Projective synchronisation with non-delayed and delayed coupling in complex networks consisting of identical nodes and different nodes*

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This paper first investigates the projective synchronisation problem with non-delayed and delayed coupling between drive-response dynamical networks consisting of identical nodes and different nodes. Based on Lyapunov stability theory, several nonlinear controllers are applied to achieve the projective synchronisation between the drive-response dynamical networks; simultaneously the topological structure of the drive dynamical complex networks can be exactly identified. Moreover, numerical examples are presented to verify the feasibility and effectiveness of the theorems.

Keywords: adaptive projective synchronisation, drive-response dynamical networks, non-delayed and delayed coupling, identical and different nodes

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1. Introduction

Complex networks have been shown to exist in many different fields in the real world and have been intensively studied in the last few years. Common examples of complex networks include the Internet, the World Wide Web (WWW), food webs, scientific citation webs, as well as many other systems made up of a large number of intricately connected parts. Indeed, complex networks are the important part of our daily lives.[1−8] Generally speaking, a complex network is a large set of interconnected nodes, in which each node is a nonlinear dynamical system. One of the most interesting and significant phenomena is synchronisation in complex dynamical networks. In application, there are many types of concepts of synchronisation, for example, complete synchronisation (CS), lag synchronisation (LS), generalised synchronisation (GS), projective synchronisation (PS), and so on.[9−10] Several schemes can be used to achieve synchronisation, such as coupling control,[11,12] pinning control,[13,14] impulsive control[15,16] and adaptive control.[17−21]

The PS characterised by a scaling factor (a constant λ) such that two systems synchronise proportionally, is a generalised method of synchronisation. Most existing studies pertaining to PS have mainly concentrated on two coupled chaotic systems.[22,23] In Ref. [22], the authors proposed the PS in two coupled partial linear systems. In Ref. [23], Jia discussed the PS of a new hyper-chaotic Lorenz system. Later, interest has been shown in the investigation of the PS between two networks.[24,25] Hu *et al.*[24] introduced the PS properties of the drive-response dynamical networks (DRDNs) model without time delay. Subsequently, Sun *et al.*[25] designed a linear controller and an updated law to realise the PS in DRDNs of partially linear systems with time-varying coupling delay. However, in many circumstances, there exist various uncertain informations such as unknown or uncertain topological structure and node dynamics. Zhou *et al.*[26] showed an effectively adaptive synchronisation approach based on an uncertain complex network model consisting of identical nodes without time de-

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lay. Zhou and Lu[27] proposed a linear controller and updated laws to identify the exact topology of a rather general weighted complex network model consisting of identical or different nodes without time delay. More recently, Liu *et al.*[28] initiated a novel approach for simultaneously identifying the topological structure and unknown parameters of general uncertain complex networks consisting of identical or different nodes with coupling delay and node delay. Additionally, in most real-world complex networks, it is noticed that there exist widespread complex networks consisting of identical nodes and different nodes, in which the information communication of nodes exist not only at time t but also at time $t-\tau$. Therefore, the new idea of adopting nonlinear controllers to achieve PS and identify network structure for such kind of DRDNs consisting of identical and different nodes as well as with non-delayed and delayed coupling has theoretical and practical contribution in practical application.

The layout of this paper is as follows. Section 2 describes the network model and some necessary mathematical preliminaries. Section 3 details the designs of several nonlinear controllers and simple updated laws in order to investigate the PS in DRDNs consisting of identical and different nodes as well as with non-delayed and delayed coupling. Section 4 gives some numerical examples to demonstrate the feasibility and effectiveness of the proposed approach. Finally some concluding remarks are given in Section 5.

2. Model description and preliminaries

In this section, we introduce the network models mentioned in this paper and give some useful mathematical preliminaries.

Now consider the drive complex dynamical networks with non-delayed and delayed coupling consisting of N identical nodes or two different node dynamics, which are described as

$$\begin{aligned}&\dot{\boldsymbol{x}}_i(t)\\ =\ &\boldsymbol{A}\boldsymbol{x}_i(t)+f(t,\boldsymbol{x}_i(t))+\sum_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)\\ &+\sum_{j=1}^{N}c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t-\tau(t)),\ i=1,2,...,N,\end{aligned}\tag{1}$$

$$\begin{cases}\dot{\boldsymbol{x}}_i(t)\\ =\ \boldsymbol{A}\boldsymbol{x}_i(t)+g(t,\boldsymbol{x}_i(t))+\sum_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)\\ \quad+\sum_{j=1}^{N}c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t-\tau(t)),\ 1\le i\le N^*,\\ \dot{\boldsymbol{x}}_i(t)\\ =\ \boldsymbol{A}\boldsymbol{x}_i(t)+h(t,\boldsymbol{x}_i(t))+\sum_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)\\ \quad+\sum_{j=1}^{N}c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t-\tau(t)),\ N^*+1\le i\le N,\end{cases}\tag{2}$$

where $\boldsymbol{x}_i(t)=(x_{i1}(t),x_{i2}(t),...,x_{in}(t))^{\mathrm{T}}\in R^n$ is the state vector of the ith node, $\boldsymbol{A}\in R^{n\times n}$ is a constant matrix, $f,g,h:R\times R^n\to R^n$ are smooth nonlinear functions, $\tau(t)\ge 0$ is the time-varying coupling delay, $\boldsymbol{\Gamma}\in R^{n\times n}$ is inner-coupling matrix and $\boldsymbol{B}=(b_{ij})_{N\times N}\in R^{N\times N},\boldsymbol{C}=(c_{ij})_{N\times N}\in R^{N\times N}$ are the unknown or uncertain weight configuration matrices. If there is a connection from node i to node $j(j\neq i)$, then $b_{ij}\neq 0$, $c_{ij}\neq 0$; otherwise, $b_{ij}=c_{ij}=0(j\neq i)$, and the diagonal elements of matrices $\boldsymbol{B},\boldsymbol{C}$ are defined as $b_{ii}=-\sum_{j=1,j\neq i}^{N}b_{ij},c_{ii}=-\sum_{j=1,j\neq i}^{N}c_{ij}$, $i=1,2,...,N$.

In the following, we present three useful Assumptions and a Lemma, which are required throughout this paper.

Assumption 1 (A1) Suppose that there exists a constant $\alpha>0$, such that $\|f(t,\boldsymbol{y}(t))-f(t,\boldsymbol{x}(t))\|\le\alpha\|\boldsymbol{y}(t)-\boldsymbol{x}(t)\|$ holds for any time-varying vectors $\boldsymbol{x}(t),\boldsymbol{y}(t)$, and the norm $\|\bullet\|$ of a vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\|=(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{1/2}$.

Assumption 2 (A2) Time delay $\tau(t)$ is a differential function with $0\le\dot{\tau}(t)\le\varepsilon<1$. Clearly, this assumption is certainly ensured if the coupling delay $\tau(t)$ is a constant.

Assumption 3 (A3) Suppose that there exist constants $\beta,\ \gamma>0$, such that $\|g(t,\boldsymbol{y}(t))-g(t,\boldsymbol{x}(t))\|\le\beta\|\boldsymbol{y}(t)-\boldsymbol{x}(t)\|$, $\|h(t,\boldsymbol{y}(t))-h(t,\boldsymbol{x}(t))\|\le\gamma\|\boldsymbol{y}(t)-\boldsymbol{x}(t)\|$ hold for any time-varying vectors $\boldsymbol{x}(t),\boldsymbol{y}(t)$, and the norm $\|\bullet\|$ of a vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\|=(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{1/2}$.

Lemma (See Ref. [29]) For any vectors $\boldsymbol{x},\boldsymbol{y}\in R^n$ and positive definite matrix $\boldsymbol{Q}\in R^{n\times n}$, the following matrix inequality holds: $2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}\le\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x}+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{y}$.

3. The PS with non-delayed and delayed coupling in DRDNs consisting of identical and different nodes

3.1. The PS with non-delayed and delayed coupling in DRDNs consisting of identical nodes

Now we will design several nonlinear controllers and updated laws to achieve PS with non-delayed and delayed coupling in DRDNs consisting of identical nodes. For simplicity, we refer to model Eq. (1) as the drive network, and consider a response network described as follows:

$$\dot{\boldsymbol{y}}_i(t) = \boldsymbol{A}\boldsymbol{y}_i(t) + f(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} \hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) + u_i, \quad i = 1, 2, ..., N, \tag{3}$$

where $\boldsymbol{y}_i(t) = (y_{i1}(t), y_{i2}(t), ..., y_{in}(t))^{\mathrm{T}} \in R^n$ is the response state vector of the ith node, $u_i (i = 1, 2, ..., N)$ are nonlinear controllers to be designed, and $\hat{\boldsymbol{B}} = (\hat{b}_{ij})_{N\times N}, \hat{\boldsymbol{C}} = (\hat{c}_{ij})_{N\times N}$ are the estimation of the weight matrices $\boldsymbol{B}$ and $\boldsymbol{C}$, respectively.

Let $\boldsymbol{e}_i(t) = \boldsymbol{x}_i(t) - \lambda\boldsymbol{y}_i(t)$ ($\lambda(\neq 0)$ is a scaling factor), $\tilde{b}_{ij} = \hat{b}_{ij} - b_{ij}$ and $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$, we derive from Eqs. (1) and (3) the following error dynamical network:

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) = {} & \boldsymbol{A}\boldsymbol{e}_i(t) + f(t, \boldsymbol{x}_i(t)) - \lambda f(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) - \lambda\sum_{j=1}^{N} \tilde{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) \\ & + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t)) - \lambda\sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) - \lambda u_i, \quad i = 1, 2, ..., N.\end{aligned} \tag{4}$$

Theorem 1 Suppose A1 and A2 hold. Use the following nonlinear controllers and updated laws:

$$u_i = \frac{1}{\lambda}[\boldsymbol{d}_i\boldsymbol{e}_i(t) + f(t, \lambda\boldsymbol{y}_i(t)) - \lambda f(t, \boldsymbol{y}_i(t))], \quad i = 1, 2, \cdots, N, \tag{5}$$

$$\dot{\hat{b}}_{ij} = \lambda\delta_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t), \dot{\hat{c}}_{ij} = \lambda\delta_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)), \quad \dot{\boldsymbol{d}}_i = k_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad i, j = 1, 2, \cdots, N, \tag{6}$$

where $\boldsymbol{d} = (d_1, d_2, ..., d_N)^{\mathrm{T}} \in R^N$ is the adaptive feedback gain vector to be designed, $\delta_{ij} > 0, k_i > 0 \quad (i, j = 1, 2, ..., N)$ are arbitrary constants, then the response network Eq. (3) can synchronise with the drive network Eq. (1), and the weight configuration matrices $\boldsymbol{B}$ and $\boldsymbol{C}$ of network Eq. (1) can be identified by $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{C}}$, that is ,we will have $\lim_{t\to\infty} \boldsymbol{e}_i(t) = \lim_{t\to\infty}(\hat{b}_{ij} - b_{ij}) = \lim_{t\to\infty}(\hat{c}_{ij} - c_{ij}) = 0,\ i, j = 1, 2, ..., N$.

Proof Construct the following Lyapunov function:

$$\begin{aligned}V = {} & \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{b}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{c}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{k_i}(\boldsymbol{d}_i - d_i^*)^2 \\ & + \frac{1}{2(1-\varepsilon)}\int_{t-\tau(t)}^{t} \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(\theta)\boldsymbol{e}_i(\theta)\mathrm{d}\theta,\end{aligned} \tag{7}$$

where d_i^* is a sufficiently large positive constant to be determined.

Calculating the derivative of Eq. (7) along the trajectories of Eq. (4), and with nonlinear controllers Eq. (5) and updated laws Eq. (6), we obtain

$$\begin{aligned}\dot{V}\big|_{(4)} = {} & \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\dot{\boldsymbol{e}}_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{b}_{ij}\dot{\tilde{b}}_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{c}_{ij}\dot{\tilde{c}}_{ij} + \sum_{i=1}^{N} \frac{1}{k_i}(\boldsymbol{d}_i - d_i^*)\dot{\boldsymbol{d}}_i \\ & + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t)) \\ = {} & \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)[\boldsymbol{A}\boldsymbol{e}_i(t) + f(t, \boldsymbol{x}_i(t)) - \lambda f(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_i(t) - \lambda\sum_{j=1}^{N} \tilde{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t))-\lambda\sum_{j=1}^{N}\tilde{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t))-\boldsymbol{d}_i\boldsymbol{e}_i(t)-f(t,\lambda\boldsymbol{y}_i(t))+\lambda f(t,\boldsymbol{y}_i(t))]\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda\tilde{b}_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda\tilde{c}_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t))+\sum_{i=1}^{N}(\boldsymbol{d}_i-d_i^*)\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)\\
&+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t))\\
=&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}_i(t)+\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)[f(t,\boldsymbol{x}_i(t))-f(t,\lambda\boldsymbol{y}_i(t))]+\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_i(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t))-\sum_{i=1}^{N}d_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)\\
&-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t)).
\end{aligned}
$$

Let $\boldsymbol{e}(t)=(e_1^{\mathrm{T}}(t),e_2^{\mathrm{T}}(t),\cdots,e_N^{\mathrm{T}}(t))^{\mathrm{T}}\in R^{nN}$, $\boldsymbol{P}=(\boldsymbol{B}\otimes\boldsymbol{\Gamma})$, $\boldsymbol{Q}=(\boldsymbol{C}\otimes\boldsymbol{\Gamma})$, where $\otimes$ represents the Kronecker product. Then by A1 and Lemma, we know

$$
\begin{aligned}
\dot{V}\big|_{(4)}\leq&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}_i(t)+\alpha\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t))\\
&-\sum_{i=1}^{N}d_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t))\\
=&\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}(t)+\alpha\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t)+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}(t-\tau(t))-\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{D}^*\boldsymbol{e}(t)\\
&+\frac{1}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t))\\
\leq&\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{A}+\boldsymbol{P}+\alpha\boldsymbol{I}-\boldsymbol{D}^*)\boldsymbol{e}(t)+\frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{e}(t)+\frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t))\\
&+\frac{1}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t)).
\end{aligned}
$$

From A2, we get

$$\frac{1}{2}\leq\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)},$$

thus we have

$$
\begin{aligned}
\dot{V}\big|_{(4)}\leq&\boldsymbol{e}^{\mathrm{T}}(t)\bigg(\boldsymbol{A}+\boldsymbol{P}+\frac{1}{2}\boldsymbol{Q}\boldsymbol{Q}^{\mathrm{T}}+\alpha\boldsymbol{I}\\
&+\frac{1}{2(1-\varepsilon)}\boldsymbol{I}-\boldsymbol{D}^*\bigg)\boldsymbol{e}(t),
\end{aligned}
$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{D}^*=\mathrm{diag}(d_1^*,d_2^*,...,d_N^*)$.

That is, the constants $d_i^*(i=1,2,...,N)$ can be properly chosen to make $\dot{V}\leq 0$. Therefore, according to the Lyapunov stability theory, we have $\boldsymbol{e}(t)\to 0$, $\hat{\boldsymbol{B}}\to\boldsymbol{B}$, and $\hat{\boldsymbol{C}}\to\boldsymbol{C}$ as $t\to\infty$. This implies that the unknown weights b_{ij} and c_{ij} can be successfully estimated by using the nonlinear adaptive controllers Eq. (5) and updated laws Eq. (6), and synchronisation is achieved at the same time. This completes the proof.

Remark 1 According to this theorem, it is shown that using the above controlling approach, the weights of the complex networks Eq. (1) can be estimated by receiving the evolution of network nodes. Namely, the exact topology of model Eq. (1) can be identified.

Remark 2 In the above method, the inner-coupling matrix $\boldsymbol{\Gamma}$ is not necessarily symmetric, and the weight configuration matrices $\boldsymbol{B},\boldsymbol{C}$ need not to be symmetric or irreducible. To this end, this theorem is applicable to a great many complex dynamical networks.

Remark 3 In this theorem, we design nonlinear controllers to achieve the PS, which are different from the usual linear feedback controllers. However, in practice, the linear feedback controller with large enough strength is not realizable.

Remark 4 The coupling delay $\tau = \tau(t)$ is time-varying in this theorem, which includes constant time delay as a special case. Actually, we can also use discrete delays $\tau_j (j = 1, 2, \cdots, N)$ for different nodes, which is very practical. Taking $\tau(t) = \max(\tau_j) =$constant in Eq. (7), we can find that the proof is straightforward.

If there exists the information communication of nodes via the edges only at time t or at time $t - \tau(t)$, i.e., the weight configuration matrix $\boldsymbol{C} = 0$ or $\boldsymbol{B} = 0$, then we have the following corollaries.

Corollary 1 When $C = 0$, network Eq. (1) is translated into

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t) &= \boldsymbol{A}\boldsymbol{x}_i(t) + f(t, \boldsymbol{x}_i(t)) \\ &\quad + \sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t), \quad i = 1, 2, ..., N. \end{aligned} \tag{8}$$

Use the same nonlinear controllers as shown in Eq. (5) and the following updated laws:

$$\begin{aligned}\dot{\hat{b}}_{ij} &= \lambda\delta_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t), \dot{\boldsymbol{d}}_i \\ &= k_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad i, j = 1, 2, ..., N, \end{aligned} \tag{9}$$

where $\delta_{ij} > 0, k_i > 0$ $(i, j = 1, 2, ..., N)$ are arbitrary constants, then the response network

$$\begin{aligned}&\dot{\boldsymbol{y}}_i(t) \\ &= \boldsymbol{A}\boldsymbol{y}_i(t) + f(t, \boldsymbol{y}_i(t)) \\ &\quad + \sum_{j=1}^{N} \hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) + u_i, \quad i = 1, 2, ..., N \end{aligned} \tag{10}$$

can synchronise with the drive network Eq. (8), and the weight configuration matrix $\boldsymbol{B}$ of network Eq. (8) can be identified by $\hat{\boldsymbol{B}}$, i.e., $\lim_{t\to\infty} \boldsymbol{e}_i(t) = \lim_{t\to\infty}(\hat{b}_{ij} - b_{ij}) = 0, \quad i, j = 1, 2, ..., N$.

Corollary 2 When $\boldsymbol{B} = 0$, network Eq. (1) is translated into

$$\begin{aligned}&\dot{\boldsymbol{x}}_i(t) \\ &= \boldsymbol{A}\boldsymbol{x}_i(t) + f(t, \boldsymbol{x}_i(t)) \\ &\quad + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t - \tau(t)), \quad i = 1, 2, ..., N. \end{aligned} \tag{11}$$

Use the same nonlinear controllers as shown in Eq. (5) and the following updated laws:

$$\begin{aligned}\dot{\hat{c}}_{ij} &= \lambda\delta_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{y}_j(t - \tau(t)), \\ \dot{\boldsymbol{d}}_i &= k_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \quad i, j = 1, 2, ..., N, \end{aligned} \tag{12}$$

where $\delta_{ij} > 0, \quad k_i > 0$ $(i, j = 1, 2, ..., N)$ are arbitrary constants, then the response network

$$\begin{aligned}&\dot{\boldsymbol{y}}_i(t) \\ &= \boldsymbol{A}\boldsymbol{y}_i(t) + f(t, \boldsymbol{y}_i(t)) \\ &\quad + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t - \tau(t)) + u_i, \quad i = 1, 2, ..., N \end{aligned} \tag{13}$$

can synchronise with the drive network Eq. (11), and the weight configuration matrix $\boldsymbol{C}$ of network Eq. (11) can be identified by $\hat{\boldsymbol{C}}$, i.e., $\lim_{t\to\infty} \boldsymbol{e}_i(t) = \lim_{t\to\infty}(\hat{c}_{ij} - c_{ij}) = 0, \ i, j = 1, 2, ..., N$.

Remark 5 Similarly to Theorem 1, suppose A1 and A2 hold, consider the same updated laws as shown in Eq. (6) and the following nonlinear controllers:

$$\begin{aligned}u_i &= \frac{1}{\lambda}[\boldsymbol{d}_i\boldsymbol{e}_i(t) + f(t, \boldsymbol{x}_i(t)) \\ &\quad - \lambda f(t, \boldsymbol{y}_i(t))], \quad i = 1, 2, \cdots, N, \end{aligned} \tag{14}$$

or

$$\begin{aligned}u_i &= \frac{1}{\lambda}\left[\boldsymbol{d}_i\boldsymbol{e}_i(t) + f(t, \lambda\boldsymbol{y}_i(t)) - \lambda f\left(t, \frac{1}{\lambda}\boldsymbol{x}_i(t)\right)\right], \\ &\quad i = 1, 2, \cdots, N, \end{aligned} \tag{15}$$

or

$$\begin{aligned}u_i &= \frac{1}{\lambda}\left[\boldsymbol{d}_i\boldsymbol{e}_i(t) + f(t, \boldsymbol{x}_i(t)) - \lambda f\left(t, \frac{1}{\lambda}\boldsymbol{x}_i(t)\right)\right], \\ &\quad i = 1, 2, \cdots, N, \end{aligned} \tag{16}$$

we can also achieve the PS between Eqs. (1) and (3). Therefore, we propose four different nonlinear controllers for the PS in DRDNs, which noticeably expand the varieties of nonlinear controllers. Besides, it is not difficult to check that the above nonlinear controllers also satisfy Corollary 1 and Corollary 2.

3.2. The PS with non-delayed and delayed coupling in DRDNs consisting of different nodes

Here, we investigate the PS with non-delayed and delayed coupling in DRDNs consisting of two different node dynamics. For simplicity, we refer to model Eq. (2) as the drive network, and use the following response network:

$$\begin{cases} \dot{\boldsymbol{y}}_i(t) = \boldsymbol{A}\boldsymbol{y}_i(t) + g(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} \hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) + u_i, \quad 1 \le i \le N^*, \\ \dot{\boldsymbol{y}}_i(t) = \boldsymbol{A}\boldsymbol{y}_i(t) + h(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} \hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) + u_i, \quad N^*+1 \le i \le N. \end{cases} \tag{17}$$

Therefore, we have the following error dynamical network:

$$\begin{cases} \dot{\boldsymbol{e}}_i(t) = \boldsymbol{A}\boldsymbol{e}_i(t) + g(t, \boldsymbol{x}_i(t)) - \lambda g(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) - \lambda \sum_{j=1}^{N} \tilde{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) \\ \qquad + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t)) - \lambda \sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) - \lambda u_i, \quad 1 \le i \le N^*, \\ \dot{\boldsymbol{e}}_i(t) = \boldsymbol{A}\boldsymbol{e}_i(t) + h(t, \boldsymbol{x}_i(t)) - \lambda h(t, \boldsymbol{y}_i(t)) + \sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) - \lambda \sum_{j=1}^{N} \tilde{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t) \\ \qquad + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t)) - \lambda \sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t)) - \lambda u_i, \quad N^*+1 \le i \le N. \end{cases} \tag{18}$$

Theorem 2 Suppose A2 and A3 hold. Use the same nonlinear controllers and updated laws as shown in Eqs. (5) and (6), the response network Eq. (17) can synchronise with the drive network Eq. (2), and the weight configuration matrices $\boldsymbol{B}$ and $\boldsymbol{C}$ of network Eq. (2) can be identified by $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{C}}$, that is, we will have $\lim_{t\to\infty} \boldsymbol{e}_i(t) = \lim_{t\to\infty}(\hat{b}_{ij} - b_{ij}) = \lim_{t\to\infty}(\hat{c}_{ij} - c_{ij}) = 0, \quad i, j = 1, 2, ..., N$.

Proof Construct the following Lyapunov function:

$$\begin{aligned} V = {} & \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{b}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{c}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{k_i}(\boldsymbol{d}_i - d_i^*)^2 \\ & + \frac{1}{2(1-\varepsilon)} \int_{t-\tau(t)}^{t} \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(\theta)\boldsymbol{e}_i(\theta)\mathrm{d}\theta, \end{aligned} \tag{19}$$

where d_i^* is a sufficiently large positive constant to be determined.

Calculating the derivative of Eq. (19) along the trajectories of Eq. (18), and with A3, nonlinear controllers Eq. (5) and updated laws Eq. (6), we obtain

$$\begin{aligned} \dot{V}\big|_{(18)} = {} & \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\dot{\boldsymbol{e}}_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{b}_{ij}\dot{\tilde{b}}_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}}\tilde{c}_{ij}\dot{\tilde{c}}_{ij} + \sum_{i=1}^{N} \frac{1}{k_i}(\boldsymbol{d}_i - d_i^*)\dot{\boldsymbol{d}}_i + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & - \frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t)) \\ \le {} & \beta\sum_{i=1}^{N^*} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \gamma\sum_{i=N^*+1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_i(t) \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_j(t-\tau(t)) - \sum_{i=1}^{N} d_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & - \frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_i(t-\tau(t)). \end{aligned}$$

Let $\boldsymbol{e}(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \cdots, e_N^{\mathrm{T}}(t))^{\mathrm{T}} \in R^{nN}$, $\boldsymbol{P} = (\boldsymbol{B}\otimes\boldsymbol{\Gamma})$, $\boldsymbol{Q} = (\boldsymbol{C}\otimes\boldsymbol{\Gamma})$, where $\otimes$ represents the Kronecker product. Then by Lemma and A2, we know

$$\dot{V}\big|_{(18)} \le \beta\sum_{i=1}^{N^*} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \lambda\sum_{i=N^*+1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}(t) + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}(t-\tau(t)) - \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{D}^*\boldsymbol{e}(t)$$

$$
\begin{aligned}
&+\frac{1}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t))\\
&\leq \boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{A}+\boldsymbol{P}+\boldsymbol{R}-\boldsymbol{D}^{*})\boldsymbol{e}(t)+\frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{e}(t)+\frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t))+\frac{1}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)\\
&\quad-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t))\\
&\leq \boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{A}+\boldsymbol{P}+\boldsymbol{R}+\frac{1}{2}\boldsymbol{Q}\boldsymbol{Q}^{\mathrm{T}}+\frac{1}{2(1-\varepsilon)}\boldsymbol{I}-\boldsymbol{D}^{*})\boldsymbol{e}(t),
\end{aligned}
$$

where $\boldsymbol{R}=\begin{pmatrix}\beta\boldsymbol{I}_{nN^{*}} & 0\\ 0 & \gamma\boldsymbol{I}_{n(N-N^{*})}\end{pmatrix}$.

The constants $d_i^{*}(i=1,2,...,N)$ can be properly chosen to make $\dot{V}\leq 0$. Therefore, according to Lyapunov stability theory, this theorem has been proofed.

Remark 6 From this theorem, by using the above similar controlling scheme, the exact topology of model Eq. (2) can be identified.

Remark 7 In this theorem, the DRDNs with non-delayed and delayed coupling are built up of two types of different nodes. For networks with more types of nodes, the similar work can be generalised easily. On account of the widespread circumstances in which considerable complex networks with different nodes exist, this mechanism is of great significance in practice.

Remark 8 Similarly, using the same nonlinear controllers and updated laws as shown in Eq. (14) (or Eq. (15), or Eq. (16)) and Eq. (6), we can also achieve the PS between Eq. (2) and Eq. (17).

Corollary 3 When $\boldsymbol{C}=0$, network Eq. (2) is translated into

$$
\begin{cases}
\dot{\boldsymbol{x}}_i(t)=\boldsymbol{A}\boldsymbol{x}_i(t)+g(t,\boldsymbol{x}_i(t))+\sum\limits_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t), & 1\leq i\leq N^{*},\\
\dot{\boldsymbol{x}}_i(t)=\boldsymbol{A}\boldsymbol{x}_i(t)+h(t,\boldsymbol{x}_i(t))+\sum\limits_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t), & N^{*}+1\leq i\leq N,
\end{cases}
\tag{20}
$$

using the same nonlinear controllers and updated laws as shown in Eq. (5) (or Eq. (14), or Eq. (15), or Eq. (16)) and Eq. (12), we find that the response network

$$
\begin{cases}
\dot{\boldsymbol{y}}_i(t)=\boldsymbol{A}\boldsymbol{y}_i(t)+g(t,\boldsymbol{y}_i(t))+\sum\limits_{j=1}^{N}\hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t)+u_i, & 1\leq i\leq N^{*},\\
\dot{\boldsymbol{y}}_i(t)=\boldsymbol{A}\boldsymbol{y}_i(t)+h(t,\boldsymbol{y}_i(t))+\sum\limits_{j=1}^{N}\hat{b}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t)+u_i, & N^{*}+1\leq i\leq N
\end{cases}
\tag{21}
$$

can synchronise with the drive network Eq. (22), and the weight configuration matrix $\boldsymbol{B}$ of network Eq. (20) can be identified by $\hat{\boldsymbol{B}}$.

Corollary 4 When $\boldsymbol{B}=0$, network Eq. (2) is translated into

$$
\begin{cases}
\dot{\boldsymbol{x}}_i(t)=\boldsymbol{A}\boldsymbol{x}_i(t)+g(t,\boldsymbol{x}_i(t))+\sum\limits_{j=1}^{N}c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t-\tau(t)), & 1\leq i\leq N^{*},\\
\dot{\boldsymbol{x}}_i(t)=\boldsymbol{A}\boldsymbol{x}_i(t)+h(t,\boldsymbol{x}_i(t))+\sum\limits_{j=1}^{N}c_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t-\tau(t)), & N^{*}+1\leq i\leq N,
\end{cases}
\tag{22}
$$

using the same nonlinear controllers and updated laws as shown in Eq. (5) (or Eq. (14), or Eq. (15), or Eq. (16)) and Eq. (12), we find that the response network

$$
\begin{cases}
\dot{\boldsymbol{y}}_i(t)=\boldsymbol{A}\boldsymbol{y}_i(t)+g(t,\boldsymbol{y}_i(t))+\sum\limits_{j=1}^{N}\hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t))+u_i, & 1\leq i\leq N^{*},\\
\dot{\boldsymbol{y}}_i(t)=\boldsymbol{A}\boldsymbol{y}_i(t)+h(t,\boldsymbol{y}_i(t))+\sum\limits_{j=1}^{N}\hat{c}_{ij}\boldsymbol{\Gamma}\boldsymbol{y}_j(t-\tau(t))+u_i, & N^{*}+1\leq i\leq N
\end{cases}
\tag{23}
$$

can synchronise with the drive network Eq. (22), and the weight configuration matrix $\boldsymbol{C}$ of network Eq. (22) can be identified by $\hat{\boldsymbol{C}}$.

4. Numerical examples

In this section, to verify and demonstrate the effectiveness of the above theory, we consider two numerical examples, that is, the Qi chaotic system[30] and the Lü chaotic system.[31] It is well known that the Qi chaotic system is described by

$$\begin{pmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \dot{\boldsymbol{x}}_3 \end{pmatrix} = \begin{pmatrix} a(x_2 - x_1) + x_2 x_3 \\ cx_1 - x_2 - x_1 x_3 \\ -bx_3 + x_1 x_2 \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + f(t, \boldsymbol{x}), \tag{24}$$

which has a chaotic attractor when $a = 35$, $b = 8/3$, $c = 80$, where $\boldsymbol{A} = \begin{pmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{pmatrix}$, $f(t, \boldsymbol{x}) = \begin{pmatrix} x_2 x_3 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix}$, while the Lü chaotic system is

$$\begin{pmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \dot{\boldsymbol{x}}_3 \end{pmatrix} = \begin{pmatrix} d(x_2 - x_1) \\ fx_2 - x_1 x_3 \\ -ex_3 + x_1 x_2 \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + f(t, \boldsymbol{x}), \tag{25}$$

where $a = 36, b = 3, c = 20, \boldsymbol{A} = \begin{pmatrix} -d & d & 0 \\ 0 & f & 0 \\ 0 & 0 & -e \end{pmatrix}, f(t, \boldsymbol{x}) = \begin{pmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix}$.

In the following, we will investigate these two chaotic systems in detail to validate the effectiveness of Theorems 1 and 2. In the numerical simulations throughout this paper, all the initial values are given as follows: $\boldsymbol{\Gamma} = \boldsymbol{I}$, $\lambda = -0.5$, $\tau(t) = e^t/(1 + e^t), k_i = 3$, $\boldsymbol{x}_i(0) = (0.1 + i \times 0.1, 0.2 + i \times 0.1, 0.4 + i \times 0.1)^{\mathrm{T}}, \boldsymbol{y}_i(0) = (1.6 + i \times 0.1, 0.6 + i \times 0.1, 0.7 + i \times 0.1)^{\mathrm{T}}$, where $i = 1, 2, \ldots, 5$. And we choose the coupling configuration matrices as follows:

$$\boldsymbol{B} = \begin{pmatrix} 0 & -3 & 2 & 0 & 1 \\ 3 & -4 & 1 & 0 & 0 \\ 2 & 1 & -3 & 0 & 0 \\ 1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 & -5 \end{pmatrix}$$

and

$$\boldsymbol{C} = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -5 & 2 & 1 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 & -4 \end{pmatrix}.$$

4.1. Network with identical node dynamics

Now, we consider a weighted linearly coupled complex dynamical network Eq. (1) with non-delayed and delayed coupling consisting of 5 identical Qi chaotic systems. Figure 1 displays the synchronisation errors $\|\boldsymbol{e}_i(t)\| (i = 1, 2, \cdots, 5)$ of DRDNs with non-delayed and delayed coupling consisting of identical nodes. Furthermore, when the coupling matrix $\boldsymbol{C} = 0$ or $\boldsymbol{B} = 0$, network Eq. (1) translates into network Eq. (8) or Eq. (11), then the simulation results are shown in Figs. 2 and 3. From these figures, we can see that the DRDNs achieve PS.

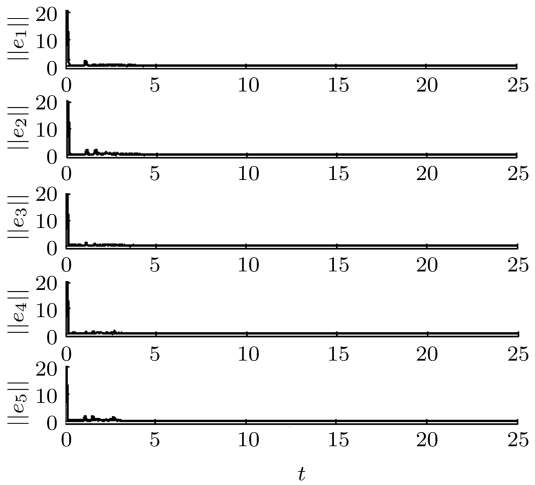


Fig. 1. Synchronisation errors $\|\boldsymbol{e}_i(t)\| (i = 1, 2, \cdots, 5)$ of the drive network Eq. (1) and the response network Eq. (3).

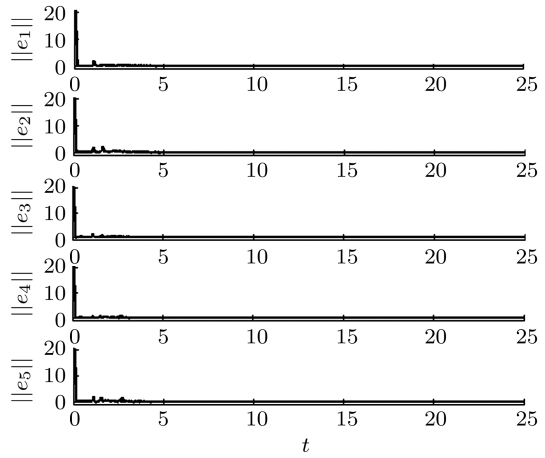


Fig. 2. Synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of the drive network Eq. (8) and the response network Eq. (10).

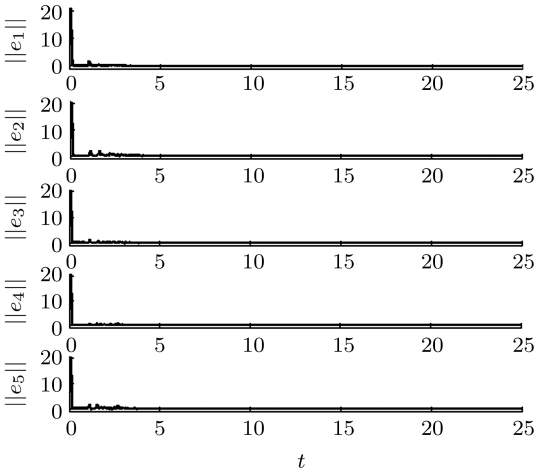


Fig. 3. Synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of the drive network Eq. (11) and the response network Eq. (13).

4.2. Network with different node dynamics

In this subsection, we consider a weighted complex network Eq. (2) with non-delayed and delayed coupling consisting of 3 Qi systems (nodes I–III) and 2 Lü systems (nodes IV–V). The synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of DRDNs with non-delayed and delayed coupling consisting of different nodes are presented in Fig. 4. Furthermore, when the coupling matrix $\boldsymbol{C} = 0$ or $\boldsymbol{B} = 0$, network Eq. (2) translates into network Eq. (20) or Eq. (22), then the simulation results are shown in Figs. 5 and 6. From these figures, we can see that the DRDNs achieve PS.

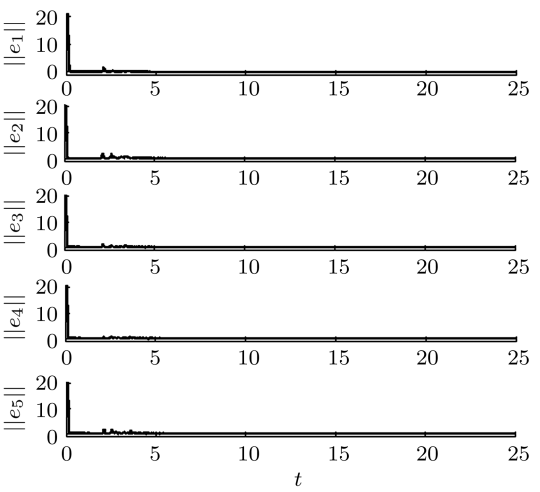


Fig. 4. Synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of the drive network Eq. (2) and the response network Eq. (17).

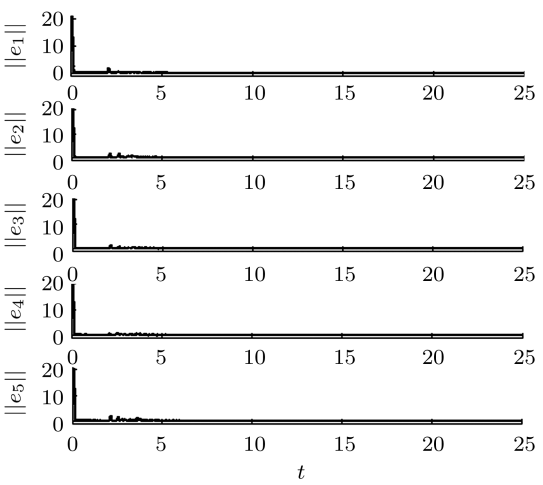


Fig. 5. Synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of the drive network Eq. (20) and the response network Eq. (21).

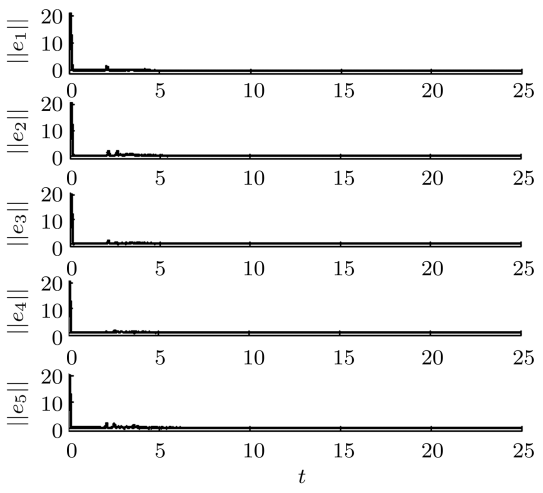


Fig. 6. Synchronisation errors $\|\boldsymbol{e}_i(t)\|\,(i = 1, 2, \cdots, 5)$ of the drive network Eq. (22) and the response network Eq. (23).

5. Conclusion

This paper investigates the problem of PS with non-delayed and delayed coupling in DRDNs consisting of identical and different nodes. Based on the Lyapunov stability theory, under the designed nonlinear controllers and updated laws, we obtain the analytical result that the PS can be achieved and the topology of the drive dynamical complex networks can be exactly identified. In particular, the coupling matrices are not assumed to be symmetric or irreducible. Moreover, the feasibility of the proposed scheme is proved in theory, and numerical simulations further demonstrate its effectiveness.

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