boldsymbol{Q}^{-1}$. Then we can denote $\boldsymbol{G}_i = \boldsymbol{A}_i + \boldsymbol{B}\boldsymbol{K}_i$ instead of $\boldsymbol{A}_i$ in Eq. (6).

According to Lemma 1, we can derive the following approximatively equivalent discrete-time T–S fuzzy model corresponding to the continuous-time T–S fuzzy non-smooth-air-gap PMSM system Eq. (6).

$$\begin{aligned} \boldsymbol{x}(k+1) \approx & \begin{pmatrix} \sum\limits_{i=1}^{r} \mu_i^2(p(k)) \dfrac{\alpha\,(T_i)_{11}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{11}} x_1(k)\right) \\ \sum\limits_{i=1}^{r} \mu_i^2(p(k)) \dfrac{\alpha\,(T_i)_{21}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{21}} x_2(k)\right) \\ \sum\limits_{i=1}^{r} \mu_i^2(p(k)) \dfrac{\alpha\,(T_i)_{31}}{a'_{i0}} \sin\left(\dfrac{\beta\pi}{(T_i)_{31}} x_3(k)\right) \end{pmatrix} \\ & + \begin{pmatrix} \sum\limits_{i=1}^{r} \sum\limits_{j=1, j\neq i}^{r} \mu_i(p(k))\mu_j(p(k)) \dfrac{\alpha\,(T_i)_{11}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{11}} x_1(k)\right) \\ \sum\limits_{i=1}^{r} \sum\limits_{j=1, j\neq i}^{r} \mu_i(p(k))\mu_j(p(k)) \dfrac{\alpha\,(T_i)_{21}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{21}} x_2(k)\right) \\ \sum\limits_{i=1}^{r} \sum\limits_{j=1, j\neq i}^{r} \mu_i(p(k))\mu_j(p(k)) \dfrac{\alpha\,(T_i)_{31}}{a'_{i0}} \sin\left(\dfrac{\beta\pi\left(T_j^{-1}T_i\right)_{11}}{(T_i)_{31}} x_3(k)\right) \end{pmatrix}. \end{aligned} \tag{9}$$

Next, in order to prove that the discrete-time T–S fuzzy system will generate chaos in the sense of Li–Yorke, some definitions and lemmas are given below.

Consider a general n-dimensional discrete-time autonomous system with the form

$$\boldsymbol{x}(\boldsymbol{k}+1) = \boldsymbol{g}(\boldsymbol{x}(k)), \tag{10}$$

where $\boldsymbol{g}$ is a C^1 nonlinear map. Let $\boldsymbol{g}^{\mathrm{t}}$ denote the t times of compositions of $\boldsymbol{g}$ with itself. A point, $\boldsymbol{x}^*$, is said to be a p-periodic point of $\boldsymbol{g}$ if $\boldsymbol{g}^p(\boldsymbol{x}^*) = \boldsymbol{x}^*$ but $\boldsymbol{g}^{\mathrm{t}}(\boldsymbol{x}^*) \neq \boldsymbol{x}^*$ for $p > t \geq 1$. If $p = 1$, i.e., $\boldsymbol{g}(\boldsymbol{x}^*) = \boldsymbol{x}^*$, $\boldsymbol{x}^*$ is called a fixed point. Let $\boldsymbol{g}'(\boldsymbol{x})$ and $\det(\boldsymbol{g}'(\boldsymbol{x}))$ be the Jacobian of $\boldsymbol{g}$ at the point $\boldsymbol{x}$ and its determinant respectively.

Definition 1[33] Suppose $\boldsymbol{x}^*$ is a fixed point of $\boldsymbol{g}$ with all eigenvalues of $\boldsymbol{g}'(\boldsymbol{x}^*)$ exceeding 1 in magnitude, and suppose there exists a point $\boldsymbol{x}^0 \neq \boldsymbol{x}^*$ in a repelling neighbourhood of $\boldsymbol{x}^*$, such that for some pos-

itive integer m, $\boldsymbol{g}^m(\boldsymbol{x}^0) = \boldsymbol{x}^*$ and $\det((\boldsymbol{g}^m)'(\boldsymbol{x}^0)) \neq 0$. Then $\boldsymbol{x}^*$ is said to be a snap-back repeller of $\boldsymbol{g}$.

Lemma 2[33] If system (10) has a snap-back repeller then the system is chaotic in the sense of Li-Yorke, namely,

1) there exists a positive integer n such that for every integer $p \geq n$, Eq. (10) has p periodic points;

2) there exists a scrambled set (an uncountable invariant set $\boldsymbol{S}$ containing no periodic points) such that

(a) $\boldsymbol{g}(\boldsymbol{S}) \subset \boldsymbol{S}$,

(b) for every $\boldsymbol{y} \in \boldsymbol{S}$ and any periodic point $\boldsymbol{x}$ of Eq. (10)

$$\lim_{k\to\infty} \sup \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| > 0,$$

(c) for every $x, y \in S$ with $x \neq y$

$$\lim_{k\to\infty} \sup \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| > 0;$$

3) there exists an uncountable subset $S_0 \subset S$ such that for any $x, y \in S_0$,

$$\lim_{k\to\infty} \inf \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| = 0.$$

Theorem 2 If the parameter β is given, then there exists a sufficiently large positive constant α, such that the discrete-time T–S fuzzy system Eq. (9) is chaotic in the sense of Li–Yorke.

Proof In Eq. (9), define $\boldsymbol{x}(k+1) = \boldsymbol{g}(\boldsymbol{x}(k))$. Obviously, $\boldsymbol{x}^* = 0$ is the fixed point of g, which will be proven to be a snap-back repeller later. First, the boundedness property of the discrete-time T–S fuzzy system Eq. (9) is proved. Noting that the sinusoid function is bounded and the magnitude of the membership function $\boldsymbol{\mu}_i(\boldsymbol{p}(k))$ could not be greater than 1, then we can derive

$$\begin{aligned}
\lim_{k\to\infty} \|\boldsymbol{x}(k)\| = & \lim_{k\to\infty} \left\| \sum_{i=1}^{r} \boldsymbol{\mu}_i^2(\boldsymbol{p}(k-1)) \frac{\alpha\,(\boldsymbol{T}_i)_1}{a'_{i0}} \sin\left(\frac{\beta\pi}{(\boldsymbol{T}_i)_1}\boldsymbol{x}(k-1)\right) \right. \\
& \left. + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(k-1))\mu_j(\boldsymbol{p}(k-1)) \frac{\alpha\,(\boldsymbol{T}_i)_1}{a'_{i0}} \sin\left(\frac{\beta\pi\left(\boldsymbol{T}_j^{-1}\boldsymbol{T}_i\right)_1}{(\boldsymbol{T}_i)_1}\boldsymbol{x}(k-1)\right) \right\| \\
\leq & \lim_{k\to\infty} \left\| \sum_{i=1}^{r} \boldsymbol{\mu}_i^2(\boldsymbol{p}(k-1)) \frac{\alpha\,(\boldsymbol{T}_i)_1}{a'_{i0}} + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r} \boldsymbol{\mu}_i(\boldsymbol{p}(k-1))\boldsymbol{\mu}_j(\boldsymbol{p}(k-1)) \frac{\alpha\,(\boldsymbol{T}_i)_1}{a'_{i0}} \right\| \\
\leq & \left\| \frac{\alpha\,(\boldsymbol{T}_i)_1}{a'_{i0}} \right\|.
\end{aligned}$$

Therefore, the bounded property of the discrete-time T–S fuzzy system Eq. (9) is guaranteed.

Next, we will prove that the fixed point $\boldsymbol{x}^* = 0$ is a snap-back repeller when the controller magnitude α is sufficiently large. Differentiating Eq. (9) at the fixed point $\boldsymbol{x}^* = 0$ yields

$$\boldsymbol{g}'(\boldsymbol{x}^*) = \pi\alpha\beta\delta I.$$

where $\boldsymbol{I}$ denotes the identity matrix with appropriate dimensions,

$$\delta = \sum_{i=1}^{r} \frac{\boldsymbol{\mu}_i^2(\boldsymbol{x}^*)}{a'_{i0}} + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r} \boldsymbol{\mu}_i(\boldsymbol{x}^*)\boldsymbol{\mu}_j(\boldsymbol{x}^*) \frac{\left(\boldsymbol{T}_j^{=1}\boldsymbol{T}_i\right)_{11}}{a'_{i0}}.$$

If $\alpha > \alpha_1 = 1/\beta\delta\pi$, then it is guaranteed that all the eigenvalues of $\boldsymbol{g}'(\boldsymbol{x}^*)$ exceed 1 in magnitude.

Finally, we will prove that the second condition of the snap-back repeller can be satisfied when the controller magnitude α is chosen large enough. i.e., there exists a point $\boldsymbol{x}^0 \neq \boldsymbol{x}^*$ in a repelling neighbourhood of $\boldsymbol{x}^*$, such that for some positive integer m, $\boldsymbol{g}^m(\boldsymbol{x}^0) = \boldsymbol{x}^*$ and $\det((\boldsymbol{g}^m)'(\boldsymbol{x}^0)) \neq 0$. From Ref. [34], we obtain the following conclusion. i.e., for the following controlled discrete dynamical system,

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{u}(k) := \boldsymbol{F}(\boldsymbol{x}(k)), \\ \boldsymbol{u}(k) = B\varphi(\sigma\boldsymbol{x}(k)) = B\left[\sin(\sigma x_1(k)), \sin(\sigma x_2(k)), \ldots, \sin(\sigma x_n(k))\right]^{\mathrm{T}}, \end{cases} \tag{11}$$

if B is chosen as

$$B > \max\left\{ \frac{\sqrt{289\pi^2 + 256\sigma^2}(N+1)}{16\sigma^2}, \frac{17\pi(N+1)}{16\sigma\sin\left(\frac{\pi}{16}\right)} \right\},$$

where N is determined by $\|\boldsymbol{f}'(\boldsymbol{x}(k))\| \leq N < \infty$, then there exists a point $\boldsymbol{x}_0 \neq \boldsymbol{x}^*$ in a repelling neighbourhood of $\boldsymbol{x}^*$, such that $\boldsymbol{F}^2(\boldsymbol{x}_0) = \boldsymbol{x}^*$ and $\det((\boldsymbol{F}^2)'(\boldsymbol{x}_0)) \neq 0$. Based on the above fact, we can consider the latter part of the right hand side of Eq. (9) as $\boldsymbol{f}(\boldsymbol{x}(k))$ in Eq. (11) and consider the first part of the right hand side of Eq. (9) as $u(k)$ in Eq. (11). Then, we can derive the following results, i.e., if $\alpha > \max\{\alpha_2, \alpha_3\}$, we can get $\boldsymbol{g}^2(\boldsymbol{x}_0) = \boldsymbol{x}^*$ and $\det((\boldsymbol{g}^2)'(\boldsymbol{x}_0)) \neq 0$, where

$$\alpha_2 = \frac{\sqrt{289\pi^2 + 256\rho^2}\theta(N+1)}{16\rho^2},$$

$$\alpha_3 = \frac{17\pi\theta(N+1)}{16\rho\sin\left(\dfrac{\pi}{16}\right)}, \quad \rho = \min\left\{\frac{\beta\pi}{(T_i)_{l1}}\right\},$$

$$\theta = \max\left\{\frac{a'_{i0}}{\sum\limits_{i=1}^{r}\boldsymbol{\mu}_i^2(p(k))(T_i)_{l1}}\right\}, \quad l = 1, 2, 3.$$

According to Definition 1, we can conclude that if the controller magnitude α is chosen as $\alpha > \max\{\alpha_1, \alpha_2, \alpha_3\}$, then $\boldsymbol{x}^* = 0$ is a snap-back repeller of the map $\boldsymbol{g}$ defined in Eq. (9). Consequently, by applying Lemma 2, we can prove that the controlled discrete-time T–S fuzzy system (9) is chaotic in the sense of Li–Yorke. The proof is completed.

Remark 3 Since the asymptotically approximate relationship between the continuous-time T–S fuzzy delayed system Eq. (8) and a discrete-time T–S fuzzy model Eq. (9) has already been established, therefore the continuous-time T–S fuzzy delayed system Eq. (8) will generate chaotic behaviours in the sense of Li–Yorke. In other words, the stable non-smooth-air-gap PMSM system (5) is chaotified successfully.

5. Simulation study

To visualize the effectiveness of the proposed chaotification method, the following practical example is presented for illustration. Firstly, the T–S fuzzy model of the non-smooth-air-gap PMSM system to be chaotified is established. Here, assume that $\gamma = 20$, $\sigma = 8$ and $\varepsilon = 0.5$. The non-smooth-air-gap PMSM system is stable in this situation. The initial conditions are chosen as $\boldsymbol{x}(0) = [0.5,\ 0.5,\ 0.5]^{\mathrm{T}}$. From the simulation result, we can get $c_1 = 2.1768$, $d_1 = 1.8285$, $c_2 = 3.6497$ and $d_2 = 3.1700$. Then, according to Theorem 1, the T–S fuzzy representation of the original non-smooth-air-gap PMSM system can be derived as follows:

R^1: IF $x_1(t)$ is M_{11} and $x_2(t)$ is M_{12}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{u}(t),$$

R^2: IF $x_1(t)$ is M_{21} and $x_2(t)$ is M_{22}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_2\boldsymbol{x}(t) + \boldsymbol{B}_2\boldsymbol{u}(t),$$

R^3: IF $x_1(t)$ is M_{31} and $x_2(t)$ is M_{32}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_3\boldsymbol{x}(t) + \boldsymbol{B}_3\boldsymbol{u}(t),$$

R^4: IF $x_1(t)$ is M_{41} and $x_2(t)$ is M_{42}, THEN

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_4\boldsymbol{x}(t) + \boldsymbol{B}_4\boldsymbol{u}(t),$$

where

$$\boldsymbol{A}_1 = \begin{pmatrix} -1 & 0 & 5 \\ 0 & -1 & 8.0106 \\ 8 & 4.0053 & -8 \end{pmatrix},$$

$$\boldsymbol{A}_2 = \begin{pmatrix} -1 & 0 & 5 \\ 0 & -1 & 0.6966 \\ 8 & 0.3483 & -8 \end{pmatrix},$$

$$\boldsymbol{A}_3 = \begin{pmatrix} -1 & 0 & -8.6394 \\ 0 & -1 & 0 \\ 8 & 0 & -8 \end{pmatrix},$$

$$\boldsymbol{A}_4 = \begin{pmatrix} -1 & 0 & 4.0406 \\ 0 & -1 & 0 \\ 8 & 0 & -8 \end{pmatrix},$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}_3 = \boldsymbol{B}_4 = B = [1, 1, 0]^{\mathrm{T}}.$$

The membership functions are chosen as

$$M_{11}(x_1(t)) = 0.5\left(1 - \frac{2.1768 - x_1(t)}{1.8285}\right),$$

$$M_{21}(x_1(t)) = 0.5\left(1 + \frac{2.1768 - x_1(t)}{1.8285}\right),$$

$$M_{32}(x_2(t)) = 0.5\left(1 - \frac{3.6497 - x_2(t)}{3.1700}\right),$$

$$M_{42}(x_2(t)) = 0.5\left(1 + \frac{3.6497 - x_2(t)}{3.1700}\right),$$

$$M_{12}(x_2(t)) = M_{22}(x_2(t))$$
$$= M_{31}(x_1(t)) = M_{41}(x_1(t)) = 1.$$

Then the whole output of the T–S fuzzy system can be derived as

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{4}\boldsymbol{\mu}_i(\boldsymbol{p}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\}. \qquad (12)$$

The state response curves and the corresponding state evolution trajectory of the uncontrolled T–S fuzzy non-smooth-air-gap PMSM system are presented in Figs. 1 and 2, respectively.

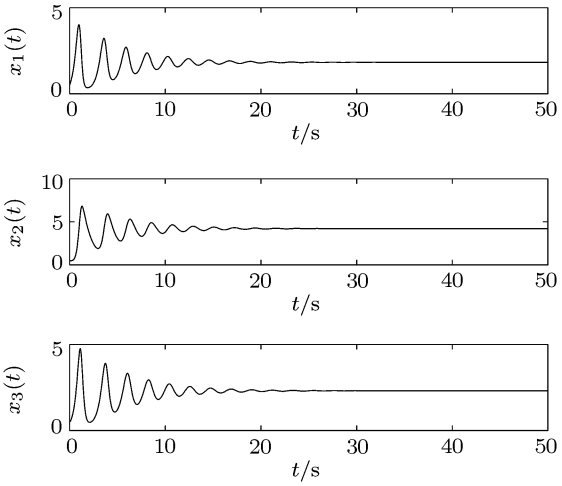


Fig. 1. The state response curves of the fuzzy non-smooth-air-gap PMSM system without control.

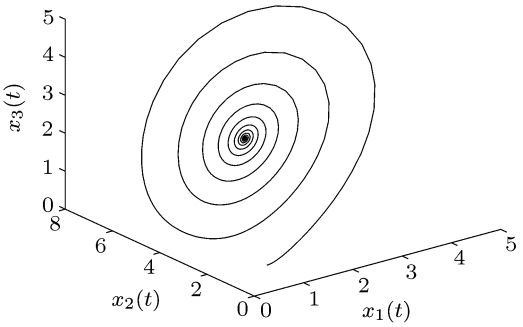


Fig. 2. The state evolution trajectory of the fuzzy non-smooth-air-gap PMSM system without control.

It is obvious that all the state matrices $\boldsymbol{A}_i$ are not Hurwitz stable. Therefore, a fuzzy state feedback subcontroller is needed. According to Remark 3, the controller gain matrices $\boldsymbol{K}_i$ can be derived by linear matrix inequalities (LMI) toolbox in Matlab 7.0 as follows:

$$K_1 = [2.5578, -46.8642, -529.8862],$$
$$K_2 = [2.6549, -32.4411, -358.3441],$$
$$K_3 = [0.5202, -29.0718, -332.5233],$$
$$K_4 = [2.5133, -30.9273, -341.3422].$$

Then, the non-smooth-air-gap PMSM system to be chaotified becomes:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{4} \boldsymbol{\mu}_i(\boldsymbol{p}(t)) \{\boldsymbol{G}_i x(t) + \boldsymbol{B}\boldsymbol{u}(t)\}, \qquad (13)$$

where $\boldsymbol{G}_i = \boldsymbol{A}_i + \boldsymbol{B}\boldsymbol{K}_i$.

Next, we will chaotify the continuous-time T–S fuzzy non-smooth-air-gap PMSM system Eq. (13), The controller is designed in Eq. (7). The matrices T_i^{-1} can be derived as follows:

$$\boldsymbol{T}_1^{-1} = \begin{pmatrix} -0.0276 & 0.0276 & 0.0000 \\ 0.0275 & -0.0275 & 0.0833 \\ 0.6385 & 0.3615 & -0.7497 \end{pmatrix},$$

$$\boldsymbol{T}_2^{-1} = \begin{pmatrix} 0.0278 & -0.0278 & 0.0000 \\ -0.0278 & 0.0278 & 0.1198 \\ 0.9851 & 0.0149 & -1.0781 \end{pmatrix},$$

$$\boldsymbol{T}_3^{-1} = \begin{pmatrix} -0.0145 & 0.0145 & 0.0000 \\ 0.0145 & -0.0145 & 0.1250 \\ 0.9859 & 0.0141 & -1.1250 \end{pmatrix},$$

$$\boldsymbol{T}_4^{-1} = \begin{pmatrix} 0.0309 & -0.0309 & 0.0000 \\ -0.0309 & 0.0309 & 0.1250 \\ 1.0316 & -0.0316 & -1.1250 \end{pmatrix}.$$

In this simulation, the parameter β of the control input is arbitrarily chosen to be $\beta = 200$. The time delay is chosen as $\tau = 1.5$ s. The initial value is given as $\boldsymbol{x}(0) = [0.5,\ 0.5,\ 0.5]^{\mathrm{T}}$. The controller magnitude α is chosen as $\alpha = 50$. The state response curves and the corresponding chaotic attractor of the controlled system are presented in Figs. 3 and 4.

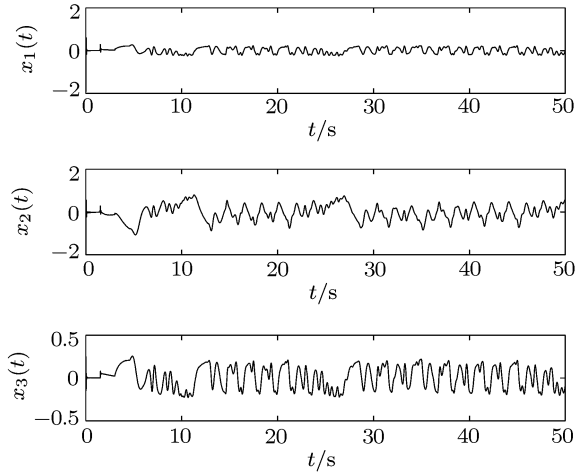


Fig. 3. The state response curves of the controlled fuzzy non-smooth-air-gap PMSM system at $\alpha = 50$.

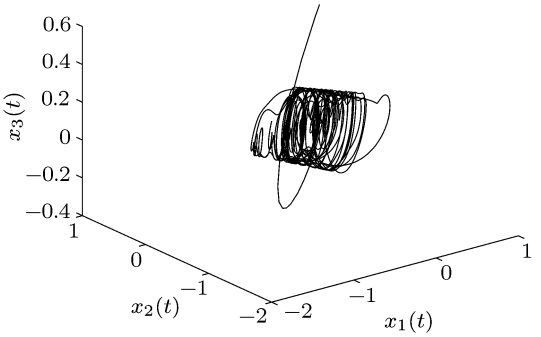


Fig. 4. The chaotic attractor of the controlled fuzzy non-smooth-air-gap PMSM system at $\alpha = 50$.

From these numerical results, it can be seen that the designed controller realizes the chaotification of the above T–S fuzzy non-smooth-air-gap PMSM system.

6. Conclusions

In this work, the chaotification problem of a class of stable continuous-time T–S fuzzy models is investigated. Furthermore, as a practical example to be chaotified, the non-smooth-air-gap PMSM systems are employed. The exact T–S fuzzy model of the non-smooth-air-gap PMSM system is given in this paper. Based on the T–S fuzzy model, we design a simple nonlinear delayed state feedback controller for this purpose. It is proved that the controlled system indeed becomes chaotic in the sense of Li–Yorke by rigorous mathematical analysis. Simulation study is carried out to verify the validity of the proposed chaotification method.

References

[1] Wang X Y and Wang M J 2008 *Phys.* A **387** 3751
[2] Zhang H G, Zhao Y, Yu W and Yang D S 2008 *Chin. Phys.* B **17** 4056
[3] Yang D S, Zhang H G, Li A P and Meng Z Y 2007 *Acta Phys. Sin.* **56** 3121 (in Chinese)
[4] Wang X Y and He Y J 2008 *Acta Phys. Sin.* **57** 1485 (in Chinese)
[5] Ito S and Narikiyo T 1998 *Journal of the Japan Society for Precision Engineering* **64** 748
[6] Chau K, Ye S and Gao Y 2004 *39th IAS Annual Meeting, Conference Record of the 2004 IEEE* **3** 1874
[7] Harnby N, Edwards M and Nienow A 1992 *Mixing in the Process Industries* 2nd ed. (Oxford: Butterworth-Heinemann) p1
[8] Lu J G 2005 *Chin. Phys.* **14** 703
[9] Wang X Y and Meng J 2008 *Acta Phys. Sin.* **57** 726 (in Chinese)
[10] Liu N and Guan Z H 2009 *Chin. Phys.* B **18** 1769
[11] Zhao Y, Zhang H G and Zheng C D 2008 *Chin. Phys.* B **17** 529
[12] Lu J G 2005 *Chin. Phys.* **14** 1088
[13] Wang X Y and Meng J 2009 *Acta Phys. Sin.* **58** 3780 (in Chinese)
[14] Lu J G 2005 *Chin. Phys.* **14** 67
[15] Wu Z M, Lu J G and Xie J Y 2007 *Chin. Phys.* **16** 1258
[16] Ren H P, Liu D and Han C Z 2006 *Acta Phys. Sin.* **55** 2694 (in Chinese)
[17] Wang X Y and He Y J 2008 *Phys. Lett.* A **372** 435
[18] Guan X P, Fan Z P, Zhang Q L and Wang Y Q 2002 *Acta Phys. Sin.* **51** 2216 (in Chinese)
[19] Feng Y L and Zhang X H 2009 *Chin. Phys.* B **18** 5212
[20] Wang X Y and Meng J 2008 *Chaos* **18** 033102
[21] Ma T D, Zhang H G and Wang Z L 2007 *Acta Phys. Sin.* **56** 3796 (in Chinese)
[22] Zhang H G, Ma T D, Yu W and Fu J 2008 *Chin. Phys.* B **17** 3616
[23] Ma T D, Zhang H G and Fu J 2008 *Chin. Phys.* B **17** 4407
[24] Zhang H G, Fu J, Ma T D and Tong S C 2009 *Chin. Phys.* B **18** 969
[25] Zhang H G, Ma T D, Fu J and Tong S C 2009 *Chin. Phys.* B **18** 3751
[26] Li W L and Song Y Z 2008 *Acta Phys. Sin.* **57** 51 (in Chinese)
[27] Zhang H G, Wang Z L and Liu D R 2005 *Int. J. Bifur. Chaos* **15** 2603
[28] Li J, Chen J H and Chau K T 2006 *Proc. CSEE* **26** 77
[29] Meng Z J, Sun C Z, An Y J, Cao J W and Gao P 2007 *Int. Conf. Electrical Machines and Systems* p.1748
[30] Takagi T and Sugeno M 1985 *IEEE Trans. Syst. Man and Cybern.* **15** 116
[31] Ren H P and Liu D 2006 *IEEE Trans. Circ. Syst.* II **53** 45
[32] Kim T R, Joo Y H, Park J B and Chen G R 2005 *Int. J. Bifur. Chaos* **15** 3883
[33] Marotto F R 2005 *Chaos, Solitons and Fractals* **25** 25
[34] Zheng Y A and Chen G R 2004 *Int. J. Bifur. Chaos* **14** 279