Dynamical analysis and circuit simulation of a new three-dimensional chaotic system

Wang Ai-Yuan(王爱元)[a)b)†] and Ling Zhi-Hao(凌志浩)[a)]

[a)] *Department of Automation, East China University of Science and Technology, Shanghai* 200237, *China*
[b)] *School of Electric Engineering, Shanghai Dianji University, Shanghai* 200240, *China*

(Received 26 July 2009; revised manuscript received 16 January 2010)

This paper reports a new three-dimensional autonomous chaotic system. It contains six control parameters and three nonlinear terms. Two cross-product terms are respectively in two equations. And one square term is in the third equation. Basic dynamic properties of the new system are investigated by means of theoretical analysis, numerical simulation, sensitivity to initial, power spectrum, Lyapunov exponent, and Poincaré diagrams. The dynamic properties affected by variable parameters are also analysed. Finally, the chaotic system is simulated by circuit. The results verify the existence and implementation of the system.

Keywords: chaotic system, dynamical properties, circuit simulation, nonlinear analysis

PACC: 0545

1. Introduction

Chaos has the complex nonlinear dynamical properties and widely exists in nature. Since Lorenz discovered the first three-dimensional (3D) chaotic system in 1963,[1] the research of chaos has allured much interest. And it centralises in two aspects. One is to discover or purposely generate new chaotic system, and describes its dynamic properties and method of realisation. Another is to apply chaos in biological medicine, telecommunication, information processing, etc. This research adheres to the first aspect.

After Lorenz, Rössler constructed an even simpler 3D chaotic system in 1976.[2] The system contains one cross-product term and one constant term. In the last decade, there has been increasing interest in creating chaotic system aroused by Chen and Ueta in 1999.[3] They found a new chaotic system called Chen system by feedback control, which is not topologically equivalent to Lorenz system. In 2002, Lü and Chen further constructed a new chaotic system called Lü system which unifies Lorenz system and Chen system.[4,5] Liu discovered another chaotic system by modifying Lorenz system in 2004.[6] Recently, some studies have carried out which gradually enriches the Lorenz system family.[7−16]

In this paper, we construct another chaotic system which has unique algebraic structure and is not equivalent to any of the reported chaotic systems in Refs. [1]–[16]. The system contains six control parameters and three nonlinear terms. Two cross-product terms are respectively in two equations. And one square term is in the third equation. It has the basic properties of chaos. The new chaotic system has also been verified by circuit simulation.

2. Evolution of the new chaotic system

In Ref. [7], a chaotic system by adding one cross-product term in first equation of Lorenz system is described as

$$\begin{aligned} \dot{x} &= a(y - x + yz), \\ \dot{y} &= cx - y + xz, \\ \dot{z} &= -bz + xy. \end{aligned} \tag{1}$$

Another chaotic system reported in Ref. [8] is

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= cx - kxz, \\ \dot{z} &= -bz + hx^2. \end{aligned} \tag{2}$$

Inspired by Eqs. (1) and (2), we construct the new system as bellow

$$\begin{aligned} \dot{x} &= a(y - x) + lyz, \\ \dot{y} &= cx - mxz, \\ \dot{z} &= -bz + nx^2. \end{aligned} \tag{3}$$

†Corresponding author. E-mail: wang_aiyuan@sohu.com

© 2010 Chinese Physical Society and IOP Publishing Ltd

where a, b, c, l, m and n are positive real constant. When $a = 10$, $b = 8$, $c = 20$, $l = 25$, $m = 7$, $n = 3$, the new system exists a chaotic attractor as shown in Fig. 1.

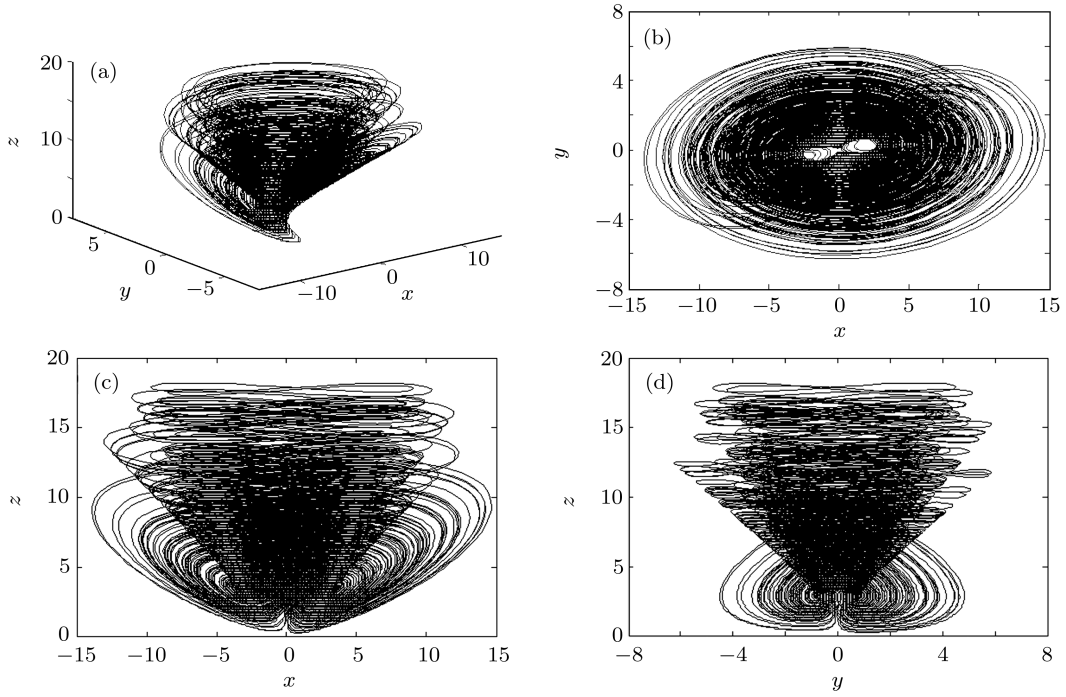


Fig. 1. Chaotic attractor of the new system (3). (a) phase portrait in x–y–z; (b) phase portrait in x–y; (c) phase portrait in x–z; (d) phase portrait in y–z.

The new system (3) has different attractor and algebraic structure to Eqs. (1) and (2). So it is not equivalent to them. Also the new system has more control parameters which show more complex chaotic behaviour.

3. The basic dynamical analysis

3.1. Equilibrium and stability

Let

$$\begin{aligned} &a(y-x)+lyz=0,\\ &cx-mxz=0,\\ &-bz+nx^2=0, \end{aligned} \tag{4}$$

we get the following three equilibrium points:

$$\begin{aligned} &s_0=(0,0,0),\\ &s_1=(x_1,y_1,z_1),\\ &s_2=(x_2,y_2,z_2), \end{aligned}$$

where $x_{1,2}=\pm\sqrt{bc/mn}$, $y_{1,2}=\pm a\sqrt{bcm/n}/(am+lc)$, and $z_1=z_2=c/m$.

For equilibrium point s_0, we linearise Eq. (3) and obtain the Jacobian matrix

$$\boldsymbol{J}_0=\begin{bmatrix} -a & a & 0\\ c & 0 & 0\\ 0 & 0 & -b \end{bmatrix}.$$

Let $\det(\boldsymbol{J}_0-\lambda\boldsymbol{I})=0$, we get the following eigenvalues:

$$\begin{aligned} &\lambda_{01}=-b,\quad \lambda_{02}=\frac{-a+\sqrt{a^2+4ac}}{2},\\ &\lambda_{03}=\frac{-a-\sqrt{a^2+4ac}}{2}. \end{aligned}$$

Here as the parameters in Eq. (3) are positive real, λ_{01} and λ_{03} are negative real, and λ_{02} are positive real. So, the equilibrium point s_0 is an unstable saddle.

By the same reasoning, the Jacobian matrix and corresponding characteristic equation for equilibrium point s_1 are

$$\boldsymbol{J}_1=\begin{bmatrix} -a & a+lz_1 & lx_1\\ c & -mz_1 & -mx_1\\ 2nx_1 & 0 & -b \end{bmatrix},$$

$$f(\lambda)=\lambda^3+A\lambda^2+B\lambda+C. \tag{5}$$

where

$$A = \frac{a^2m^2 + amlc + mblc + m^2ba}{m^2a^2 + mlc},$$
$$B = \frac{-mablc + m^2a^2b}{m^2a^2 + mlc},$$
$$C = \frac{2a^2bcm^2 + 4abc^2ml + 2bc^3l^2}{m^2a^2 + mlc}.$$

According to the Routh–Hurwitz condition, if and only if $A > 0$, $B > 0$, $C > 0$, and $AB - C > 0$, all the eigenvalues have positive real parts which insure that the equilibrium point is stable. Whereas the parameters with $a = 10$, $b = 8$, $c = 20$, $l = 25$, $m = 7$, $n = 3$ cannot meet the stability condition of the Routh–Hurwitz. Thus, the equilibrium point is unstable. Furthermore, there are two eigenvalues of conjugate complex with the same positive real part. So, the equilibrium point s_1 is unstable saddle-foci.

By the symmetry of Eq. (3), the stability of equilibrium point s_2 is as same as that of s_1.

3.2. Time domain waveform, sensitivity to initial value, and power spectrum

Using Matlab program, we have completed the numerical simulation. Figure 2 shows the time domain waveform of x with initial value of $x_0 = 0$, $y_0 = -0.8$, and $z_0 = 3$. It indicates that the new system is bounded, ergodic, and non-periodic, all these are the properties of chaos. Figure 3 is the time domain waveforms of x corresponding to different two initial values. The two initial values only differ with 0.001 of x. They are $(0, -0.8, 3)$ and $(0.001, -0.8, 3)$. The two waveforms in Fig. 3 are adjacent to each other at the initial time and obviously separate with each other after $t > 1.5$. It exhibits initial sensitivity of chaos for the new system (3).

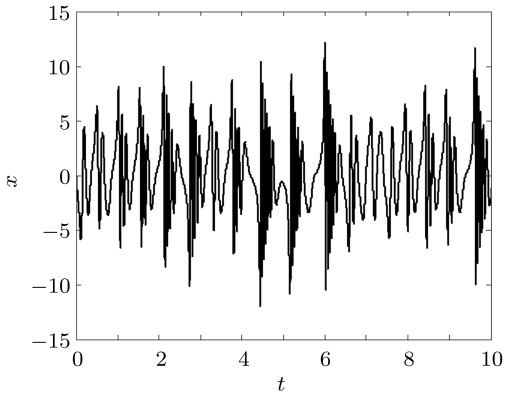


Fig. 2. Waveform of x for new system (3).

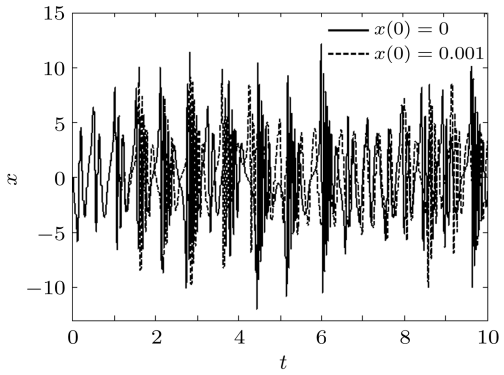


Fig. 3. Initial sensitivity of x for new system (3).

Figure 4 is the power spectrum of x. It shows that the peaks of spectrum joint together. There is no obvious peak. The spectrum sequence is continuous and wide, which is distinct from that of periodic signal and quasi-periodic signal. All these properties are the spectrum properties of chaotic signal.

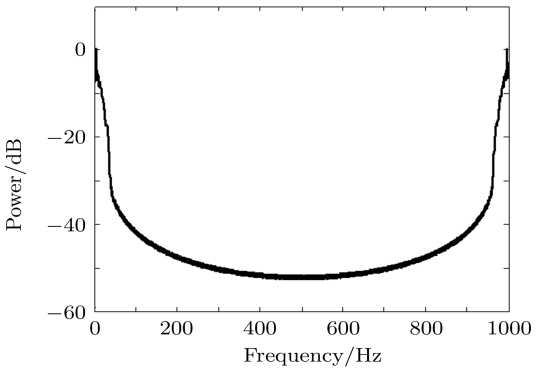


Fig. 4. Power spectrum of x for new system (3).

3.3. Lyapunov exponent and Poincaré diagrams

As it is well known, the Lyapunov exponents measure the exponential rates of divergence or convergence of nearby trajectories. Based on Wolf method, we get the Lyapunov exponents of the new system (3) when $a = 10$, $b = 8$, $c = 20$, $l = 25$, $m = 7$, and $n = 3$

$$\lambda_{L1} = 4.621714,$$
$$\lambda_{L2} = 0.016387,$$
$$\lambda_{L3} = -22.638011.$$

There are two positive Lyapunov exponents. It indicates that the system has the hyper-chaotic property.

As all of the three equilibrium points are unstable, there are no trajectories going through the equilibrium points or tangential to them. Thus, we select the plane included the three equilibrium points as the Poincaré section. By Eq. (3), the plane is

$$x - 8.14y = 0. \quad (6)$$

Poincaré mappings crossed by the selected plane are these points of the confusion as shown in Fig. 5.

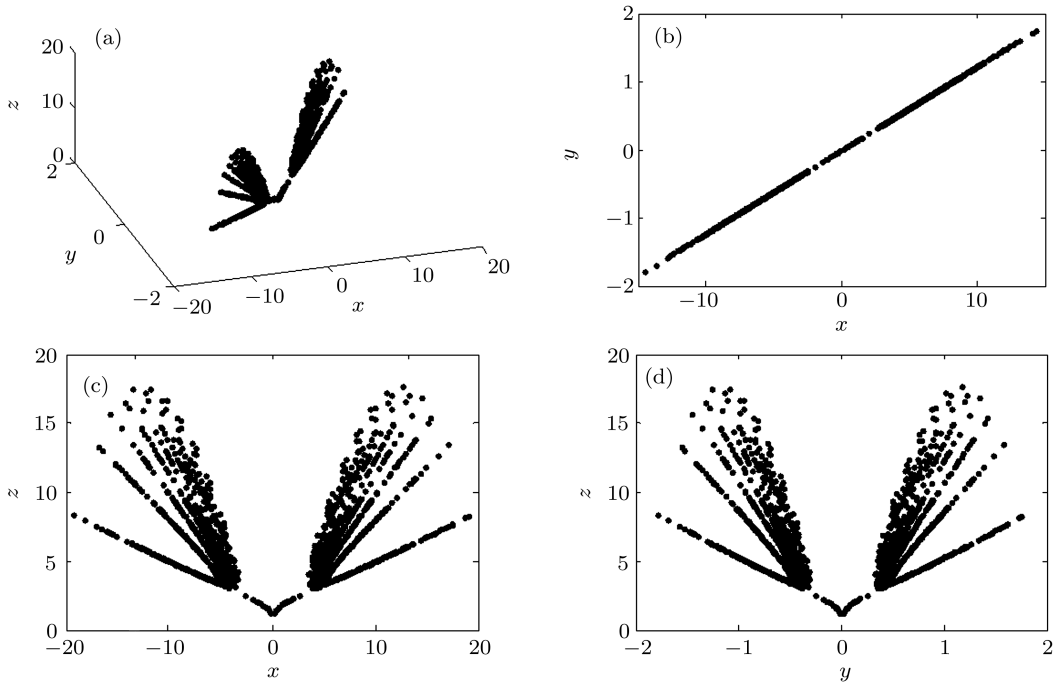


Fig. 5. Poincaré map of the new system (3). (a) cross section in x–y–z space; (b) projection on x–y plane; (c) projection on x–z plane; (d) projection on y–z plane.

3.4. Dynamical properties affected by parameters variation

The new system (3) has six parameters. Each parameter can affect the dynamical properties of the system. Here, as an example we let c vary and fix other parameters. And we do not discuss the influence of the other parameters variation for the paper length limitation.

When $c \in [0.2, 30]$ and $a = 10$, $b = 8$, $l = 25$, $m = 7$, $n = 3$, the Lyapunov-exponents spectrum shows in Fig. 6.

According to the figure, we give the following analysis.

For $c \in [0.2, 1.6)$, all the Lyapunov exponents are negative. So, the system (3) is convergent. The convergent point is (–0.6172, –0.4548, 0.1429) or (0.6172, 0.4548, 0.1429) which depends on the initial point.

While for $c \in [1.6, 3.6]$, the maximum Lyapunov exponent equals zero, implying that the system (3) has a periodic orbit. Figure 7(a) shows the orbit when $c = 2.5$.

And for $c \in (3.6, 4.4]$, the maximum Lyapunov exponent is positive. And the system (3) is in chaos. Figure 7(b) shows the chaotic attractor when $c = 4$.

For $c \in (4.4, 4.8]$, the maximum Lyapunov exponent equals zero. And the system (3) has a periodic orbit. Figure 7(c) shows the orbit when $c = 4.6$.

For $c \in (4.8, 24.5]$, the maximum Lyapunov exponent is positive. The middle Lyapunov exponent is around zero. Thus, the system (3) is in chaos or hyper-chaos. Figure 1 shows the chaotic attractor when $c = 20$.

For $c \in (24.5, 30]$, the maximum Lyapunov exponent equals zero. Thus, the system (3) has a periodic orbit. Figure 7(d) shows the orbit when $c = 28$.

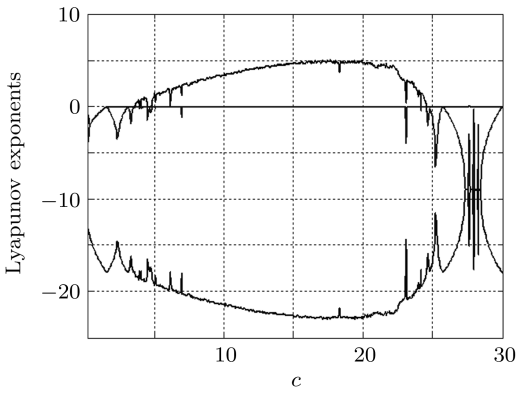


Fig. 6. Lyapunov-exponents spectrum with c variation.

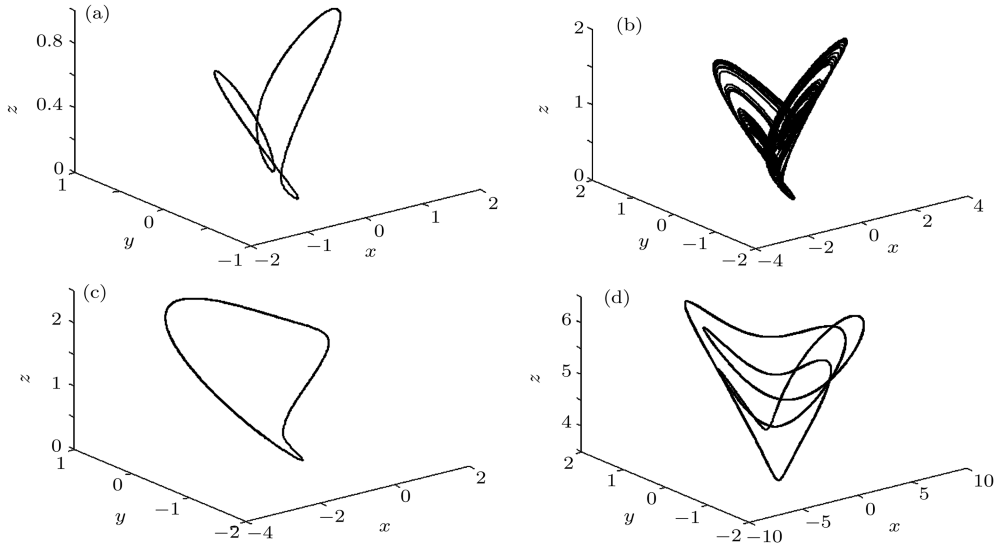


Fig. 7. The system (3) evolving with the variation of c. (a) $c = 2.5$; (b) $c = 4$; (c) $c = 4.6$; (d) $c = 28$.

4. Circuit design and simulation for the new chaotic system (3)

A circuit is designed to implement the new chaotic system (3) based on Multisim10. Figure 8 shows the circuit diagram. In the figure, operational amplifiers and the associated circuity jointly perform the operations of addition, subtraction and integration. Analogous multipliers are employed to perform multiplication or square operation.

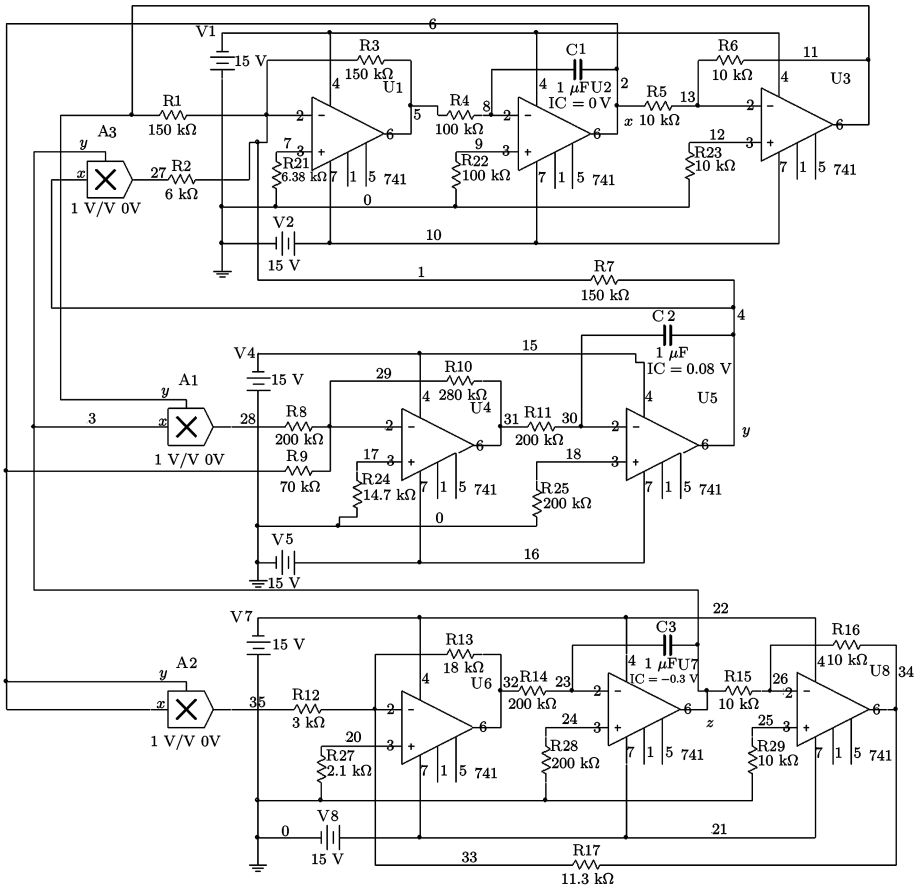


Fig. 8. Circuit simulation diagram.

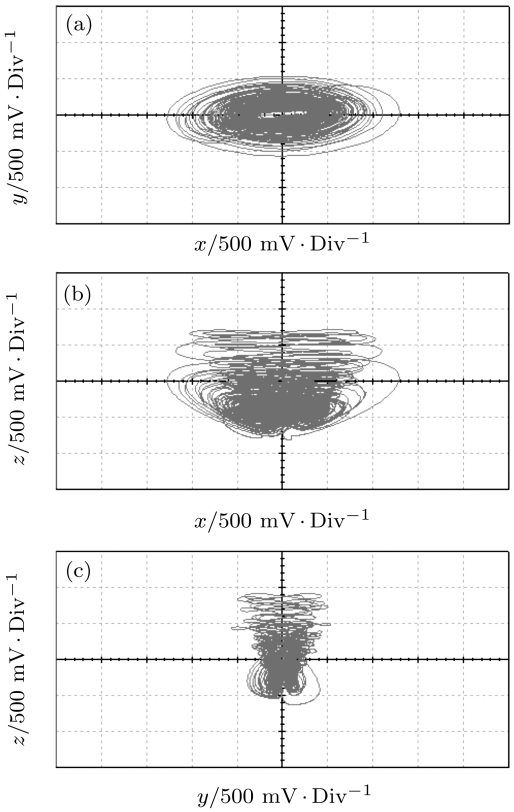


Fig. 9. Circuit simulation results. (a) phase portrait in $x-y$; (b) phase portrait in x–z; (c) phase portrait in y–z.

Considered the saturation of operational amplifier and multiplier, the variable amplitude in Eq. (3) is decreased to one tenth by variable change. In this way, the system (3) can be changed as follows:

$$\dot{x} = a(y - x) + 10lyz,$$
$$\dot{y} = cx - 10mxz,$$
$$\dot{z} = -bz + 10nx^2. \tag{7}$$

According to Eq. (7) and $a = 10$, $b = 8$, $c = 20$, $l = 25$, $m = 7$, $n = 3$, the values of resistances and capacitors are selected as shown in Fig. 8. The initial voltages of the capacitor c1, c2, and c3 respectively represent the initial value of $x_0 = 0$, $y_0 = -0.08$, and $z_0 = 0.3$ to the system (7). The three variables of x, y and z are obtained respectively from the output voltage of operational amplifier marked with x, y and z in the figure.

Figure 9 shows simulated phase portraits. They meet respectively the phase portraits in Figs. 1(b), 1(c) and 1(d) which are obtained by numerical simulation. Thus, the new chaotic system (3) have been verified and realised.

5. Conclusions

In this paper, we have introduced a new chaotic system. It has six control parameters and each equation has one controlled nonlinear term. Some basic dynamical behaviours are further explored by investigating its stability, sensitivity to initial, power spectrum, Lyapunov exponent, and Poincaré diagrams. Furthermore, a circuit simulation has been implemented. The circuit simulation results show agreement with numerical simulation.

Further work on the new chaotic system will be going on its control and application.

References

[1] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[2] Rössler O E 1976 *Phys. Lett.* A **57** 397
[3] Chen G R and Ueta T 1999 *Int. J. Bifurc. Chaos* **9** 1465
[4] Lü J H and Chen G R 2002 *Int. J. Bifurc. Chaos* **12** 659
[5] Lü J H, Chen G R and Celikovshý S 2002 *Int. J. Bifurc. Chaos* **12** 2917
[6] Liu C X, Liu L and Liu K 2004 *Chaos, Solitons and Fractals* **22** 1031
[7] Wang G Y, Qiu S S and Xu Z Y2006 *Acta Phys. Sin.* **55** 3295 (in Chinese)
[8] Wang J Z, Chen Z Q and Yuan Z Z 2006 *Acta Phys. Sin.* **55** 3956 (in Chinese)
[9] Liu L, Su Y C and Liu C X 2007 *Acta Phys. Sin.* **56** 1966 (in Chinese)
[10] Cai G L, Tan Z H, Zhou W H and Tu W T 2007 *Acta Phys. Sin.* **56** 6230 (in Chinese)
[11] Wang F Z, Chen Z Q, Wu W J and Yuan Z Z 2007 *Chin. Phys.* **16** 3238
[12] Liu Y Z, Jiang C S, Lin C S and Sun H 2007 *Acta Phys. Sin.* **56** 5131 (in Chinese)
[13] Zhang J X, Tang W S and Xu Y 2008 *Acta Phys. Sin.* **57** 6799 (in Chinese)
[14] Tang L R, Li J, Fan B and Zhai M Y 2009 *Acta Phys. Sin.* **58** 785 (in Chinese)
[15] Liu C X 2009 *Chaos, Solitons and Fractals* **39** 1037
[16] Li C B, Chen S and Zhu H Q 2009 *Acta Phys. Sin.* **58** 2255 (in Chinese)