Fault diagnosis of time-delay complex dynamical networks using output signals*

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This paper proposes a novel approach for fault diagnosis of a time-delay complex dynamical network. Unlike the other methods, assuming that the dynamics of the network can be described by a linear stochastic model, or using the state variables of nodes in the network to design an adaptive observer, it only uses the output variable of the nodes to design an observer and an adaptive law of topology matrix in the observer of a complex network, leading to simple design of the observer and easy realisation of topology monitoring for the complex networks in real engineering. The proposed scheme can monitor any changes of the topology structure of a time-delay complex network. The effectiveness of this method is successfully demonstrated by virtue of a complex networks with Lorenz model.

Keywords: time-delay complex dynamical networks, fault diagnosis, observer, output variable

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1. Introduction

So far, complex dynamical networks exist in every corner of the real world, such as cellular networks,[1,2] electrical networks, communication networks, social networks, Internet and World Wide Web.[3,4] The complex dynamical networks and its control and synchronisation have drawn more and more attention. The most studies on this issue are based on the known topology of the complex dynamical networks, but in the real world, the topology of a complex dynamical network is often uncertain, so the monitoring of the topology and fault diagnosis of a network is a basic and important topic in the field of complex networks.

Recently, there has been much work in the topology monitoring for dynamical systems. In Ref. [1], it assumes that the dynamics of the network can be described by a linear stochastic model, but if a more complex network is considered, it may not be true. In Ref. [2], it proposes an adaptive observer approach using a state variable to identify and monitor the topology of complex dynamical network. It is effective for the special dynamics of node, but it is difficult to extend to a general case, where the node dynamics is a general nonlinear system. By using the state variables of node, a state observer has been constructed to synchronise a complex dynamical network in Ref. [5], the extension to weighted complex dynamical networks and time-delay complex dynamical networks can be found in Refs. [6] and [7]. In Ref. [8], synchronisation between two non-identical topological networks has been considered based on the observer design with the state variables of nodes. Due to the use of the state variables of network nodes, the state observer of the network is very complicated, and moreover, if some state variables of node are not measureable, it may not be practical in real engineering.

In this paper, we study the synchronisation of two time-delay networks coupling with output signals and monitor the change of network topology, where only a scalar coupling signal is required between the nodes of network, which is simpler to design and practical in real engineering. An observer using the output variable (scalar) will be constructed to obtain the synchronisation between the targeted time-delay network and our observer network. The output variable is the linear combination of the state variables. Based on the Lyapunov stability theorem, some conditions are derived. The effectiveness of the approach is demonstrated by using a time-delay dynamical network with each node being the Lorenz system.

The rest of this paper is organised as follows. The complex dynamical network model with time delay is proposed in Section 2. In Section 3, an observer using output variable is designed to monitor the topology of the targeted time-delay network. In Section 4, the targeted time-delay network, with each node being the

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Lorenz system, is simulated to illustrate the effectiveness of the proposed method.

2. Dynamical network model with time delay

Consider a dynamical network consisting of N identical nodes coupling by output variable, with each node being an n-dimensional dynamical system. The dynamical network model can be described by[9]

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i) + \sum_{j=1}^{N} c_{ij}\boldsymbol{L}y_j. \tag{1}$$

Unlike the common diagonally coupling networks,[5−8,10−12] where full state coupling is typically needed between two directly connected nodes, only a scalar coupling signal is required here. It is more simple to design and practical in real engineering.[9]

Considering that time-delay exists between two connected nodes of the network Eq. (1), we assume that the interconnection time-delay between any two connected nodes is same. Then, the so-called time-delay dynamical network coupling with output signals can be constructed as follows:

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}(t, \boldsymbol{x}_i(t)) + \sum_{j=1}^{N} c_{ij}\boldsymbol{L}y_j(t-\tau(t)), \tag{2}$$

where $\boldsymbol{x}_i(t) = [x_{i1}(t) \;\; x_{i2}(t) \;\; ... \;\; x_{in}(t)]^{\mathrm{T}} \in R^n$, $i = 1, 2, ..., N$ is the state variable of node i, $y_i(t) \in R$ is the output variable (scalar) of node i, $\boldsymbol{L} = [l_1 \;\; l_2 \;\; ... \;\; l_n]^{\mathrm{T}}$ is the gain matrix, $\boldsymbol{f} : R \times R^n \to R^n$ is the nonlinear field, $\tau(t)$ is the time delay between two connected nodes, and $\boldsymbol{C} = (c_{ij})_{N\times N}$ is the coupling configuration matrix representing the coupling strength and the topological structure of the network, in which c_{ij} is defined as follows. If there is a connection between node i and node $j (j \neq i)$, then $c_{ij} = c_{ji} > 0$, otherwise, $c_{ij} = c_{ji} = 0$, and the diagonal elements of matrix $\boldsymbol{C}$ are defined by

$$c_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij} < 0, \;\; i = 1, 2, ..., N. \tag{3}$$

Let

$$y_i(t) = \boldsymbol{H}\boldsymbol{x}_i(t), \tag{4}$$

where $\boldsymbol{H} = [h_1 \;\; h_2 ... h_n]$. Thus, $y_i(t)$ is a linear combination of the state variables of node i.

3. State observer design

In this section, we will design an observer to monitor the time-delay dynamical network using output variable. For the purpose of design, firstly two useful assumptions and one lemma are given.

Assumption 1 (A1) For system (2), we suppose that there exists a positive constant α satisfying

$$\|\boldsymbol{f}(t, \boldsymbol{y}(t)) - \boldsymbol{f}(t, \boldsymbol{x}(t))\| \leq \alpha \|\boldsymbol{y}(t) - \boldsymbol{x}(t)\|, \tag{5}$$

where $\boldsymbol{y}(t)$, $\boldsymbol{x}(t)$ are time-varying vectors, $\|\cdot\|$ represents 2-norm.

Assumption 2 (A2) For system (2), $\tau(t)$ is a differential function with[7]

$$0 \leq \dot{\tau}(t) \leq \varepsilon < 1. \tag{6}$$

In real engineering, the physical meaning of A2 is that the changing rate of time-delay is not large. This assumption can be easily satisfied in real engineering, because the time-delay often does not change quickly.

Lemma 1 (L1) For any vectors $\boldsymbol{x}, \boldsymbol{y} \in R^n$ and positive definite matrix $\boldsymbol{Q} \in R^{n\times n}$, the following inequality holds:[7]

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \leq \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{y}. \tag{7}$$

Assuming that in the network Eq. (2), the model of node and time delay $\tau(t)$ is known in advance. According to the observer design in control theory, using the output signal error between the system (2) and observer system, we construct an observer system as follows:

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}_i(t) &= \boldsymbol{f}(t, \hat{\boldsymbol{x}}_i(t)) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{L}\hat{y}_j(t-\tau(t)) \\ &\quad - \boldsymbol{k}_i(\hat{y}_i(t) - y_i(t)), \\ \dot{\hat{c}}_{ij} &= -e_{y_i}(t)\boldsymbol{H}\boldsymbol{L}\hat{y}_j(t-\tau(t)), \end{aligned} \tag{8}$$

where $\dot{\hat{\boldsymbol{x}}}_i(t) = [\hat{x}_{i1}(t) \;\; \hat{x}_{i2}(t) \;\; \ldots \;\; \hat{x}_{in}(t)]^{\mathrm{T}} \in R^n$ is the response state vector of i-th node, $i = 1, 2, ..., N, j = 1, 2, ..., N$, $\hat{c}_{ij}$ is the estimation of c_{ij}, $\hat{y}_i(t) = \boldsymbol{H}\hat{\boldsymbol{x}}_i(t), \boldsymbol{k}_i = [k_{i1} \;\; k_{i2} \; \ldots \; k_{in}]^{\mathrm{T}} \in R^n$.

Assume that $\boldsymbol{e}_i(t) = (\hat{\boldsymbol{x}}_i(t) - \boldsymbol{x}_i(t))$, $e_{y_i}(t) = \hat{y}_i(t) - y_i(t) = \boldsymbol{H}\boldsymbol{e}_i(t)$, $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$, the error system can be written as follows:

$$\begin{aligned} \dot{\boldsymbol{e}}_i(t) &= \boldsymbol{f}(t, \hat{\boldsymbol{x}}_i(t)) - \boldsymbol{f}(t, \boldsymbol{x}_i(t)) + \sum_{j=1}^{N} c_{ij}\boldsymbol{L}e_{y_j}(t-\tau(t)) \\ &\quad + \sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{L}\hat{y}_j(t-\tau(t)) - \boldsymbol{k}_i e_{y_i}(t). \end{aligned} \tag{9}$$

Theorem 1 Suppose that A1 and A2 hold. The topology matrix $\boldsymbol{C}$ of complex dynamical network Eq. (2) can be monitored by the estimation matrix $\hat{\boldsymbol{C}}$ using the state observer Eq. (8).

Proof We have $\|\boldsymbol{f}(t,\hat{\boldsymbol{x}}_i(t))-\boldsymbol{f}(t,\boldsymbol{x}_i(t))\|\leq\alpha\|\boldsymbol{e}_i(t)\|$, $i=1,2,...,N$. Consider a positive Lyapnov function V as

$$V=\frac{1}{2}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)e_{y_i}(t)+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2+\frac{1}{2(1-\varepsilon)}\int_{t-\tau(t)}^{t}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(\theta)e_{y_i}(\theta)\,\mathrm{d}\theta, \tag{10}$$

$$\begin{aligned}
\dot{V}&=\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)\dot{e}_{y_i}(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\tilde{c}}_{ij}+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)e_{y_i}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t-\tau(t))e_{y_i}(t-\tau(t))\\
&=\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}(\boldsymbol{f}(t,\hat{\boldsymbol{x}}_i(t))-\boldsymbol{f}(t,\boldsymbol{x}_i(t))+\sum_{i=1}^{N}\sum_{j=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}c_{ij}\boldsymbol{L}e_{y_j}(t-\tau(t))+\sum_{i=1}^{N}\sum_{j=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}\tilde{c}_{ij}\boldsymbol{L}\hat{y}_j(t-\tau(t))\\
&\quad-\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{k}_ie_{y_i}(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\tilde{c}}_{ij}+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)e_{y_i}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}e_{y_i}^{\mathrm{T}}(t-\tau(t))e_{y_i}(t-\tau(t))\\
&\leq\sum_{i=1}^{N}\alpha\|e_{y_i}(t)\|^2+\sum_{i=1}^{N}\sum_{j=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}c_{ij}\boldsymbol{L}e_{y_j}(t-\tau(t))-\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{k}_ie_{y_i}(t)\\
&\quad+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t)e_{y_i}(t)-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t-\tau(t))e_{y_i}(t-\tau(t))\\
&=\sum_{i=1}^{N}\left(\alpha-\boldsymbol{H}\boldsymbol{k}_i+\frac{1}{2(1-\varepsilon)}\right)\|e_{y_i}(t)\|^2+\sum_{i=1}^{N}\sum_{j=1}^{N}e_{y_i}^{\mathrm{T}}(t)\boldsymbol{H}c_{ij}\boldsymbol{L}e_{y_j}(t-\tau(t))\\
&\quad-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}e_{y_i}^{\mathrm{T}}(t-\tau(t))e_{y_i}(t-\tau(t))\\
&\leq\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}_y(t)+\frac{1}{2}\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{e}_y(t)+\frac{1}{2}\boldsymbol{e}_y^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_y(t-\tau(t))-\frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}\boldsymbol{e}_{y_i}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}_{y_i}(t-\tau(t))\\
&\leq\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{A}\boldsymbol{e}_y(t)+\frac{1}{2}\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{e}_y(t)\\
&\leq(\lambda_{\max}(\boldsymbol{A})+\lambda_{\max}\left(\frac{1}{2}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\right))\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{e}_y(t)\\
&=\left(\alpha-\min(\boldsymbol{H}\boldsymbol{k}_i)+\frac{1}{2(1-\varepsilon)}+\lambda_{\max}\left(\frac{1}{2}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\right)\right)\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{e}_y(t),
\end{aligned}$$

where

$$\boldsymbol{A}=\begin{bmatrix}
\alpha+\dfrac{1}{2(1-\varepsilon)}-\boldsymbol{H}\boldsymbol{k}_1 & & & & \\
& \alpha+\dfrac{1}{2(1-\varepsilon)}-\boldsymbol{H}\boldsymbol{k}_2 & & & \\
& & \cdot & & \\
& & & \cdot & \\
& & & & \cdot \\
& & & & \alpha+\dfrac{1}{2(1-\varepsilon)}-\boldsymbol{H}\boldsymbol{k}_N
\end{bmatrix},$$

$$\boldsymbol{B}=\boldsymbol{C}\otimes\boldsymbol{H}\boldsymbol{L}=\boldsymbol{H}\boldsymbol{L}\boldsymbol{C},$$

$$\boldsymbol{e}_y(t)=[e_{y_1}(t)\ \ e_{y_2}(t)\ \ \ldots\ \ e_{y_N}(t)]^{\mathrm{T}}.$$

The $\lambda_{\max}(\boldsymbol{P})$ denotes the maximal eigenvalue for symmetric matrix $\boldsymbol{P}$, and $\min(\boldsymbol{Hk}_i)$ denotes the minimum value of all $\boldsymbol{Hk}_i$, $i = 1, 2, ..., N$. One can make $\dot{V} \leq 0$ if he can obtain the suitable $\boldsymbol{k}_i$ making $\min(\boldsymbol{Hk}_i)$ positive large enough. Therefore, based on the Lyapnov stability theorem, one obtain $\lim_{t\to\infty} e_{y_i}(t) = 0$ and $\lim_{t\to\infty} \dot{\tilde{c}}_{ij} = 0$, so $\tilde{c}_{ij}$ converges to a constant, where $i = 1, 2, ..., N$, $j = 1, 2, ..., N$. As a result, the topology matrix $\boldsymbol{C}$ can be monitored by the matrix $\hat{\boldsymbol{C}}$. □

4. Simulation results

In this section, a simple example is provided to verify the effectiveness of the fault diagnosis of time-delay network Eq. (2) using the observer system Eq. (8).

We use the Lorenz system as the model of each node. The Lorenz system can be expressed as follows:

$$\begin{aligned} \dot{x}_{i1} &= a\,(x_{i2} - x_{i1})\,, \\ \dot{x}_{i2} &= c x_{i1} - x_{i2} - x_{i1} x_{i3}, \\ \dot{x}_{i3} &= x_{i1} x_{i2} - b x_{i3}. \end{aligned} \tag{11}$$

when $a = 16, b = 4, c = 45$. The chaotic characteristics can be shown in Fig. 1.

In general, a complex network can be represented by the evolution of all its nodes. In the simulation, an unweighted dynamical connection delay network of four nodes is constructed, as shown in Fig. 2, where $c_{ij} = c_{ji} = 1$ if there is a connection between node i and node j, otherwise, $c_{ij} = c_{ji} = 0$. It is assumed that each node's output variable is measurable, the model of node and the time delay $\tau(t)$ are known.

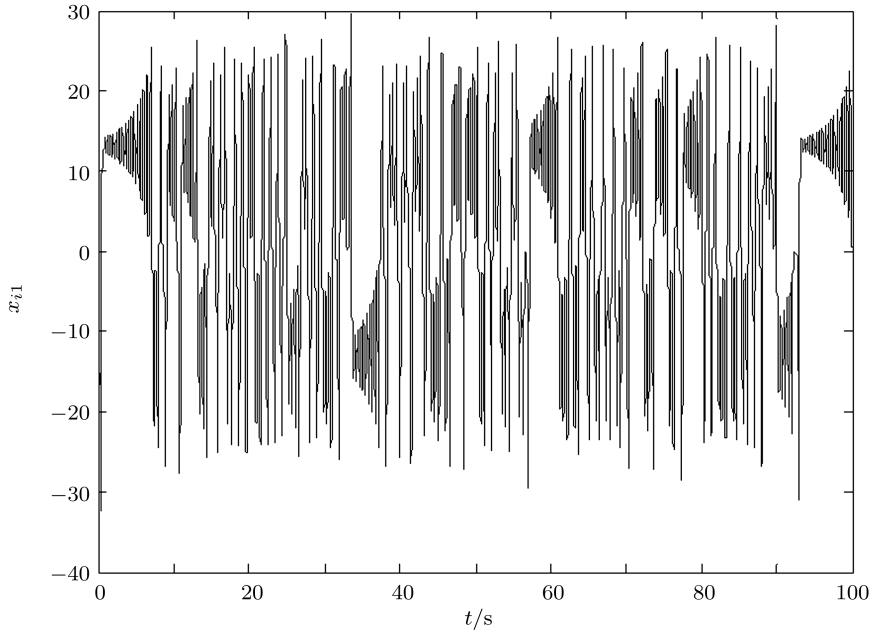


Fig. 1. Lorenz system.

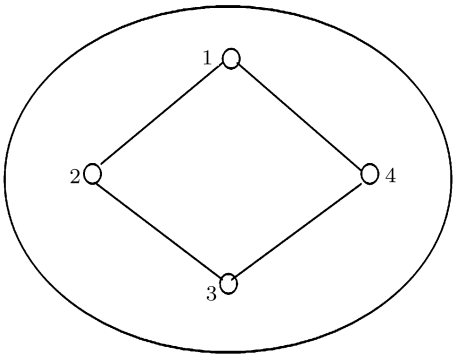


Fig. 2. Topology of targeted network.

Then, using the complex dynamical network model Eq. (2), a dynamical connection delay network with four identical nodes can be constructed by

$$\begin{aligned} \dot{x}_{i1}(t) &= a\,(x_{i2}(t) - x_{i1}(t)) + l_1 \sum_{j=1}^{4} c_{ij} y_j(t - \tau(t)), \\ \dot{x}_{i2}(t) &= c x_{i1}(t) - x_{i2}(t) - x_{i1}(t) x_{i3}(t) \\ &\quad + l_2 \sum_{j=1}^{4} c_{ij} y_j(t - \tau(t)), \\ \dot{x}_{i3}(t) &= x_{i1}(t) x_{i2}(t) - b x_{i3}(t) \end{aligned}$$

$$+ l_3 \sum_{j=1}^{4} c_{ij} y_j(t - \tau(t)), \tag{12}$$

where $y_i = \boldsymbol{H}\boldsymbol{x}_i = [h_1 h_2 h_3][x_{i1}(t)x_{i2}(t)x_{i3}(t)]^{\mathrm{T}}, i = 1, 2, 3, 4.$

The topology matrix $\boldsymbol{C}$ can be written as

$$\boldsymbol{C} = (c_{ij})_{4\times 4} = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & -2 \end{bmatrix}.$$

Since A1 and A2 hold, according to Theorem 1, the topology matrix $\boldsymbol{C}$ can be monitored by the estimation matrix $\hat{\boldsymbol{C}}$ using the observer Eq. (8) as

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}_i(t) &= \boldsymbol{f}(t, \hat{x}_i(t)) + \sum_{j=1}^{4} \hat{c}_{ij} \boldsymbol{L} \hat{y}_j(t - \tau(t)) \\ &\quad - \boldsymbol{k}_i(\hat{y}_i(t) - y_i(t)), \\ \dot{\hat{c}}_{ij} &= -e_{y_i}(t) \boldsymbol{H}\boldsymbol{L}\hat{y}_j(t - \tau(t)), \end{aligned} \tag{13}$$

where $\boldsymbol{f}(t, \hat{\boldsymbol{x}}_i(t)) = [a(\hat{x}_{i2}(t) - \hat{x}_{i1}(t))c\hat{x}_{i1}(t) - \hat{x}_{i2}(t) - \hat{x}_{i1}(t)\hat{x}_{i3}(t)\hat{x}_{i1}(t)\hat{x}_{i2}(t) - b\hat{x}_{i3}(t)]^{\mathrm{T}}$, $\hat{\boldsymbol{x}}_i = [\hat{x}_{i1}(t)\hat{x}_{i2}(t)\hat{x}_{i3}(t)]^{\mathrm{T}}, \boldsymbol{L} = [l_1 l_2 l_3]^{\mathrm{T}}, \boldsymbol{k}_i = [k_{i1}k_{i2}k_{i3}]^{\mathrm{T}}$, $\hat{y}_i(t) = \boldsymbol{H}\hat{\boldsymbol{x}}_i(t) = [h_1 h_2 h_3][\hat{x}_{i1}(t)\hat{x}_{i2}(t)\hat{x}_{i3}(t)]^{\mathrm{T}}, i = 1, 2, 3, 4.$

In the simulation, for simplicity, we set $\tau(t) = 1$. We assume that the connection between nodes 1 and 2 is broken at $t = 50$ s. Some elements of $\hat{\boldsymbol{C}}$ are shown in Fig. 3, the estimation of C_{12} and the differential coefficient of error $\dot{\tilde{C}}_{12}$ are shown in Fig. 4, and the synchronous errors $e_{y_i}(t)$ are shown in Fig. 5.

From Fig. 4, we can see that the estimation of C_{12} changes when the connection between nodes 1 and 2 changes, and the error $\tilde{C}_{12}$ converges to a constant. Assuming that the connection between nodes 1 and 2 is broken, we can duly capture the event in the observer. So, when the estimation of C_{12} decreases, we can know that the value of C_{12} changes from 1 to 0, since the value of C_{12} can only be 1 or 0. So, using the observer Eq. (13), we can monitor the changes of connection between nodes and identify the connection if the changes exactly happen in the time-delay network.

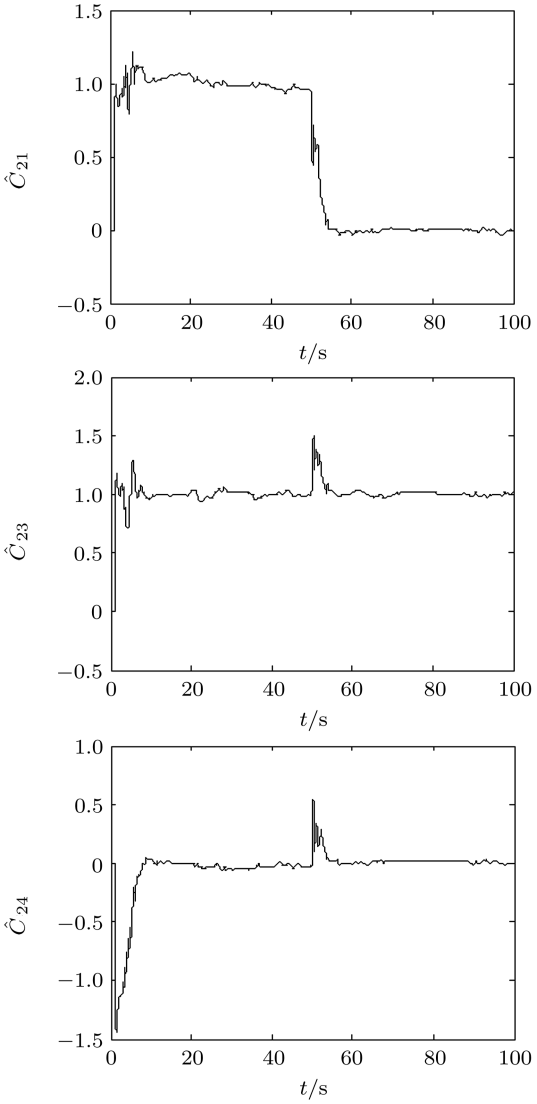
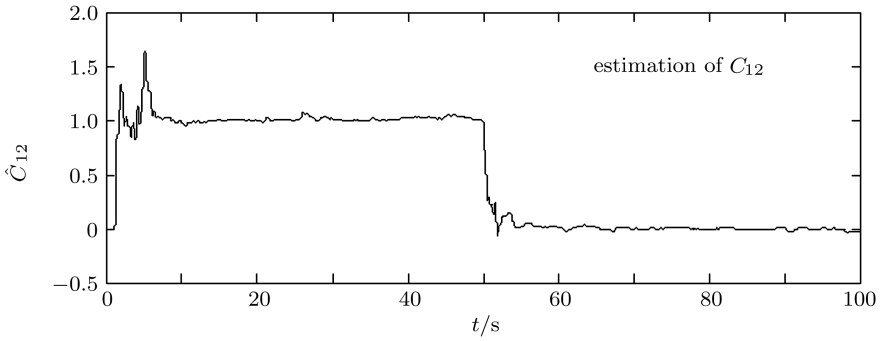


Fig. 3. The connections of node 2.



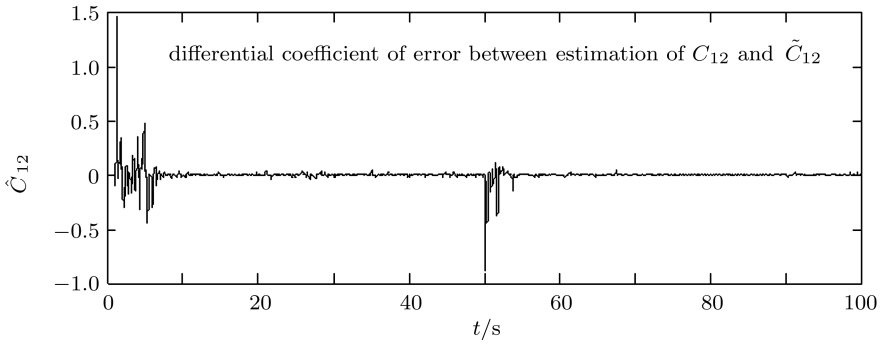


Fig. 4. Estimation of C_{12} and the differential coefficient of error $\tilde{C}_{12}$.

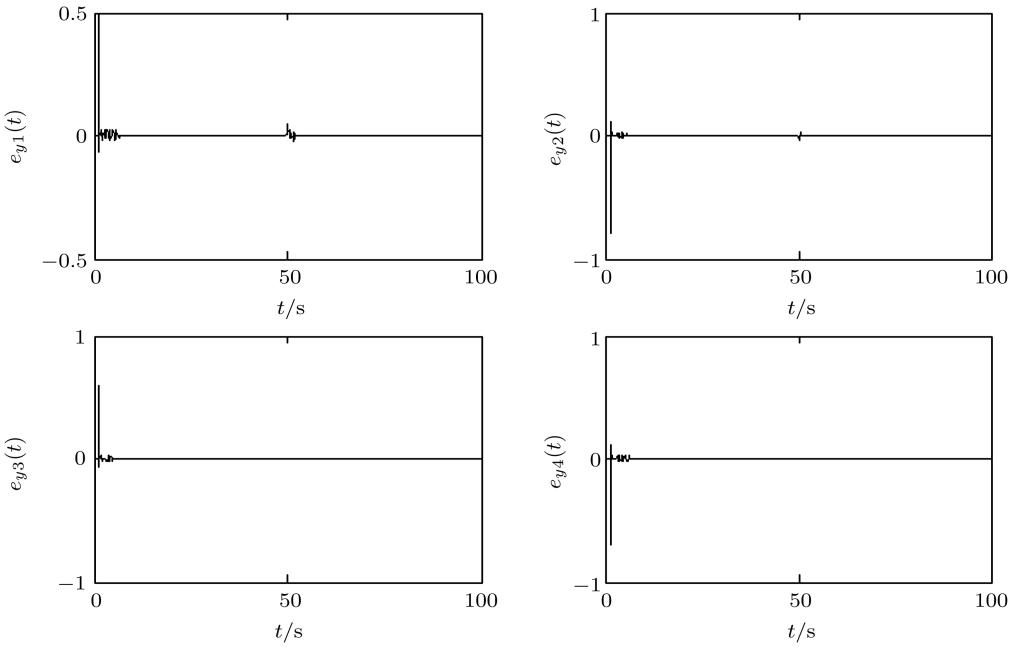


Fig. 5. Synchronous errors ($e_{y_i}(t) = \hat{y}_i(t) - y_i(t)$).

5. Conclusions

The synchronisation between a time-delay network and the observer has been addressed by using output variable to monitor the topology of time-delay complex dynamical networks. A time-delay network with each node being a Lorenz system has been constructed to illustrate the effectiveness of the observer. The simulation results show that the proposed observer can duly monitor the changes in the time-delay complex network.

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