Characterization of the radiation from single-walled zig-zag carbon nanotubes at terahertz range*

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This paper investigates the radiation characteristics of metal single-walled zig-zag carbon nanotubes as a dipole antenna at terahertz wave range. The current distribution, input impedance and mutual impedance are calculated for various geometrical parameters of vertically-aligned carbon nanotubes. The numerical results demonstrate the properties of the antenna depending strongly on the geometrical parameters such as the radius, the lengths of carbon nantobues, and the spacing between nanotubes. It is found that the zig-zag carbon nanotubes exhibit very high input impedance and the mutual impedances for antenna array applications. These unique high impedance properties are different from the conventional metal thin wire antenna. The far-field patterns and gain of antenna array are also calculated. The maximum gain of array of 100-element array is up to 20.0 dB, which is larger than the gain of 0.598 dB of single dipole antenna at distance $d = 0.5\lambda$.

Keywords: zig-zag carbon nanotube, dipole antenna, terahertz, impedance

PACC: 7800, 7865, 4110H

1. Introduction

Since the first discovery of single-walled carbon nanotubes (SWNTs),[1,2] experiments and theories have revealed a great deal about their intriguing electronic and mechanical properties. Scanning tunnel microscopy measurements of individual tubes have verified that they are either a one-dimensional conductor with excellent conductivity or semiconductor with a band gap inversely proportional to the diameter of the tube, depending on their different helicity.[3] Just as based on their outstanding electronic properties, unique nanostructure and vertically-aligned carbon nanotubes and array synthesized successfully, carbon nanotubes are extremely attractive for a wide field of applications such as field emitters,[4,5] transistors,[6,7] chemical sensors[8] and unusual nano antennas[9] for communications between microscopic and macroscopic fields.

In recent years, there has been increasing interest to develop carbon nanotubes as an antenna for visible light and infrared frequency regions. In previous research, light scattering properties of an array of aligned randomly multiwalled carbon nanotubes have firstly been investigated and the basic antenna effects of carbon nanotubes have been performed in experiments.[10] Further researches show that the radiation characteristics of a single multi-walled carbon nanotubes acts as optical antenna, whose response is consistent with traditional radio antenna theory and simulations in quantity.[11] Until recently, the infrared radiation properties of finite-length carbon nanotubes dipole antenna have been studied based on the Hallén integral equation, and the current on an infinite-length carbon nanotubes antenna has been analysed by a Fourier transform technique under the condition of neglecting the edge effects.[12,13]

The generations of THz wave[14,15] with photoconductive switching and optical rectification have been extensively investigated.[16−18] However, the researches of single-walled carbon nanotubes and array proposed for THz transmission are at initial stages and their THz radiation properties have not been investigated thoroughly.[19] To complete the preliminary

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studies of THz wave generated from nanotubes antenna, the method of moment based on the integral equation is utilized to perform the detailed antenna radiation properties in the THz range. The fundamental characteristics such as current distribution, input impedance and mutual impedance of zig-zag carbon nanotubes antenna are studied in the THz range by the integral function. The overall results will serve as the development and the design of carbon nanotubes antenna.

2. Modeling description

2.1. Modeling for a carbon nanotube antenna and antenna array

The nanotubes are essentially a sheet of graphitic carbon rolled into a cylindrical tube. The way of the rolling graphene sheet determines the fundamental properties of the tube. In order to describe such a fundamental characteristic of the nanotubes, we introduce a vector $\boldsymbol{C}_{\rm h}$ which defines the circumference on the surface of the tube connecting two equivalent carbon atoms. Here, $C_{\rm h} = na_1 + ma_2$, a_1 and a_2 are the two basis vectors of graphene, and n and m are integers, respectively. Indexes n and m determine the chiral angle $\theta = \tan^{-1}\sqrt{3}(n/(2n+m))$, which is used to separate carbon nanotubes into three classes differentiated by their electronic properties: armchair ($n = m$, $\theta = 30°$), zig-zag ($m = 0$, $n > 0$, $\theta = 0°$), and chiral ($0 < |m| < n$, $0 < \theta < 30°$) carbon nanotubes. Armchair carbon nanotubes are always metallic, as are zig-zag carbon nanotubes with the value $(n - m)$ divided by three. The radius of carbon nanotubes is given by

$$R = \frac{C_{\rm h}}{2\pi} = \frac{\sqrt{3}}{2\pi} a_{\rm C-C}\sqrt{n^2 + nm + m^2}, \qquad (1)$$

where $a_{\rm C-C} = 0.142$ nm is the C–C bond length. Here attention will be focused on metallic single-walled zig-zag carbon nanotubes

The typical structure of carbon nanotubes antenna is shown in Fig. 1. The radius and total length of the antenna are R and L, respectively, and the gap between dipole is D ($D \ll L$). Theoretically, a simple dipole thin antenna maximizes its response to a wavelength λ under the condition $L = m\lambda/2$, where m is a positive integer.

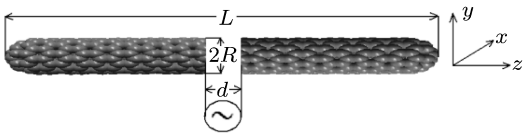


Fig. 1. Structure of carbon nanotubes (zig-zag) antenna.

Figure 2 depicts the linear antenna array, where d is the intertube distance in the x direction.

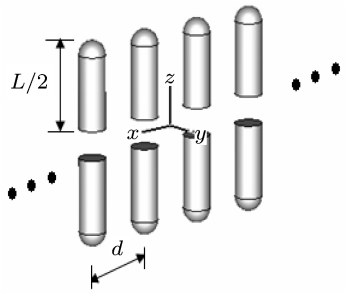


Fig. 2. Structure of carbon nanotubes antenna array.

2.2. Formulation

As depicted in Fig. 1, we can regard a zig-zag carbon nanotubes dipole antenna having large aspect ratio L/R as a sufficiently thin tube. So the transverse current can be neglected, and just only the longitudinal current $I_z(z)$ distributes on the carbon nanotubes. The axial component $A_z(z)$ of the vector potential $\boldsymbol{A}$ on the surface of a carbon nanotubes antenna that has the surface impedance per unit length $z^{\rm i}$ and carries a total tangential current $I_z(z)$ satisfies the one-dimensional Pocklington's integral equation[20]

$$\left(\frac{{\rm d}^2}{{\rm d}z^2} + k_0^2\right) A_z(z) = {\rm i}\frac{k_0^2}{\omega}[z^{\rm i} I_z(z) - E_z^{\rm i}(z)], \qquad (2)$$

where k_0 is free-space wave number, ω is the frequency, and $E_z^{\rm i}(z)$ is incident electric field. Because of the symmetry of carbon nanotubes antenna, the radius R and k_0 satisfied $k_0 R \ll 1$, and neglecting edge effects (carbon nantobues having high aspect ratio L/R), we obtain approximately the vector potential on the surface of carbon nanotubes antenna

$$A_z(z) = \mu_0 \int_{-L/2}^{+L/2} I_z(z') K(z, z')\, {\rm d}z', \qquad (3)$$

where μ_0 is the permeability of free space, z and z' represent the observation point and the source point, respectively, and

$$K(z, z') \approx \frac{1}{4\pi}\frac{{\rm e}^{-{\rm i}k_0\sqrt{R^2+(z-z')^2}}}{\sqrt{R^2+(z-z')^2}}. \qquad (4)$$

Let us assume that the carbon nanotubes antenna is fed by an electromotive force $V_z^{\rm i}$, thus the incident electric field can be expressed by

$$E_z^{\rm i}(z) = \frac{V_z^{\rm i}}{D}\delta(z), \qquad (5)$$

where D is the smaller width gaps, $\delta(z)$ is delta gap source. Then, equation (1) can also be expressed in a

more detailed form as

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}+k_0^2\right)\int_{-L/2}^{+L/2} I_z(z')\frac{\mathrm{e}^{-\mathrm{i}k_0\sqrt{R^2+(z-z')^2}}}{\sqrt{R^2+(z-z')^2}}\mathrm{d}z' = \mathrm{i}\frac{4\pi k_0^2}{\mu_0\omega}\left[z^{\mathrm{i}}I_z(z)-\frac{V_z^{\mathrm{i}}}{D}\delta(z)\right]. \quad (6)$$

This equation relates the unknown current distribution on the carbon nanotubes antenna to the voltage impressed across the gap, which cannot be evaluated analytically. To resolve such problems, we assume that the surface current $I_z(z)$ is approximated by a series of expansion function. Therefore, we will adopt the method of moment to solve the integral equation numerically by writing N equation in N unknown quantity.

3. Numerical results and discussions

3.1. Current distribution characteristics

One of the most fundamental parameters of any antenna is the current distribution on the antenna, which determines the radiation pattern, impedance, and other parameters of antenna.

As presented in Ref. [21], the movements of the free charges on the carbon nanotubes create a traveling wave which makes the carbon nanotubes act differently from a traditional metallic line. For the model of carbon nanotubes transmission line, the parameters of kinetic inductance and electrostatic capacitance must be taken into account, which leads to the wave propagation velocity on the carbon nanotubes transmission line being the plasmon velocity v_{p} rather than the speed of the light c. In Refs. [21] and [22], $v_{\mathrm{p}} \approx 0.01c$. Therefore, the wave number is much larger on the carbon nanotubes in comparison with that on the typical metallic thin wire. In the metallic zig-zag carbon nanotubes, the surface impedance z^{i} is specified by

$$z^{\mathrm{i}} = \frac{1}{2\pi R\sigma_{\mathrm{CN}}^z}, \quad (7)$$

and

$$\sigma_{\mathrm{CN}}^z = \mathrm{i}\frac{2e^2\gamma_0}{n\pi\hbar^2(\omega+\mathrm{i}\nu)}, \quad (8)$$

where σ_{CN}^z is the conductivity of the zig-zag carbon nanotubes, e is the electric quantity, γ is the overlap integral for electron wave functions between two nearest atoms (here $\gamma_0 = 2.7$ eV), n is the index of zig-zag carbon nanotubes, h is the Planck constant, and ν is the relaxation frequency and $\nu = 0.25 \times 10^{-14}$ Hz for the zig-zag carbon nanotubes.[23]

To further illustrate the point on the variation of the current distribution on a carbon nanotubes dipole, based on Eqs. (6), (7) and (8), it has been calculated by method of moment and plotted in Fig. 3 for $L = \lambda/2$ with a carbon nanotube radius of $R = 0.705$ nm, $R = 2.466$ nm, and $R = 4.932$ nm, respectively.

In Fig. 3, we can see that the current waveforms are approximately a half sinusoid. It is apparent that the changes in the radius of the carbon nanotubes do not influence to a large extent the distribution of the $L = \lambda/2$ antenna, but it has a major impact on the magnitude of current distribution at the feed point, especially for Fig. 3(b).

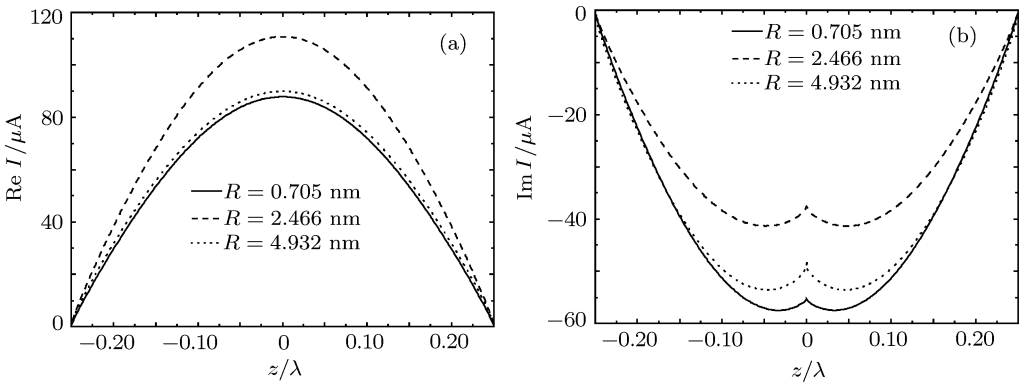


Fig. 3. Current distributions along the zig-zag carbon nanotubes antenna, corresponding to the frequency of $f = 2.4$ THz, the radius of $R = 0.705$ nm (solid line), $R = 2.466$ nm (dashed line), and $R = 4.932$ nm (dotted line). (a) The real part of current, and (b) the imaginary part of current.

Figure 4 depicts the current distribution on the carbon nanotubes antenna having length of $L = 3\lambda/2$ when the frequency $f = 5.0$ THz. As can be seen from Fig. 4(a), the current distributions along the carbon nanotube

are approximately an ideal standing wave pattern with a null at the ends, and the influence of the changes of radius of the carbon nanotubes on the real parts of the current distributions is not obvious, as plotted in Fig. 3(a). However, it has a profound effect on the imaginary of the current distributions of $L = 3\lambda/2$ at the feed point [see Fig. 4(b)]. From Figs. 3, 4, and 5, it can also be found that the current profiles depend on the length of the carbon nanotubes. Therefore, the input impedance of carbon nanotubes antenna is quite different in the cases of Figs. 3 and 4, which will be discussed in the following contents.

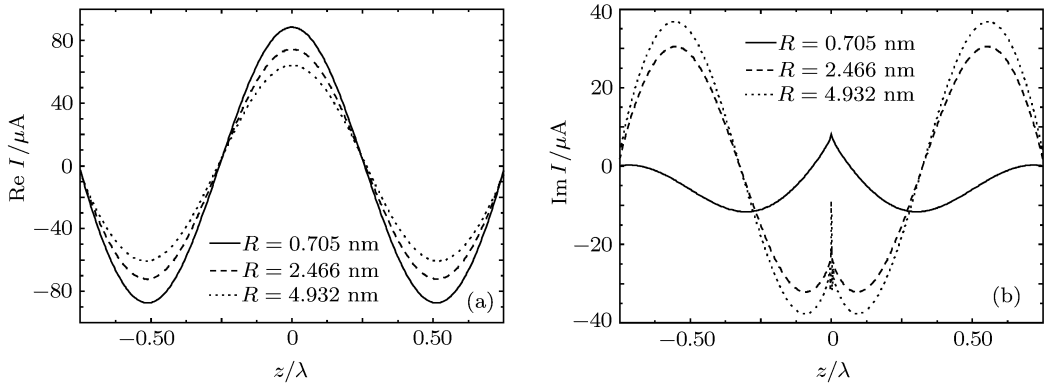


Fig. 4. Current distribution along the zig-zag carbon nanotube antenna, corresponding to $f = 5.0$ THz, the radius is the same as thtat in Fig. 3. (a) The real part of current, and (b) the imaginary part of current.

In order to illustrate the influence of the frequency on the current on the antenna further examples are shown in Fig. 5, where the current patterns of the $L = \lambda$ and $L = 2\lambda$ antenna with a given radius of $R = 2.466$ nm have been compared and the results are given in Figs. 5(a) and 5(b), respectively. Due to the increase of the length of carbon nanotubes, the current distributions oscillate heavily, especially at higher frequencies.

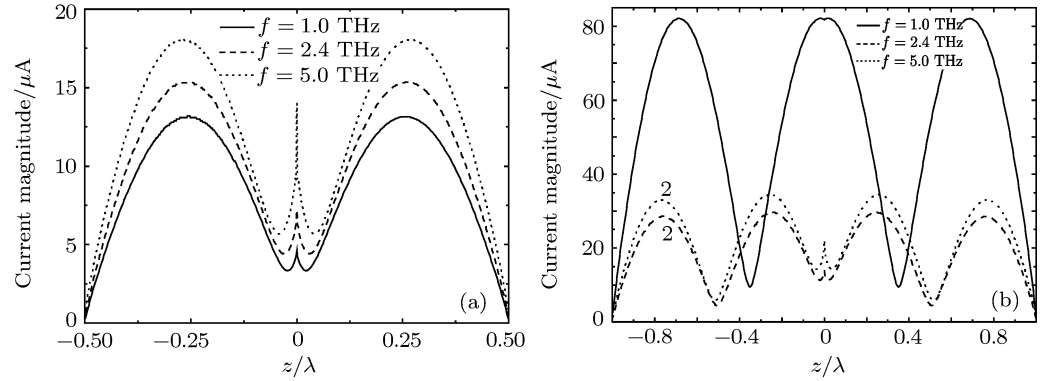


Fig. 5. Comparison of current forms of the different operating frequencies of $f = 1.0$ THz (solid line), $f = 2.4$ THz (dashed line), and $f = 5.0$ THz (dotted line) at $R = 2.466$ nm. (a) $L = \lambda$, and (b) $L = 2\lambda$.

3.2. Self-impedance and mutual impedance characteristics

In practice, the calculation of the input (or self) impedance of each radiator and the mutual impedance of an antenna array need to be considered. For the conventional thin wire antenna, sufficient data have been presented to solve the input impedance and mutual impedance. However, the similar problems of the carbon nanotubes antenna are less investigated. Therefore, it is necessary to study the input and mutual impedances for the carbon nanotubes antenna for better understanding of its characteristics.

Once the unknown current distribution has been obtained from Eq. (6), the characteristics of the carbon nanotubes antenna array, such as input impedance, mutual coupling between elements, and radiation patterns, can be obtained.

The input impedance can be expressed by

$$Z_{\text{in}} = R_{\text{in}} + \mathrm{i}X_{\text{in}}, \tag{9}$$

where R_{in} is the input resistance and X_{in} is the input reactance.

Figure 6 plots the input resistance and reactance as a function of z/λ of the carbon nanotube antenna. It can be seen from Fig. 6(a) that the input resistance is periodic. A main reason is that the current distribution patterns of the carbon nanotubes undergoes 180° phase reversals between adjoining half cycles, which leads to the current in all parts of the nanotubes having the different phase [typically in Fig. 4]. As a result, significant interference and canceling effects are observed in the formation of the impedance pattern. Furthermore, an infinite impedance (but, very large) has been gained since the zero current at the centre of the standing wave distribution.

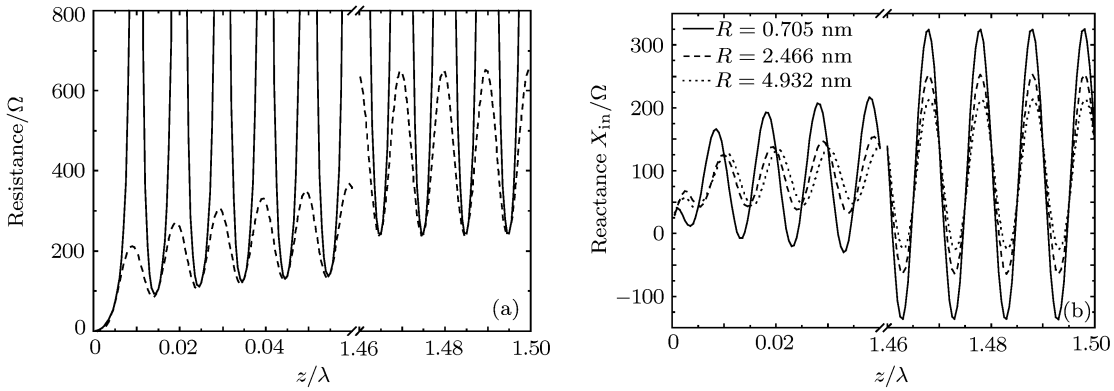


Fig. 6. The input resistance and input reactance as a function of z/λ, (a) the radiation resistance (dashed line) and input resistance (solid line) at $R = 2.466$ nm, (b) the input reactance at $f = 5$ THz with different radius of $R = 0.705$ nm (solid line), $R = 2.466$ nm (dashed line), and $R = 4.932$ nm (dotted line).

It is worth while pointing out in summary that the radius has a little effect on the input resistance because the radius of the carbon nanotubes is very little ($R \ll L$) In order to illustrate the influence of the radius on the reactance, its values have been calculated and plotted in Fig. 6(b) for the different radius at frequency of $f = 5$ THz. The values of the reactance of the zig-zag carbon nanotubes elucidate the difference of the ability of the power stored in near field of the antenna. However, though the radius is different, it is evident that the reactance will be focused on the same point at the fixed cycle.

In our discussions on the mutual impedance of the zig-zag carbon nanotubes line array, it can be seen that array parameter (the distance d between array elements in Fig. 2) must be considered in order to analyse its influence on the mutual coupling. The bottom-left part of Fig. 7(a) shows the mutual impedance values at $d/\lambda = 0.2 - 0.7$, and the top-right part shows the values at $d/\lambda = 0.7$–1.5. It is apparent that the interference effect is contributed from the different parts of the array. At $d/\lambda > 1.5$, considering different nanotubes lengths, we have calculated the mutual impedance values, and the similar phenomenon can be gained.

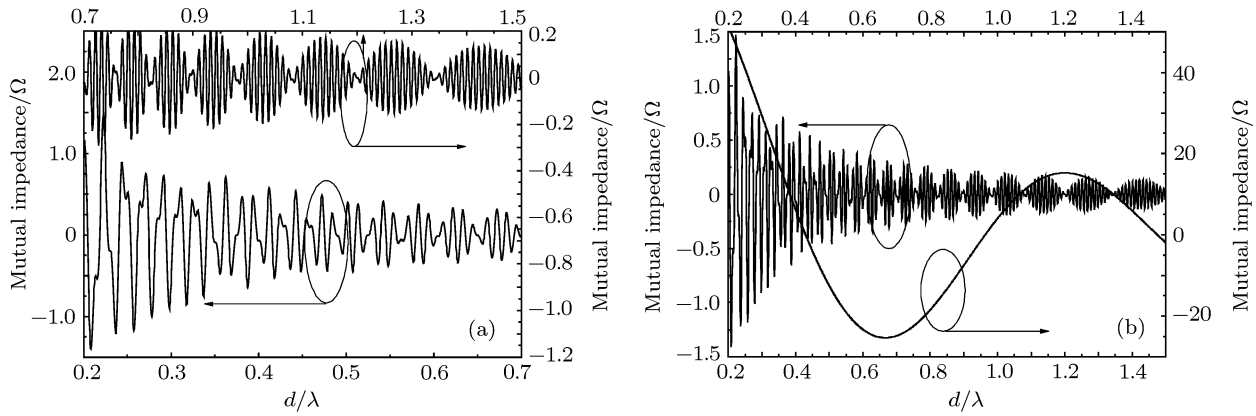


Fig. 7. (a) The mutual impedance in the THz range for $L = \lambda/2$, $R = 2.466$ nm zig-zag carbon nanotubes antenna elements. (b) The comparisons of mutual impedance values between zig-zag carbon nanotubes antenna (left arrowhead) and the conventional metal antenna (right arrowhead).

In Fig. 7(b), it is shown that the mutual impedance values for the carbon nanotubes array will have more variation than a conventional compared-sized metal array. Also, it is shown that the mutual impedance values of the carbon nanotubes array oscillate periodically and are less than the conventional array in the condition of the same distance between antenna elements.

For the practical configurations of the carbon nanotubes antenna, it is necessary to point out that no matter the input impedance or the mutual impedance that are difficult to predict analytically but must be taken into account due to their significant contributions.

3.3. Radiation characteristics

Radiation patterns describe the relative strength of the radiation field in various directions from the antenna. The field patterns of antenna can be obtained from the following expression:

$$E_\theta = \mathrm{i}\frac{30k_0\mathrm{e}^{-\mathrm{i}k_0 r}}{r}\int_{-L/2}^{+L/2} I_z(z')\mathrm{e}^{\mathrm{i}k_0 z\cos\theta}\mathrm{d}z'. \quad (10)$$

For the single carbon nanotubes dipole antenna, its field radiation pattern can be calculated and plotted in Fig. 8(a). It is apparent that its radiation pattern is isotropic and its maximum gain is 0.598 dB. In order to increase gain of carbon nanotubes antenna, the array structure will be taken into account. In the following contents, considering the array factor form as:

$$F = \sum_{n=0}^{N-1} \mathrm{e}^{\mathrm{i}n(k_0 \mathrm{d}\cos\theta+\beta)}, \quad (11)$$

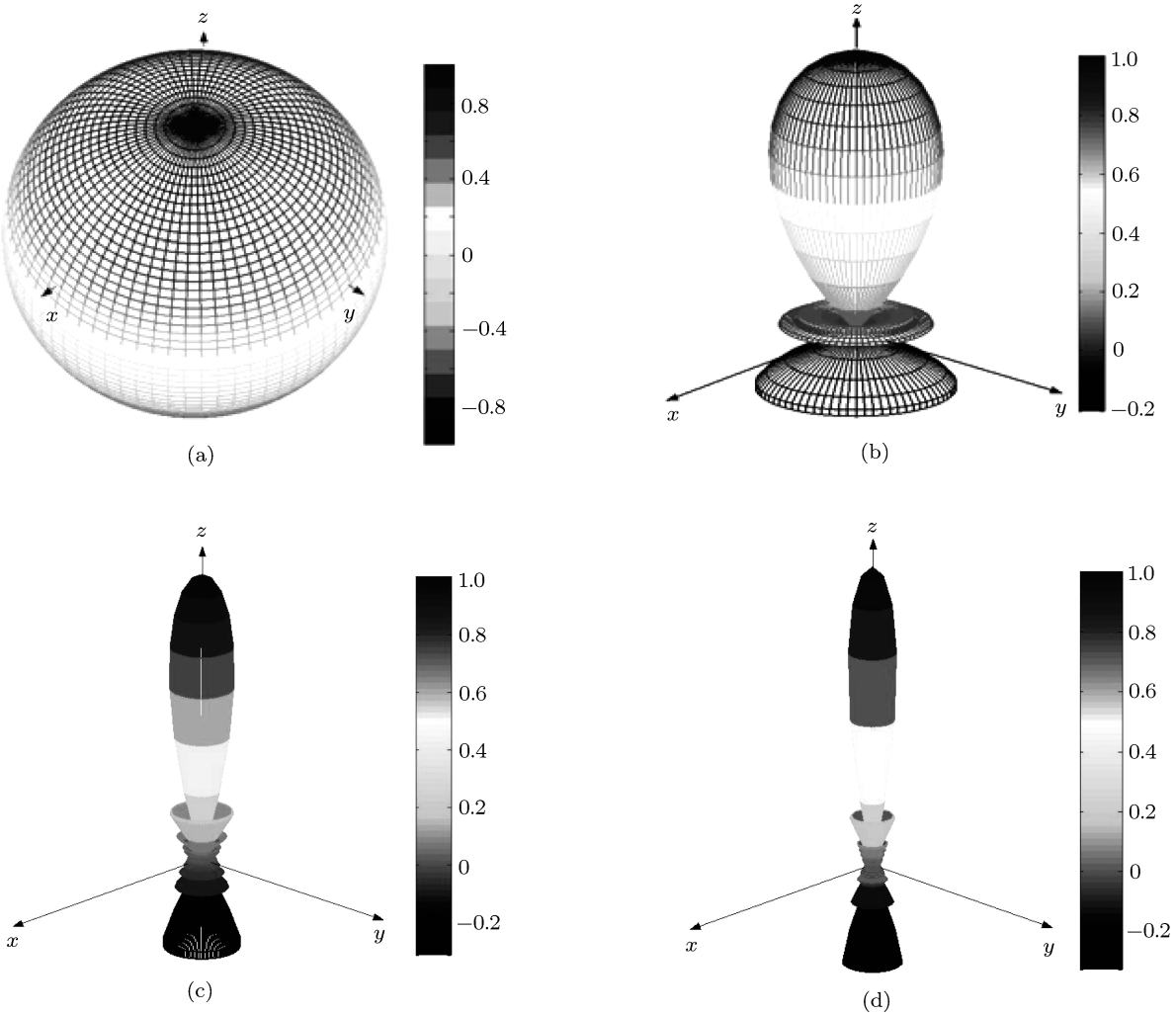


Fig. 8. Normalized three-dimensional radiation patterns of zig-zag carbon nanotubes antenna at $f = 2.4$ THz, $R = 2.466$ nm. (a) Single carbon nanotubes antenna; (b) 10 elements array antenna; (c) 50 elements array antenna and (d) 100 elements array antenna.

where F is the amplitude, N is the number of antenna element, and β is the phase between two elements, we have discussed the radiation patterns of antenna array shown in Fig. 2. The following figures, at given frequency of $f = 2.4$ THz and radius of $R = 2.466$ nm, show the typical radiation patterns of carbon nanotubes liner array of 10 elements in Fig. 8(b), 50 elements in Fig. 8(c), and 100 elements in Fig. 8(d), respectively. It is obvious that the directionality of an array can be improved notably with the increase of carbon nanotubes element. The gain of the array of different elements as a function of N is shown in Fig. 9.

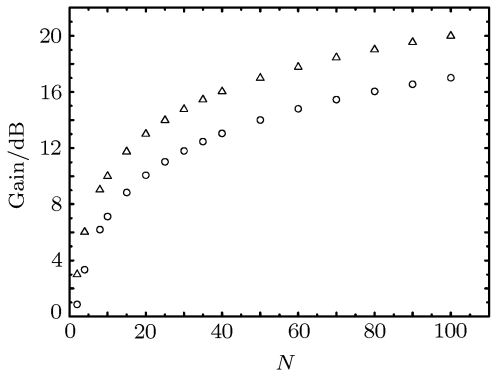


Fig. 9. Gain of antenna array at different elements of N, ($d = 0.5\lambda$ with symbol $\triangle$, $d = 0.25\lambda$ with symbol $\circ$).

4. Conclusions

In summary, the current distributions, input impedances and mutual impedance, radiation patterns of the zig-zag carbon nanotubes antenna at THz band are investigated by using the Pocklinton's integral function. The numerical results demonstrated that the carbon nanotubes antenna can operate at THz wave range. It is also found that, due to the slower wave velocity on the carbon nanotubes and the conductivity properties of the zig-zag carbon nanotubes, the carbon nanotubes antenna exhibits very high input impedance in the THz frequency that will be beneficial to match to high impedance devices, but low radiation resistance, which will lead to low gain and efficiency. On the other hand, another significant conclusion of this paper is, for the first time, that the mutual impedance of the carbon nanotubes antenna exhibits oscillation periodically and much different from the conventional metal thin wire antenna. It is obvious that the directionality of an array can be improved notably with the increase of carbon nanotubes element. Other new properties of the zig-zag carbon nanotubes antenna still need to be found for the future work.

References

[1] Iijima S and Ichihashi T 1993 *Nature* **363** 603
[2] Bethune D S, Klang C H, de Vries M S, Gorman G, Savoy R, Vazquez J and Beyers R 1993 *Nature* **363** 605
[3] O'Connell M J 2006 *Carbon Nanotubes Properties and Applications* (London: CRC Press)
[4] Gael F C, Shinichi Y and Bipul P 2008 *Nano Lett.* **8** 706
[5] Yuan Y H, Miao R C, Bai J T and Hou X 2006 *Chin. Phys.* **15** 2761
[6] Heinze S, Tersoff J, Martel R, Derycke V, Appenzeller J and Ph. Avouris 2002 *Phys. Rev. Lett.* **89** 106801
[7] Patil N, Deng J and Mitra S 2009 *IEEE Trans. Nanotech.* **149** 150
[8] Kong J, Franklin N R, Zhou C W, Chapline M G, Peng S, Cho K J and Dai H J 2000 *Science* **287** 622
[9] Dresselhaus M S 2004 *Nature* **432** 959
[10] Wang Y, Kempa K, Kimball B, Carlson J B, Benham G, Li W Z, Kempa T, Rybczynski J, Herczynski A and Ren Z F 2004 *Appl. Phys. Lett.* **85** 2607
[11] Kempa K, Rybczynski J, Huang Z P, Gregorczyk K, Vidan A, Kimball B, Carlson J, Benham G, Wang Y, Herczynski A and Ren Z F 2007 *Adv. Mater.* **19** 421
[12] Jin H and Hanson G W 2006 *IEEE Trans. Nanotech.* **5** 766
[13] Hanson G W 2006 *IEEE Trans. Antennas Propagat.* **54** 76
[14] Fan W H, Liu H L, Zhang T Y and Zhu S L 2009 *Acta Phys. Sin.* **58** 3658 (in Chinese)
[15] Wang C and Zhang Y H 2006 *Chin. Phys.* **15** 649
[16] Shi W, Qu G H and Wang X M 2009 *Acta Phys. Sin.* **58** 477 (in Chinese)
[17] Zhang T Y and Zhao W 2008 *Chin. Phys.* B **17** 4285
[18] Shi W, Qu G H, Xu M, Ji W L, Xue H, Zhang L Q and Tian L Q 2009 *Appl. Phys. Lett.* **92** 072110
[19] Wang Y, Wu Q, He X J, Zhang S Q and Zhuang L L 2009 *Chin. Phys.* B **18** 1801
[20] Wilton D R and Butler C M 1976 *IEEE Trans. Antennas. Propagat.* **1** 84
[21] Burke P J, Li S and Yu Z 2006 *IEEE Trans. Nanotech.* **5** 314
[22] Burke P J, Spielman I B, Eisenstein J P, Pfeiffer L N and West K W 2000 *Appl. Phys. Lett.* **76** 745
[23] Slepyan G Ya, Maksimenko S A, Lakhtakia A, Yevtusheno O and Gusakov A V 1999 *Phys. Rev.* B **60** 17136