Control of entanglement between two atoms by the Stark shift*

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Considering a quantum model consisting of two effective two-level atoms and a single-mode cavity, this paper investigates the entanglement dynamics between the two atoms, and studies the effect of the Stark shift on the entanglement. The results show that, on the one hand the atom–atom entanglement evolves periodically with time and the periods are affected by the Stark shift; on the other hand, the two atoms are not disentangled at any time when the Stark shift is considered, and for large values of the Stark shift parameter, the two atoms can remain in a stationary entangled state. In addition, for the initially partially entangled atomic state, the atom–atom entanglement can be greatly enhanced due to the presence of Stark shift. These properties show that the Stark shift can be used to control entanglement between two atoms.

Keywords: entanglement, Stark shift, control

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1. Introduction

Quantum information processing (QIP) provides more different ways for manipulation of information than the classical one, which is related to quantum entanglement (QE). The QE is the most intriguing feature of quantum mechanics and plays a key role in QIP.[1−8] As a result, many theoretical efforts have been made at study of entanglement. One of them is control of entanglement[9−13] because it has been realized that the controlled manipulation of such resources has practical importance in realistic QIP. Malinovsy and Sola[10] have shown that one can control entanglement at will by changing a relative phase of the pulses that drive the transitions in a two-qubit system with closed-loop couplings. Reference [12] has proposed a strategy to fight against the deterioration of the entanglement using the quantum Zeno effect. For generating multipartite entanglement, Duan and Kimble[14] have proposed an efficient scheme to engineer multiatom entanglement by detecting cavity decay through single-photon detectors. Recently, the problem of controlling entanglement by dynamic Stark effect has attracted much attention.[15−17] Ghosh *et al.*[15] have found that the Stark shift can be used to enhance the magnitude of atomic entanglement. In Ref. [16], Abdel-Aty and Moya-Cessa have shown that the Stark shift can produce sudden death of entanglement and long-lived entanglement between the qubits which are prepared initially in separable states or mixed state.

It is worth mentioning that, in the real world, QIP will be inevitably affected by decoherence which degrades the entanglement. In recent years, the description of decoherence for bipartite entangled systems has obtained theoretical[18−20] and experimental[21] results due to the introduction of entanglement sudden death. Reference [22] has investigated the entanglement existed in a quantum model consisting of two two-level atoms and a single mode cavity. There is no interaction between the two atoms. They found that sudden death effect arises when the initial state is a combination of typical Bell state. In this paper, we consider a quantum model similar to that studied in Ref. [22], which is a system consisting of two effective two-level atoms and a single-mode cavity. Each atom interacts alone with the cavity field through a two-photon process, but there is no physical interaction between the two atoms. To make the model closer to

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the experimental realization, we include the effect of the dynamic Stark shift. When the two atomic levels are coupled with comparable strength to the intermediate relay level, the Stark shift becomes significant and cannot be ignored.[23−28] We investigate the entanglement dynamics between the two two-level atoms which are initially entangled for weak few-photon field, and study the possibility of controlling and protecting the entanglement by the dynamic Stark shift.

2. The model

We consider a system consisting of two effective two-level atoms labeled A and B, and a single-mode cavity field. The two atoms are in a perfect near-resonant and lossless cavity, but there is no interaction between the two atoms (see Fig. 1). The excited and ground states of each atom will be designated by $|e\rangle$ and $|g\rangle$, respectively. The transition frequency of atom is ω_0 and the field has frequency ω. For simplicity of analysis, we assume that the two atoms are resonantly coupled with the single-mode cavity field, i.e., $\omega_0 = 2\omega$. The Hamiltonian describing such a model in the rotating wave approximation and in the interaction picture can be written as (assuming $\hbar = 1$)[24−26]

$$H_{\rm I} = \sum_{l=\rm A}^{\rm B}[a^{+}a(\beta_2 S_l^{+}S_l^{-} + \beta_1 S_l^{-}S_l^{+}) + g(a^{+2}S_l^{-} + a^{2}S_l^{+})], \quad (1)$$

where $a^{+}(a)$ is the photon creation (annihilation) operator of the cavity mode, $S^{+}_{l=\rm A,B} = |e\rangle_{ll}\langle g|$ and $S^{-}_{l=\rm A,B} = |g\rangle_{ll}\langle e|$ are the atomic flopping operators of the lth two-level atom. The β_2 and β_1 are the parameters describing the intensity-dependent Stark shifts of the two levels of each atom due to the virtual transitions to the intermediate relay level, and g is the effective two-photon coupling constant between atoms and cavity.

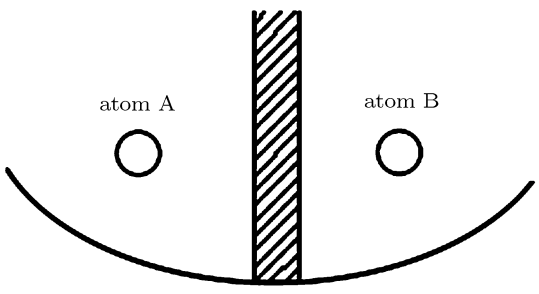


Fig. 1. The system configuration diagram used in this paper.

Several different measures have been proposed to identify entanglement between two qubits, and we choose the Wootters entanglement measure,[29,30] the concurrence C, defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (2)$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^{*}(\sigma_y \otimes \sigma_y)$, in which ρ is the density matrix representing the quantum state, and the matrix elements are taken with respect to the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$. The range of the concurrence is from 0 to 1. For unentangled qubits $C = 0$, whereas $C = 1$ for the maximally entangled qubits. In what follows, we investigate entanglement dynamics between the two atoms for two kinds of initial states. In Section 3, we assume that the two atoms are initially in an entangled state and the cavity field in the vacuum state. It is investigated in Section 4 that how the atom–atom entanglement is affected by the dynamic Stark shift if initially there is one photon in the cavity.

3. The atom–atom entanglement for the case where no photons are initially presented in the cavity

We assume that the two atoms are initially in a partially entangled atomic pure state, which is a superposition of Bell states usually denoted by $|\Psi^{\pm}\rangle \sim |e,g\rangle \pm |g,e\rangle$, and the cavity field initially in vacuum state $|0\rangle$. Then the initial state for the system is

$$|\Psi(0)\rangle = (\cos\theta|eg\rangle + \sin\theta|ge\rangle) \otimes |0\rangle. \quad (3)$$

Solving the Schrödinger equation in the interaction picture, we can obtain the time-dependent wave function

$$|\Psi_{\rm I}(t)\rangle = x_1|eg0\rangle + x_2|ge0\rangle + x_3|gg2\rangle \quad (4)$$

with probability amplitudes

$$\begin{aligned} x_1 &= \frac{\alpha_-}{2} + \frac{\alpha_+}{4\sqrt{1+\gamma_1^2}}\left(\frac{1}{\Delta_{x+}}{\rm e}^{-2\,{\rm i}\,\Delta_{x+}gt} - \frac{1}{\Delta_{x-}}{\rm e}^{-2\,{\rm i}\,\Delta_{x-}gt}\right), \\ x_2 &= -\frac{\alpha_-}{2} - \frac{\alpha_+}{4\sqrt{1+\gamma_1^2}}(\Delta_{x-}{\rm e}^{-2\,{\rm i}\,\Delta_{x+}gt} - \Delta_{x+}{\rm e}^{-2\,{\rm i}\,\Delta_{x-}gt}), \\ x_3 &= \frac{\alpha_+}{2\sqrt{2+2\gamma_1^2}}({\rm e}^{-2\,{\rm i}\,\Delta_{x+}gt} - {\rm e}^{-2\,{\rm i}\,\Delta_{x-}gt}), \end{aligned} \quad (5)$$

$$\alpha_{\pm} = \cos\theta \pm \sin\theta, \quad \Delta_{x\pm} = \gamma_1 \pm \sqrt{1+\gamma_1^2},$$
$$\gamma_1 = \beta_1/g.$$

In the course of solving the Schrödinger equation, we find both terms $a^{+}a\beta_2 S_{+}^{\mathrm{A}} S_{-}^{\mathrm{A}} |\Psi_{\mathrm{I}}(t)\rangle$ and $a^{+}a\beta_2 S_{+}^{\mathrm{B}} S_{-}^{\mathrm{B}} |\Psi_{\mathrm{I}}(t)\rangle$ are equal to zero, which means that, when the cavity is prepared initially in the vacuum state, the parameter β_2 has not any effect on atom–atom entanglement.

We can obtain atom–atom density operator $\rho_{\mathrm{I}}^{\mathrm{AB}}(t)$ by tracing over the field variables from the state given by Eq. (4). In the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$, the atom–atom density matrix $\boldsymbol{\rho}_{\mathrm{I}}^{\mathrm{AB}}(t)$ is written as

$$\boldsymbol{\rho}_{\mathrm{I}}^{\mathrm{AB}}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1 x_2^{*} & 0 \\ 0 & x_1^{*} x_2 & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 \end{pmatrix}, \tag{6}$$

where $*$ stands for the complex conjugate and the time-dependent matrix elements are given by Eq. (5). It can be shown that the concurrence of the density matrix (6) is given by

$$C^{\mathrm{AB}}(t) = 2\max\{0, |x_1 x_2^{*}|\}. \tag{7}$$

The time evolution of the concurrence C^{AB} (from Eq. (7)) versus gt is plotted in Fig. 2 when initially there is no photon in the cavity, and the atoms are in entangled state. Figure 2(a) displays that the dependence of the atom–atom entanglement on β_1/g when the two two-level atoms are initially in the maximally entangled state, where $\beta_1/g = 0$ for the solid line, $\beta_1/g = 1$ for the dashed line, $\beta_1/g = 2$ for the dotted line, and $\beta_1/g = 10$ for the dot-dashed line. It is evident from these lines that the atom–atom entanglement evolves periodically, and from Eqs. (5) and (7) we can obtain that the periods are $\pi/[(\Delta_{x+} - \Delta_{x-})g]$. Obviously, the periods are affected by the dynamic Stark shift. What is more interesting is that, when the Stark shift is considered (corresponding to $\beta_1/g \neq 0$), the two two-level atoms are no longer disentangled, the minimum value of the atom–atom concurrence C^{AB} goes up with the increase of β_1/g and long-lived entanglement can be obtained (as shown by the dashed and dotted lines). Increasing the value of the parameter β_1/g to 10 (the dot-dashed line in Fig. 1(a)) or larger, corresponding to increasing the coupling strength between the atomic ground and intermediate level, we observe steady-going atom–atom entanglement, or in other words, the two atoms can remain in a stationary entangled state and the degree of stationary entanglement equals their initial concurrence. A possible physical reason for the above behaviour of the entanglement dynamics is the nonlinear nature of the coupling in two-photon processes. When $\theta = \pi/12$, the two atoms are initially in partially entangled states, the time evolution of the concurrence C^{AB} versus gt is plotted in Fig. 2(b) for different β_1/g. We find that, in this case, the long-lived entanglement between the two atoms can still occur for $\beta_1/g \neq 0$. It is most interesting that the entanglement between partially entangled atoms is greatly enhanced owing to the presence of Stark shift, the maximum of the enhanced entanglement reaches 1 (as shown in Fig. 2(b) by the dashed line for $\beta_1/g = 2$ and dotted line for $\beta_1/g = 4$). These properties show that the entanglement dynamics are strongly influenced by the dynamic Stark shift. Thus the dynamic Stark shift can be used to control entanglement between two qubits.

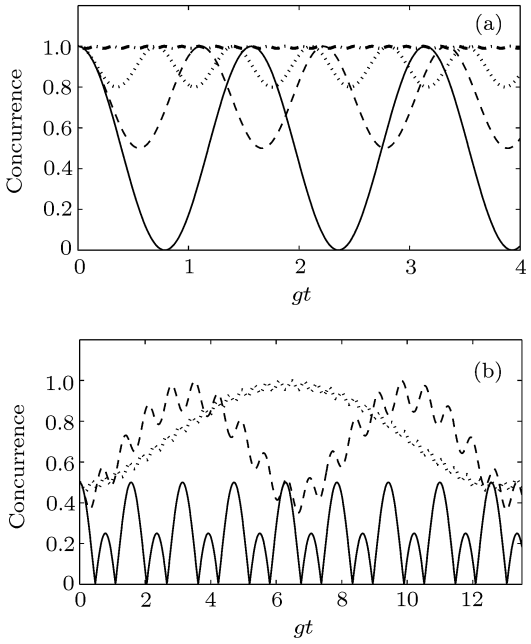


Fig. 2. Plots of the concurrences C^{AB}: (a) $\theta = \pi/4$, $\beta_1/g = 0$ (solid), $\beta_1/g = 1$ (dashed) and $\beta_1/g = 2$ (dotted); (b) $\theta = \pi/12$, $\beta_1/g = 0$ (solid), $\beta_1/g = 2$ (dashed) and $\beta_1/g = 4$ (dotted). There is initially no photon in the cavity, and the atoms are initially in entangled state.

4. The atom–atom entanglement for the case that there is one photon in the cavity

It is worth mentioning that all results in Section 3 refer to the case where no photons are initially presented in the cavity. In this section, let us consider

that how the atom–atom entanglement dynamics are affected by the dynamic Stark shift if initially there is one photon in the cavity. The atom–atom concurrence $C^{\rm AB}$ for this case can be obtained by using the same method in Section 3. We assume that initially the two atoms are still in a partially entangled atomic pure state, which is a superposition of Bell states usually denoted by $|\Psi^{\pm}\rangle \sim |e,g\rangle \pm |g,e\rangle$. Then the initial state for the system is

$$|\Phi(0)\rangle = (\cos\theta|eg\rangle + \sin\theta|ge\rangle) \otimes |1\rangle, \tag{8}$$

in this case the state of the total system at time t can be written as

$$|\Phi_{\rm I}(t)\rangle = y_1|eg1\rangle + y_2|ge1\rangle + y_3|gg3\rangle, \tag{9}$$

the corresponding coefficients are

$$\begin{aligned}
y_1 &= \left[\frac{\alpha_-}{2} - 12\frac{\alpha_+}{\varepsilon}\left(\frac{1}{\Theta_-}{\rm e}^{-{\rm i}\,\Theta_- gt/2} - \frac{1}{\Theta_+}{\rm e}^{-{\rm i}\,\Theta_+ gt/2}\right)\right]{\rm e}^{-{\rm i}(\lambda_1+\lambda_2)gt},\\
y_2 &= \left[-\frac{\alpha_-}{2} + 12\frac{\alpha_+}{\varepsilon}\left(\frac{1}{\Theta_-}{\rm e}^{-{\rm i}\,\Theta_- gt/2} - \frac{1}{\Theta_+}{\rm e}^{-{\rm i}\,\Theta_+ gt/2}\right)\right]{\rm e}^{-{\rm i}(\lambda_1+\lambda_2)gt}\\
&\quad + \frac{\alpha_+}{\varepsilon}\left[\left(6\beta_1 - \frac{1}{2}\Delta_{y-}\right){\rm e}^{-{\rm i}\,\Delta_{y-}gt/2} - \left(6\beta_1 - \frac{1}{2}\Delta_{y+}\right){\rm e}^{-{\rm i}\,\Delta_{y+}gt/2}\right],\\
y_3 &= -\frac{\sqrt{6}\alpha_+}{\varepsilon}[{\rm e}^{-{\rm i}\,\Delta_{y-}gt/2} - {\rm e}^{-{\rm i}\,\Delta_{y+}gt/2}],
\end{aligned} \tag{10}$$

where

$$\Theta_\pm = 5\lambda_1 - \lambda_2 \pm \varepsilon,\ \ \Delta_{y\pm} = 7\lambda_1 + \lambda_2 \pm \varepsilon,\ \ \varepsilon = \sqrt{25\lambda_1^2 - 10\lambda_1\lambda_2 + \lambda_2^2 + 48},\ \ \lambda_{1,2} = \beta_{1,2}/g.$$

In the bases $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$, the atom–atom density matrix $\boldsymbol{\rho}_{\rm I}^{\rm AB}(t)$ is now found to be

$$\boldsymbol{\rho}_{\rm I}^{\rm AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & |y_1|^2 & y_1y_2^* & 0\\ 0 & y_1^*y_2 & |y_2|^2 & 0\\ 0 & 0 & 0 & |y_3|^2 \end{pmatrix}, \tag{11}$$

the time-dependent matrix elements are given by Eq. (10), and the concurrence for this matrix is expressed by

$$C^{\rm AB}(t) = 2\max\{0, |y_1y_2^*|\}. \tag{12}$$

According to Eq. (12), we illustrate the concurrences $C^{\rm AB}(t)$ versus gt in Fig. 3, which shows that how the atom–atom entanglement is affected by the dynamic Stark shift when initially there is one photon in the cavity and the atoms are still in entangled state. Figure 3(a) displays that the dependence of the atom–atom entanglement on β_1/g and β_2/g when the two two-level atoms are initially in the maximally entangled state, where $\beta_1/g = \beta_2/g = 0$ for the solid line, $\beta_1/g = \beta_2/g = 1$ for the dashed line, $\beta_1/g = \beta_2/g = 2$ for the dotted line, and $\beta_1/g = \beta_2/g = 10$ for the dot-dashed line. The time evolution of the concurrence $C^{\rm AB}$ versus gt when $\theta = \pi/12$ is plotted in Fig. 3(b), where $\beta_1/g = \beta_2/g = 0$ for the solid line, $\beta_1/g = \beta_2/g = 2$ for the dashed line, and $\beta_1/g = \beta_2/g = 4$ for the dotted line. Comparing Fig. 3 with Fig. 2, we find that there are some differences. First of all, the entanglement dynamics in this case are affected by both Stark shift parameters β_1 and β_2, but if initially there are no-photons in the cavities, the entanglement is only affected by β_1. The periods of the atom–atom concurrence are shorter than those when the cavity field are initially in vacuum states. In addition, due to the influence of the parameter β_2, the minimum values of the atom–atom concurrence decrease when β_1/g has the same value. Though there are some differences, we notice that, in the two cases, the time evolution of entanglement and the effect of the Stark shift on entanglement dynamics is similar: the time evolution of the atom–atom concurrence is periodical; for initially partially entangled atomic state, the entanglement between the two atoms can be greatly enhanced due to the presence of Stark shift; and the two two-level atoms can be in long-lived entangled state due to the dynamic Stark shift.

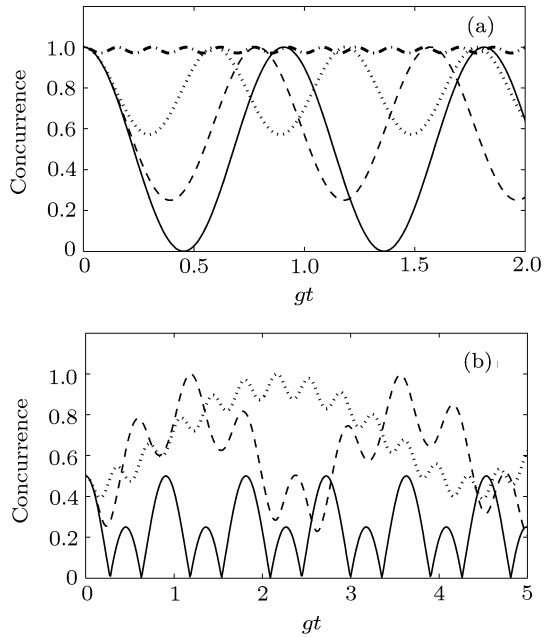


Fig. 3. Plots of the concurrences C^{AB}: (a) $\theta = \pi/4$, $\beta_1/g = \beta_2/g = 0$ (solid), $\beta_1/g = \beta_2/g = 1$ (dashed) and $\beta_1/g = \beta_2/g = 2$ (dotted); (b) $\theta = \pi/12$, $\beta_1/g = \beta_2/g = 0$ (solid), $\beta_1/g = \beta_2/g = 2$ (dashed) and $\beta_1/g = \beta_2/g = 4$ (dotted). There is initially one photon in the cavity, and the atoms are initially in entangled state.

5. Conclusion

We have considered a quantum model consisting of two effective two-level atoms and a single-mode cavity. Each atom interacts alone with the cavity field through a two-photon process, but there is no physical interaction between the two atoms. We have investigated the entanglement dynamics between the two atoms which are initially entangled for weak few-photon field, studied the effect of the dynamic Stark shift on the entanglement, and examined the possibility of controlling and protecting the entanglement by the dynamic Stark shift. The results show that, first, the atom–atom concurrence evolves periodically with time and the periods are affected by the dynamic Stark shift. Secondly, long-lived entanglement of the two two-level atoms can be obtained when the Stark shift is considered. In particular, for large values of the Stark shift parameter, the two atoms can remain in a stationary entangled state and the degree of stationary entanglement equals their initial concurrence. Finally, for initially partially entangled atomic state, the entanglement between the two atoms can be greatly enhanced due to the presence of Stark shift and the maximum of the enhanced entanglement can reach 1. A possible physical reason for the above behaviour of the entanglement dynamics is the nonlinear nature of the coupling in two-photon processes. These properties show that the entanglement dynamics are strongly influenced by the dynamic Stark shift. Thus the dynamic Stark shift can be used to control entanglement between two qubits. Here, to make the model closer to the experimental realization, we include the effect of the dynamic Stark shift generated in atomic transitions inside cavity in the two-photon process. Moreover, a two-photon micromaser has been operated successfully,[31] which offers us an opportunity to study the interaction between atoms and fields in two-photon processes. The theory of the two-photon micromaser was initiated by Brune *et al.*[32] and Davidovich *et al.*,[33], and then developed by Zubairy *et al.*[34,35] All of these make the experimental realization of the two-photon processes feasible. Our present study should motivate further investigations on the feasibility of using dynamic Stark shift in the practical manipulation of entanglement.

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