Z-scan analysis of high-order nonlinear refraction effect induced by using elliptic Gaussian beam*

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The irradiance of an elliptic Gaussian beam that is high enough to excite high-order nonlinear refraction effect is used to calculate the normalized on-axis transmittance function in the z-scan technique by introducing complex beam parameters which make the calculation simpler. The transmittance formula is applied to the first-, first two-, and first three-order nonlinearities. Numerical evaluation shows that the symmetry no longer holds when using an elliptic Gaussian beam instead of a circular Gaussian beam. A distortion is observed in the central part of the curve, which decreases as ellipticity increases. Moreover, the variation of the normalized peak-valley difference decreases as ellipticity decreases.

Keywords: z-scan, elliptic Gaussian beam, complex beam parameter, high order

PACC: 7820W, 4260H, 1110L

1. Introduction

The z-scan technique is a method with which nonlinear refractions in solids, liquids and liquid solutions can be rapidly measured through the Gaussian decomposition method given by Weaire *et al.* Simple methods of data analysis are discussed for "thin" and "thick"[1−25] nonlinear medium conditions, on-axis and off-axis conditions,[26,27] thermo-optical effect condition,[28,29] two-colour z-scan,[30,31] time-resolved excite-probe z-scan,[32,33] top-hat beam z-scan,[34,35] reflection z-scan,[36] and eclipsing z-scan (Ez-scan).[37−41]

Most of them analysed only the first-order nonlinearity,[46−48] however, higher-order nonlinearity is also involved in z-scan experiments, especially with semiconductors and organic materials,[13,42−44] as more powerful lasers are used for pump sources in this kind of experiment.

Therefore, a theoretical infrastructure for higher-order nonlinearity is desirable. The first two-order nonlinearity is discussed by Sheik-Bahae *et al.* However their analyses were restricted to circular Gaussian incident beams without dealing with the more general case of an elliptic Gaussian beam. Then, an extension of the Gaussian decomposition (GD) method to higher-order nonlinearities in the general case of an elliptic Gaussian incident beam was presented in Ref. [40], where the method was applied to the first two-order nonlinearity, and the calculation of the transmittance encountered tremendous difficulties.

In the present paper, we introduce complex beam parameters in deducing the on-axis transmittance formula which is much simpler than the other method of calculating high-order nonlinearities using a Gaussian beam. The formula is applied to the first-, first two- and first three-order nonlinearities separately.

2. Complex beam parameter method of studying high-order nonlinear refraction effect induced by using an elliptic Gaussian beam

In this section, we present a method to calculate high-order nonlinearities induced by using an elliptic Gaussian beam through introducing complex beam parameters, which makes simpler the calculation of the normalized transmittance in z-scan technique.

A general profile of an elliptic Gaussian beam is presented in Fig. 1 by referring to Ref. [39], where we set x-focus to be the origin $z = z_{0x} = 0$. z_{0x} and z_{0y} are the locations of the waist in x and y directions respectively. The y-focus is set at $z = z_{0y} = az_R$, which also denotes the distance between x-focus and y-focus. Here, a is a rational number, we set $a = 1, 2, 3, 4, 5 \ldots$

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in the calculation later. Meanwhile, the lengths of the semi-axes at the waists of the beam are set to be equal to each other ($w_{0x} = w_{0y} = w_0$), and consequently the Rayleigh lengths for each principal direction are equal ($z_{R_x} = z_{R_y} = z_R$). The electric field of the elliptic Gaussian beam is given by[45]

$$\begin{aligned} E(x,y,z,t) = {} & E_0 \left[\frac{w_{0x}w_{0y}}{w_x(z)w_y(z)}\right]^{1/2} \\ & \times \exp\left[-\frac{x^2}{w_x^2(z)} - \frac{y^2}{w_y^2(z)}\right] \\ & \times \exp\left[-\frac{\mathrm{i}kx^2}{2R_x(z)} - \frac{\mathrm{i}ky^2}{2R_y(z)}\right] \\ & \times \exp[-\mathrm{i}kz + \mathrm{i}\theta(z)], \end{aligned} \tag{1}$$

where $k = 2\pi/\lambda$ is the wave vector, and λ is the laser wavelength, both in free space. E_0 denotes the radiation electric field at the focus, here, we consider an instantaneous nonlinearity.

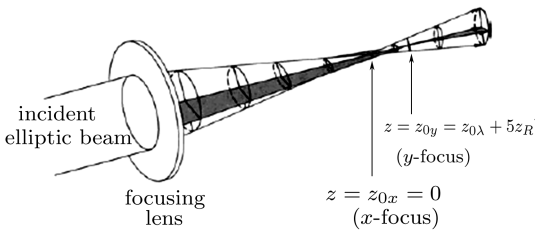


Fig. 1. General profile of an elliptic Gaussian beam traveling in the z direction.

The variations of the beam width and the radius of curvature in the two principal directions tend towards each focus in the same fashion respectively, which can be described by the following formula:

$$w_{x,y}^2(z) = w_{0x,0y}^2(1 + X_{x,y}^2)\ , \tag{2}$$

$$R_{x,y}(z) = z_{R_{x,y}} \frac{(1 + X_{x,y}^2)}{X_{x,y}}\ , \tag{3}$$

where, $X_x = z/z_{R_x}, X_y = (z - az_R)/z_{R_y}$, and $z_{R_{x,y}}$ is the Rayleigh lengths for the principal directions (x, y) given by the relationship

$$z_{R_{x,y}} = \frac{kw_{0x,y}^2}{2}. \tag{4}$$

As we are concerned about the nonlinear refraction effect, when the beam traverses the sample, the electric field encounters only a nonlinear phase shift, which leads to external self-focusing or self-defocusing. This is confirmed by the fact that the sample is thin compared with the diffraction length of the beam $z_{R_{x,y}}$.

Using the slowly varying envelope approximation (SVEA), high-order nonlinear phase shift can be given by

$$\begin{aligned} & \Delta\phi(x,y,z) \\ = {} & -\Delta\phi_0^{(1)}(z)\left(\frac{2x^2}{w_x^2} + \frac{2y^2}{w_y^2}\right) \\ & -\Delta\phi_0^{(2)}(z)\left(\frac{4x^2}{w_x^2} + \frac{4y^2}{w_y^2}\right) \\ & -\Delta\phi_0^{(3)}(z)\left(\frac{6x^2}{w_x^2} + \frac{6y^2}{w_y^2}\right)\cdots \\ & -\Delta\phi_0^{(n)}(z)\left(\frac{2nx^2}{w_x^2} + \frac{2ny^2}{w_y^2}\right)\cdots \\ & -\Delta\phi_0^{(N)}(z)\left(\frac{2Nx^2}{w_x^2} + \frac{2Ny^2}{w_y^2}\right) \end{aligned} \tag{5}$$

with $\Delta\phi_0^{(n)}(z) = \Delta\phi_0^{(n)}/(1 + z^2/z_R^2)$, where $\Delta\phi_0^{(n)} = k\Delta n^{(n)}(0,0)L$ is the n-order on-axis phase shift at focus x, N denotes the order number of the highest-order nonlinearity, and L is the sample thickness. Here,

$$\begin{aligned} \Delta n(0,0) = {} & n_2 I_0 + n_3 I_0^2 + n_4 I_0^3 + \cdots \\ & + \Delta n^{(n)}(0,0) + \cdots + n^{(N)} I_0^N, \end{aligned} \tag{6}$$

where n_{n+1} denotes the n-th order nonlinear refraction coefficient and I_0 is the on-axis intensity at x-focus.

The electric field that exits in the sample suffers a nonlinear phase distortion, given as

$$\begin{aligned} & E_{\mathrm{e}}(x,y,z,\Delta\phi) = E(x,y,z)\exp[-\mathrm{i}\Delta\phi(x,y,z)] \\ = {} & E_0\left(\frac{w_{0x}w_{0y}}{w_x(z)w_y(z)}\right)^{1/2} \exp(-\mathrm{i}kz + \mathrm{i}\theta(z)) \exp\left(-\frac{a_0 L}{2}\right) \exp\left[-\mathrm{i}\frac{kx^2}{2q_x} - \mathrm{i}\frac{ky^2}{2q_y}\right], \end{aligned} \tag{7}$$

where q_x and q_y are the introduced complex beam parameters on the two principal semi-axes, defined by

$$\begin{aligned} \frac{1}{q_x} = {} & -\mathrm{i}\frac{2}{kw_x^2(z)} + \frac{1}{R_x(z)} - \Delta\phi_0^1(z)\frac{4}{kw_x^2(z)} - \Delta\phi_0^2(z)\frac{2*4}{kw_x^2(z)} - \cdots - \Delta\phi_0^n(z)\frac{n*4}{kw_x^2(z)} - \cdots \\ & - \Delta\phi_0^N(z)\frac{N*4}{kw_x^2(z)}, \end{aligned} \tag{8}$$

$$\frac{1}{q_y} = -\mathrm{i}\frac{2}{kw_y^2(z)} + \frac{1}{R_y(z)} - \Delta\phi_0^1(z)\frac{4}{kw_y^2(z)} - \Delta\phi_0^2(z)\frac{2*4}{kw_y^2(z)} - \cdots - \Delta\phi_0^n(z)\frac{n*4}{kw_y^2(z)} - \cdots - \Delta\phi_0^N(z)\frac{N*4}{kw_y^2(z)}, \tag{9}$$

where q_x and q_y contain all the information needed to specify its characteristics in the beam propagation. The formulations of the q parameters permit the use of ray matrices to propagate the beam through the optical systems. At the exit face of the sample the electric field propagates to the aperture plane, where the final beam radius is $w_{ax,ay}(z)$ and the radius of curvature of the wave front is $R_{ax,ay}(z)$ in the two principal semi-axes directions. The complex beam parameters at the aperture plane and distance D from the sample, are related by

$$q_{ax} = q_x + D, \tag{10}$$

$$q_{ay} = q_y + D. \tag{11}$$

After combining Eqs. (8)–(11), we equate the real and the imaginary parts to obtain

$$w_{ax}^2(z+D) = w_x^2(z)\left[g_x^2 + \left(\frac{2D}{kw_x^2}\right)^2\right], \tag{12}$$

$$w_{ay}^2(z+D) = w_y^2(z)\left[g_y^2 + \left(\frac{2D}{kw_y^2}\right)^2\right], \tag{13}$$

$$R_{ax}(z+D) = D\left[1 - \frac{g_x}{g_x^2 + \left(\frac{2D}{kw_x^2}\right)^2}\right]^{-1}, \tag{14}$$

$$R_{ay}(z+D) = D\left[1 - \frac{g_y}{g_y^2 + \left(\frac{2D}{kw_y^2}\right)^2}\right]^{-1}, \tag{15}$$

where

$$g_x = 1 + \left[\frac{1}{R_x(z)} - \Delta\phi_0^1(z)\frac{4}{kw_x^2(z)} - \Delta\phi_0^2(z)\frac{2*4}{kw_x^2(z)} - \cdots - \Delta\phi_0^n(z)\frac{n*4}{kw_x^2(z)} - \cdots - \Delta\phi_0^N(z)\frac{N*4}{kw_x^2(z)}\right]D, \tag{16}$$

$$g_y = 1 + \left[\frac{1}{R_y(z)} - \Delta\phi_0^1(z)\frac{4}{kw_y^2(z)} - \Delta\phi_0^2(z)\frac{2*4}{kw_y^2(z)} - \cdots - \Delta\phi_0^n(z)\frac{n*4}{kw_y^2(z)} - \cdots - \Delta\phi_0^N(z)\frac{N*4}{kw_y^2(z)}\right]D. \tag{17}$$

Thus, the electric field at the aperture can be written as

$$E_a(z,r,\Delta\phi_0) = E_0\left(\frac{w_{0x}w_{0y}}{w_x(z)w_y(z)}\right)^{1/2}\exp\left[-\mathrm{i}kz + \mathrm{i}\theta(z)\right]\exp\left(-\frac{a_0L}{2}\right)\left(\frac{w_x(z)w_y(z)}{w_{ax}(z)w_{ay}(z)}\right)\mathrm{e}^{\mathrm{i}\eta(z)} \times \exp\left[-\frac{\mathrm{i}kx^2}{2q_{ax}} - \frac{\mathrm{i}ky^2}{2q_{ay}}\right], \tag{18}$$

where

$$\left(\frac{w_x(z)w_y(z)}{w_{ax}w_{ay}}\right)^{1/2}\mathrm{e}^{\mathrm{i}\eta(z)} = \left[\left(g_x + \mathrm{i}\frac{2D}{kw_x^2}\right)^{-1}\left(g_y + \mathrm{i}\frac{2D}{kw_y^2}\right)^{-1}\right]^{1/2}, \tag{19}$$

$$\frac{1}{q_{ax}} = \frac{1}{R_{ax}(z)} - \mathrm{i}\frac{2}{kw_{ax}^2(z)}, \tag{20}$$

$$\frac{1}{q_{ay}} = \frac{1}{R_{ay}(z)} - \mathrm{i}\frac{2}{kw_{ay}^2(z)}. \tag{21}$$

The on-axis electric field of the far field beam at the aperture plane becomes

$$E_a(z,0,\Delta\Phi_0) = E_0\,\mathrm{e}^{-\alpha L/2}\left(\frac{w_{0x}w_{0y}}{w_x(z)w_y(z)}\right)^{1/2}\mathrm{e}^{-\mathrm{i}kz+\mathrm{i}\theta(z)}\left(g_x + \mathrm{i}\frac{2D}{kw_x^2}\right)^{-1}\left(g_y + \mathrm{i}\frac{2D}{kw_y^2}\right)^{-1}. \tag{22}$$

Using Eq. (22) together with Eqs. (16), (17), (19), Eqs.(2)–(4), we have the on-axis (closed-aperture) z-scan transmittance of the far field beam at the aperture plane ($D \gg z_R$)

$$T(z,\Delta\Phi_0) = \frac{\left|E_a(z,r=0,\Delta\Phi_0^{(1)},\Delta\Phi_0^{(2)},\ldots,\Delta\Phi_0^{(n)},\ldots,\Delta\Phi_0^{(N)})\right|^2}{\left|E_a(z,r=0,\Delta\Phi_0^{(1)}=0,\Delta\Phi_0^{(2)}=0,\ldots,\Delta\Phi_0^{(n)}=0,\ldots,\Delta\Phi_0^{(N)}=0)\right|^2}$$

$$= \frac{((x-a)^2+1)(1+x^2)^3}{(AB)^{1/2}}, \tag{23}$$

where

$$x = \frac{z}{z_R}, \tag{24}$$

$$A = \left(2\Delta\Phi_0^{(1)} + 2*2*\Delta\Phi_0^{(2)} + \cdots + 2*n*\Delta\Phi_0^{(n)} + \cdots + 2*N*\Delta\Phi_0^{(N)}\right)^2 + \left(1 + 2x^2 + x^4 - 2x\Delta\Phi_0^{(1)} - 4x\Delta\Phi_0^{(2)} - \cdots - 2n\Delta\Phi_0^{(n)} - \cdots - 2Nx\Delta\Phi_0^{(N)}\right)^2, \tag{25}$$

$$B = \left[((x-a)^2+1)(1+x^2) - 2(x-a)\Delta\Phi_0^{(1)} - 4(x-a)\Delta\Phi_0^{(2)} - \cdots - 2n(x-a)\Delta\Phi_0^{(n)} - \cdots - 2N(x-a)\Delta\Phi_0^{(N)}\right]^2 + \left(2\Delta\Phi_0^{(1)} + 2*2*\Delta\Phi_0^{(2)} + \cdots + 2*n*\Delta\Phi_0^{(n)} + \cdots + 2*N*\Delta\Phi_0^{(N)}\right)^2. \tag{26}$$

When $N = 1$ (the first-order nonlinearity) and $a = 0$ (circular Gaussian beam), the on-axis (closed-aperture) z-scan transmittance tends to be

$$T^{(1)} = \frac{1}{1 - \dfrac{4x}{(x^2+1)^2}\Delta\Phi_0^{(1)} + \dfrac{4}{(x^2+1)^3}\left[\Delta\Phi_0^{(1)}\right]^2}. \tag{27}$$

Equation (27) is identical to Eq. (11) in Ref. [18].

In small nonlinear phase shift condition ($\Delta\Phi_0(t) \ll 1$) equation (27) reduces to the result of the distributed-lens model proposed by Sheik-Bahae *et al*. They also derived an expression for on-axis z-scan transmittance for a small phase shift by adopting a Gaussian decomposition method given in Ref. [7], which is expressed as

$$T^{(1)} = 1 - \frac{4x\Delta\Phi_0^{(1)}}{(1+x^2)(9+x^2)}. \tag{28}$$

Therefore, we are confident to continue the discussion in Section 3.

3. Application to the first-, first two- and first three-order nonlinearities

In this section, we apply Eq. (23) to the first-, first two- and first three-order nonlinearities.
For the first two-order nonlinearity, we have the on-axis (closed-aperture) z-scan transmittance

$$T^{(2)}(z, \Delta\Phi_0) = \frac{((x-a)^2+1)(1+x^2)^3}{(A_2B_2)^{1/2}}, \tag{29}$$

$$A_2 = \left(2\Delta\Phi_0^{(1)} + 4\Delta\Phi_0^{(2)}\right)^2 + \left(1 + 2x^2 + x^4 - 2x\Delta\Phi_0^{(1)} - 4x\Delta\Phi_0^{(2)}\right)^2, \tag{30}$$

$$B_2 = \left[((x-a)^2+1)(1+x^2) - 2(x-a)\Delta\Phi_0^{(1)} - 4(x-a)\Delta\Phi_0^{(2)}\right]^2 + \left(2\Delta\Phi_0^{(1)} + 4\Delta\Phi_0^{(2)}\right)^2. \tag{31}$$

For the first three-order nonlinearity, we have the on-axis (closed-aperture) z-scan transmittance

$$T^{(3)}(z, \Delta\Phi_0) = \frac{((x-a)^2+1)(1+x^2)^3}{(A_3B_3)^{1/2}}, \tag{32}$$

$$A_3 = \left(2\Delta\Phi_0^{(1)} + 4\Delta\Phi_0^{(2)} + 6\Delta\Phi_0^{(3)}\right)^2 + \left(1 + 2x^2 + x^4 - 2x\Delta\Phi_0^{(1)} - 4x\Delta\Phi_0^{(2)} - 6x\Delta\Phi_0^{(3)}\right)^2, \tag{33}$$

$$B_3 = \left[((x-a)^2+1)(1+x^2) - 2(x-a)\Delta\Phi_0^{(1)} - 4(x-a)\Delta\Phi_0^{(2)} - 6(x-a)\Delta\Phi_0^{(3)}\right]^2 + \left(2\Delta\Phi_0^{(1)} + 4\Delta\Phi_0^{(2)} + 6\Delta\Phi_0^{(3)}\right)^2. \tag{34}$$

In order to analyse the influence of the ellipticity of the cross section of the beam at the origin on the result, we define the ellipticity of the cross section of the beam at the origin (x-focus) as

$$\rho = \frac{w_x(0)}{w_y(0)} = \frac{w_{0x}}{w_{0y}(1+a^2)^{1/2}} = \frac{1}{(1+a^2)^{1/2}}. \quad (35)$$

Simulations of the transmittance in the close aperture z-scan experiment, concerning the first-order nonlinearity, are shown in Figs. 2 and 3 for $a = 0$ (circular Gaussian beam $\rho = 1$) and $a = 5$ ($\rho \approx 0.2$) respectively. The curves in Fig. 2 are the same as those in Fig. 1(b) in Ref. [18].

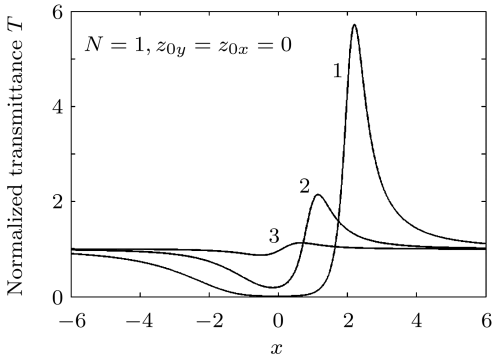


Fig. 2. Theoretical plots of closed-aperture z-scan transmittance T for only the first-order nonlinear refraction induced by using a circular Gaussian beam ($z_{0y} - z_{0x} = 0$) versus $x = z/z_R$. The curves 1, 2 and 3 correspond to the on-axis first-order nonlinear phase shift at the x-focus $\Delta\Phi_0^{(1)} = 6.0, \Delta\Phi_0^{(1)} = 1.0, \Delta\Phi_0^{(1)} = 0.1$, respectively.

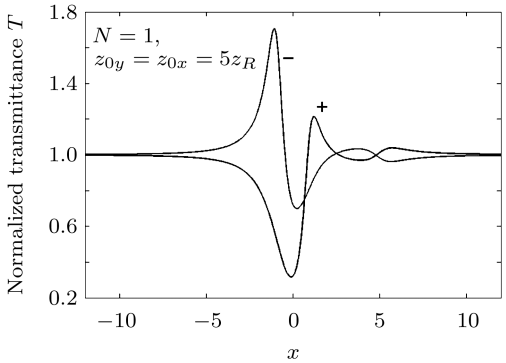


Fig. 3. Theoretical plots of closed-aperture z-scan transmittance for the first-order nonlinear refraction induced by using an elliptic Gaussian beam with the distance between x-focus and y-focus $z_{0y} - z_{0x} = 5z_R$ ($\rho \approx 0.2$) versus $x = z/z_R$. The corresponding on-axis nonlinear phase shifts at the x-focus are $\Delta\Phi_0^{(1)} = 1.0$ and $\Delta\Phi_0^{(1)} = -1.0$.

Figure 3 shows the on-axis transmittance for only the first-order nonlinear refraction induced by using an elliptic Gaussian beam with ellipticity $\rho \approx 0.2$. In contrast to Fig. 2, there are two main effects induced by using an elliptic Gaussian beam instead of a circular Gaussian beam. First, a distortion is observed in the central part of each curve, which is due to the astigmatism of the elliptic Gaussian beam. The distortion increases as the distance between x-focus and y-focus, $z_{0y} - z_{0x} = az_R$, increases, that is, the distortion decreases as the ellipticity ρ increases. The other effect is that the variation of the normalized peak–valley difference $\Delta T_{\mathrm{p-v}}$ decreases as the ellipticity ρ decreases, which we can see in Figs. 4 and 5.

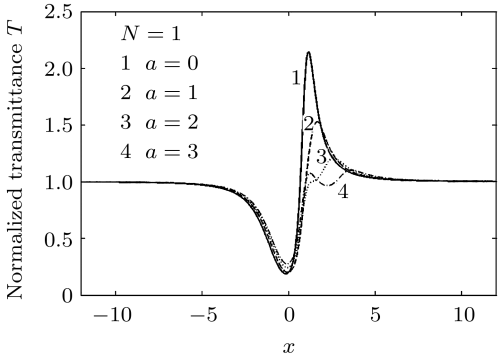


Fig. 4. Simulated closed-aperture z-scan curves for the first-order nonlinear refraction induced by using an elliptic Gaussian beam with $\Delta\Phi_0^{(1)} = 1.0$ and different values of ellipticity $\rho, \rho = 1$ (circular) (curve 1), ≈ 0.7 (curve 2), ≈ 0.4 (curve 3), ≈ 0.3 (curve 4) separately.

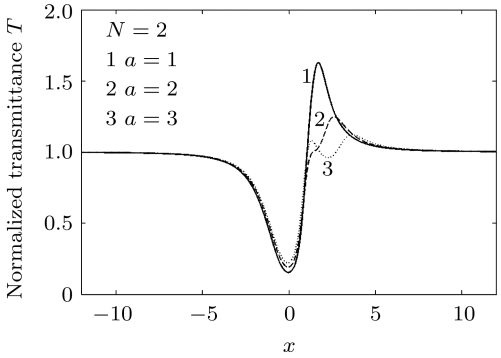


Fig. 5. Simulated closed-aperture z-scan curves for the first two-order nonlinear refraction induced by using an elliptic Gaussian beam with $\Delta\Phi_0^{(1)} = 1.0$, $\Delta\Phi_0^{(2)} = 0.1$, and different values of ellipticity ρ, $\rho \approx 0.7$ (curve 1), ≈ 0.4 (curve 2), ≈ 0.3 (curve 3) separately.

Closed-aperture z-scan transmittance for the first-order (solid line), first two-order (dotted line) and first three-order (dashed line) nonlinear refractions respectively induced by using an elliptic Gaussian beam with ellipticity $\rho \approx 0.3$ are all presented in Fig. 6. The dotted line is so close to the dashed line compared with the closeness between the solid line and the dashed line that it almost overlaps the dashed line, which indicates that the first three-order nonlinear refraction

makes less contribution to the variation of the transmittance than the first two-order nonlinear refraction.

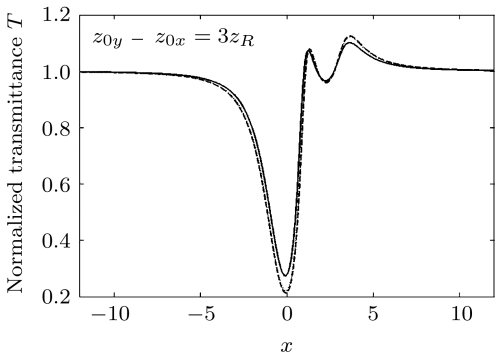


Fig. 6. Theoretical plots of closed-aperture z-scan transmittance for the first-order, first two-order and first three-order nonlinear refractions induced by using an elliptic Gaussian beam with the distance between x-focus and y-focus $z_{0y} - z_{0x} = 3z_R$ ($\rho \approx 0.3$). We set $\Delta\Phi_0^{(1)} = 1$. The solid line corresponds to $\Delta\Phi_0^{(2)}/\Delta\Phi_0^{(1)} = 0$, the dashed line to $\Delta\Phi_0^{(2)}/\Delta\Phi_0^{(1)} = 0.1$ and the dotted line to $\Delta\Phi_0^{(3)}/\Delta\Phi_0^{(1)} = 0.01$.

In comparison with the result from G Tsigaridas *et al.*[40] who analysed the first two-order nonlinear refraction induced by using an elliptic Gaussian incident beam through Gaussian decomposition, see in Figs. 7(a) and 7(b), we can find that the result that we discussed above is the same as theirs. From Figs. 7(a) and 7(b), a distortion is observed in the central part of each curve, which decreases as ellipticity increases (corresponding to the decrease of the distance between x-focus and y-focus). Moreover, the variation of the normalized peak–valley difference decreases as ellipticity decreases, which is due to the asymmetry of the elliptic Gaussian beam. The difference in the shape of distortion between their result and ours is caused by the different definitions of the location of x-focus. We can obtain the same result of the first three-order nonlinearity in the same way.

In short, the results of the first two- and first three-order nonlinearites in this paper accord with those obtained from the Gaussian-decomposition method.

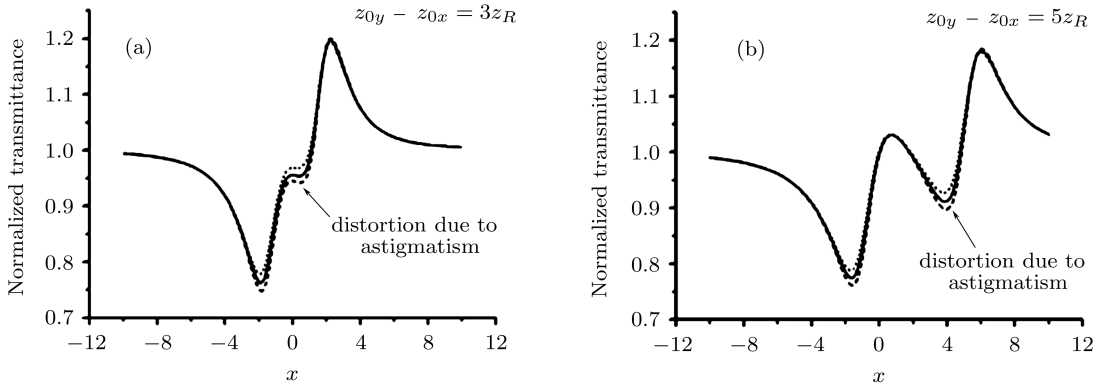


Fig. 7. Theoretical plots of closed-aperture z-scan transmittance for the first two-order nonlinear refraction induced by using an elliptic Gaussian beam, obtained through the Gaussian-decomposition method by G Tsigaridas *et al.*[40]

4. Conclusions

A transmittance formula is presented for high-order nonlinearities induced by using an elliptic Gaussian beam in z-scan technique through introducing complex beam parameters q_x and q_y. For the first-order small nonlinear phase shift and the circular Gaussian beam condition, the formula reduces to the result of on-axis z-scan transmittance proposed by Sheik-Bahae *et al.*[7] who proposed the z-scan technique. And, the results of the first two-order and first three-order nonlinearities in this paper accord with those obtained from the Gaussian-decomposition method. The formula is applied to the first-, first two- and first three-order nonlinearities separately. Numerical evaluation of Eq. (24) shows that the symmetry no longer holds due to the use of an elliptic Gaussian beam instead of a circular Gaussian beam. A distortion is observed in the central part of the curve, which decreases as ellipticity ρ increases. Moreover, the variation of the normalized peak–valley difference $\Delta T_{\mathrm{p-v}}$ decreases as ellipticity decreases, which is due to the asymmetry of the elliptic Gaussian beam.

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