A novel evolving scale-free model with tunable attractiveness*

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In this paper, a new evolving model with tunable attractiveness is presented. Based on the Barabasi–Albert (BA) model, we introduce the attractiveness of node which can change with node degree. Using the mean-field theory, we obtain the analytical expression of power-law degree distribution with the exponent $\gamma \in (3, \infty)$. The new model is more homogeneous and has a lower clustering coefficient and bigger average path length than the BA model.

Keywords: scale-free, tunable attractiveness, degree distribution, clustering coefficient
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1. Introduction

Many complex networks exist in nature and society, such as the Power grid,[1] Internet,[2] scientific collaboration networks,[3] World Wide Web (WWW),[4] etc. In the past few years, much attraction has been focused on the research of complex networks. To model the networks and study the properties of real networks, Barabasi and Albert presented a Barabasi–Albert (BA) evolving model of networks.[5] They pointed out that despite diversity, many complex networks shared the properties of power-law of the degree distribution, following $P(k) = k^{-\gamma},\ \gamma = 3$, where γ is the degree exponent. Also it is shown that many real networks are scale-free. Although the model captures the power-law tail of the degree distribution, it has other properties that may not accord with the empirical results of the real networks. For example, in many real networks, the preferential attachment mechanism of a node is not only proportional to the degree, but also related to other factors. Based on the BA model, in order to provide a more satisfactory description for empirical characteristics of some real networks, many new models based on modifying the preferential attachment mechanism have been put forward, including the nonlinear preferential attachment model,[6] the fitness model,[7,8] and the mixed attachment mechanisms model,[9] and so on.[10−16] Among these studies, a very important concept is the initial attractiveness of node[17] introduced by Barabasi and Albert. They believed that in many real networks, the node with zero degree had nonzero probability to be attached, $\Pi(0) \neq 0$, and it was also attractive to the new node which was added to the network by other factors. The preferential attachment probability of node i is modified as Eq. (1).

$$\Pi_i = \frac{k_i + \beta}{\sum_j k_j + \beta}. \tag{1}$$

From Eq. (1), the attractiveness of node β is constant for each node in the network. However, in many real networks, the attractiveness of node β is influenced by many factors, thus it should be changed. For this point, much attention has be focused on the reasearch of β. Dorogovtsev and Mendes suggeted that the probability of connectivity of the old node was related to the node age, the bigger the age of the node is, the less active the node will be, so the attractiveness of node β is altered with the age of the node.[18] Tao *et al.* defined β as the number of links that the node acquires per unit time, i.e. $\beta = n_i/\Delta t$, where Δt is the time, and n_i is the number of links in Δt.[19] However, among these studies, there is a fact which we have ignored, namely in real networks, every node has a finite capacity to hold edges, the number of degrees for each node is limited, and it has a maximum. Just as in social networks, since a person's energy and time are finite, so he just can make a finite number, not an infinite number of friends. In WWW, a hub can also hold a finite number of edges. However, the BA model

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obviously ignores this point, the number of degrees of node in the BA model is unlimited. By the analysis above, since the capacity for each node to accept links is finite, with the increase of node degree, its connection is more close to its maximum. So the node state is more and more close to its saturation and its activity is lower. The node should show as less attractive to the new links, therefore the attractiveness β should decrease with the increase of the node degree. Based on the analysis, we propose the attractiveness of node i α_i as follows:

$$\alpha_i = \beta\left(1 - \frac{k_i}{K}\right), \tag{2}$$

where K is the threshold of node degree, and it controls the attractiveness sign: positive or negative.

If $k_i < K$, node i shows attractiveness to the new node, so $\alpha_i > 0$;

If $k_i \geq K$ node i shows unattractiveness to the new node, so $\alpha_i \leq 0$;

From Eq. (2), we can see that the attractiveness is different from node to node, and it is tuned by the change of node degree and shows two properties: attractiveness when $k_i < K$, unattractiveness when $k_i \geq K$. It is more reasonable in real networks.

In our paper, we design the attractiveness α_i of node i and propose a new evolving scale-free network model with tunable attractiveness. The rest of this paper is organized as follows. In Section 2, our novel model is introduced, followed by the analytical calculation of degree distribution with the mean-field theory. In Sections 4 and 5 we compare the clustering coefficient and the average path length with those of the BA model. Finally, some conclusions are given in Section 6.

2. Scale-free model with tunable attractiveness

The BA model includes two parts: the growth of network and the preferential attachment mechanisms of node. Based on the BA model, we introduce the attractiveness of node given by Eq. (2). The new model is generated by the following algorithm.

1) Initial condition: The network consists of m_0 nodes and n_0 edges.

2) Growth: At each time step, add a new node with $m(m < m_0)$ edges.

3) Preferential attachment: Each edge of the new node is attached to existing node i. The probability Π that a new node will be connected to node i depends on the degree k_i of node i, such that Π_i is proportional to its degree k_i, that is,

$$\Pi_i = \frac{k_i + \alpha_i}{\sum\limits_j k_j + \alpha_j}, \tag{3}$$

where α_i is given by Eq. (2). After t time steps, the system develops into a network with $N = m_0 + t$ nodes.

3. Analytical expression of degree distribution

We use the mean-field theory[20] to analyse the degree distribution of the network. According to the continuum theory, we assume that the degree k_i is continuous, thus the degree change rate of i in the new evolving model is

$$\frac{\partial k_i}{\partial t} = m\frac{k_i + \alpha_i}{\sum\limits_j k_j + \alpha_j}, \tag{4}$$

where α_i is given by Eq. (2). After t steps involution, the local community has a total node number $N_G = m_0 + t$ and mt edges.

So the summation of degrees in the local community is

$$\sum_j k_j = 2m(t-1) \approx 2mt, \tag{5}$$

the degree change rate of node i is

$$\frac{\partial k_i}{\partial t} = a\frac{k_i}{t} + b\frac{1}{t}, \tag{6}$$

where

$$a = \frac{m(K-\beta)}{2m(K-\beta) + \beta K},$$

$$b = \frac{mK\beta}{2m(K-\beta) + \beta K}.$$

With the initial condition, $k_i(t_i) = m$ we have

$$k_i(t) = -\frac{b}{a} + \left(m + \frac{b}{a}\right)\left(\frac{t}{t_i}\right)^a. \tag{7}$$

Because each step, we add one node into the network, the probability density of t_i is

$$P(t_i) = \frac{1}{m_0 + t}. \tag{8}$$

We can obtain

$$\begin{aligned} P(k_i < k) &= P\left[t_i > \left(\frac{m + b/a}{k + b/a}\right)^{1/a} t\right] \\ &= 1 - \frac{1}{m_0 + t}\left(\frac{m + b/a}{k + b/a}\right)^{1/a} t. \end{aligned} \tag{9}$$

Let

$$P(k) = \frac{\partial P[k_i(t) < k]}{\partial k}, \tag{10}$$

then we will obtain the probability density of degree as follows:

$$P(k) = \frac{t}{a(m_0 + t)} \left(m + \frac{b}{a}\right)^{1/a} \left(k + \frac{b}{a}\right)^{-\gamma}. \tag{11}$$

For $m_0 \ll t$ so

$$P(k) = \frac{1}{a} \left(m + \frac{b}{a}\right)^{1/a} \left(k + \frac{b}{a}\right)^{-\gamma}, \tag{12}$$

where

$$\gamma = 1 + \frac{1}{a},$$
$$a = \frac{m(K - \beta)}{2m(K - \beta) + \beta K},$$

so

$$\gamma = 3 + \frac{\beta K}{m(K - \beta)}. \tag{13}$$

In order to make sure that $\gamma > 0$, we obtain

$$K > \beta, \tag{14}$$

or

$$0 < K < \frac{3m\beta}{3m + \beta}. \tag{15}$$

For Eq. (15), when β turns big, $0 < K < 3m$, it is too small and unreasonable for the value of K. So we assume $K > \beta$, therefore $\gamma \propto (3, \infty)$.

In Figs. 1 and 2 we can find the degree distributions of the new model at various values of β and K and we can also see that the probability density of the degree of nodes with a small degree is slightly influenced by the changes of K and β. However, for the nodes with a big degree, the probability density of degree turns smaller with K and β increasing. The bigger the degree of node is, the faster the rate of the probability density of degree decrease will be. K and β control the number of nodes with too big a degree which is created in the BA model by the principle 'The winner takes all', so the whole network becomes more homogeneous. In real networks, there should not be too many nodes with too big a degree, therefore our model provides a more satisfactory description for real networks than the BA model.

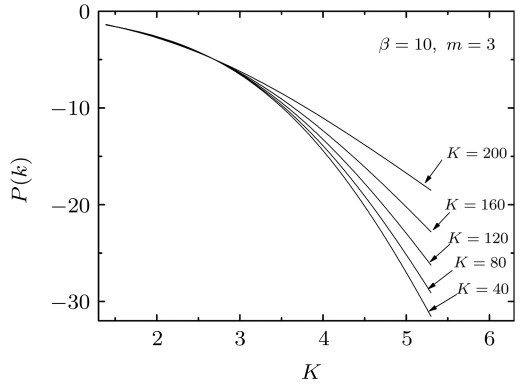


Fig. 1. Degree distributions on a logarithmic scale logarithmic scale t various values of K.

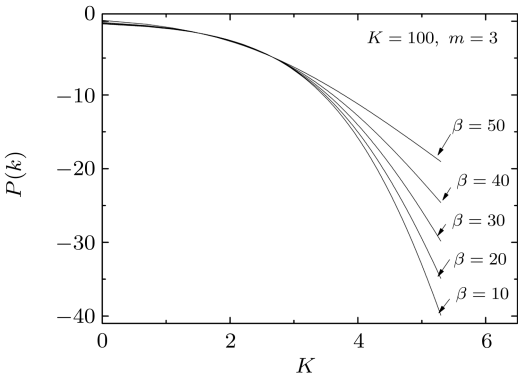


Fig. 2. Degree distributions on a logarithmic scale at various values of β.

4. Clustering coefficient

The clustering coefficient C_i of singe node i describes the density of connections in the neighbourhood of this node. It is given by the ratio of the number of links between the nearest neighbours of I, e_i, to the potential number of such links, $e_{\max} = k_i(k_i - 1)/2$, i.e

$$C_i = \frac{2e_i}{k_i(k_i - 1)}. \tag{16}$$

The clustering coefficient of the whole network is the average of all nodes clustering coefficient, i.e

$$C = \frac{1}{N} \sum C_i. \tag{17}$$

In Figs. 3 and 4, we show the clustering coefficients of our model at various values of K and β and compared them with those obtained from the BA model. In the simulation, the size of network n changes from 1000 to 5000. The data are averaged over 20 runs.

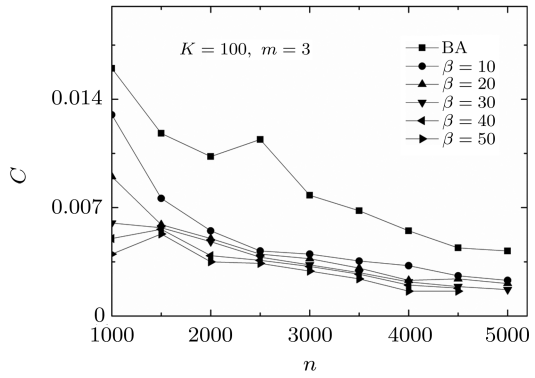


Fig. 3. Clustering coefficients at various values of β.

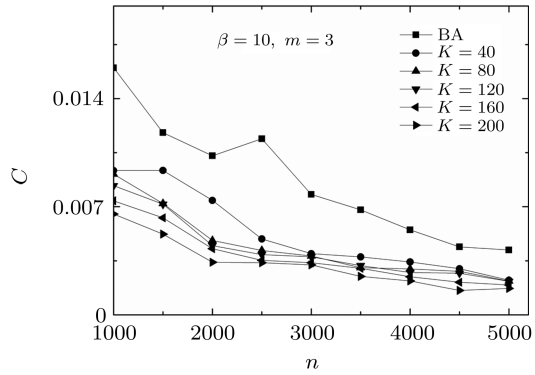


Fig. 4. Clustering coefficients at various values of K.

From the figure above, we can see that the clustering coefficient of the new model is much smaller than that of the BA model, and the clustering coefficient of the new model turns smaller with K and β increasing. This means that in the new network, there exit fewer nodes with a high clustering coefficient. Clustering coefficient reprents the spreading width of fault, so the network based on the new model has higher robustness and stability.

5. Average path length

The average distance is also one of the most important parameters for measuring the efficiency of communication networks, which is defined as the mean distance over all pairs of nodes. It plays a very important role in measuring the transmission delay. The transmission delay is approximately proportional to the number of edges which a message must pass through.

We use d_{ij} to denote the shortest path between nodes i and j, the average path length L of the network is defined as that given in Eq. (18).

$$L = \frac{1}{n(n-1)} \sum_{i,j \in n, i \neq j} d_{ij}, \tag{18}$$

where n denotes the total number of all nodes in the network.

Figures 5 and 6 describe the average path lengths of our model at various values of β and K respectively. In the simulation, the size n of network changes from 1000 to 5000. The data are averaged over 20 runs. The simulation result shows that the value of L obtained from our new model is bigger than that from the BA model and the average path length obtained from the new model turns bigger with K and β increasing.

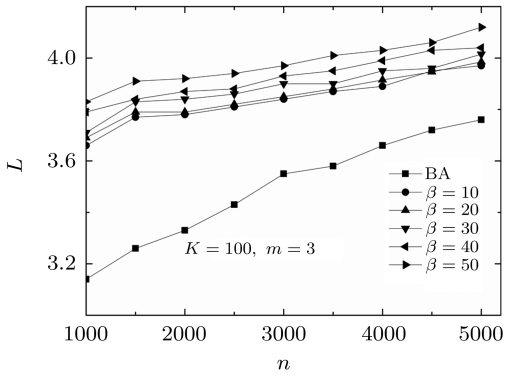


Fig. 5. Average path lengths at various values of β.

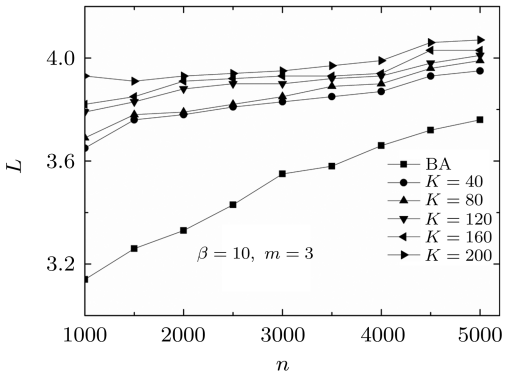


Fig. 6. Average path lengths at various values of K.

Why is the average path length obtained from our new model bigger than that obtained from the BA model? As a matter of fact, the reason is very simple. It is pointed out in Section 4 that the new model has a much lower clustering coefficient, so the nodes in the network cannot be clustered easily, and so the distance between nodes becomes longer. However, in the BA model, because of a higher clustering coefficient, it creates more 'shortcuts', therefore the distance between nodes becomes shorter.

6. Conclusion

In this paper is proposed an evolution of a scale-free network with tunable attractiveness which changes with the variation of node degree. Using the mean-field theory, we obtain the analytical expression of degree distribution with the exponent $\gamma \in (3, \infty)$. The new model is more homogeneous and has a much lower clustering coefficient and higher average path length than those of the BA model. The clustering coefficient represents the width of spreading of fault, while the average path length represents the depth of spreading of fault, so our new model has a higher robustness and stability.

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