# Phase escape of current-biased Josephson junctions*

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Switching current distributions of an Nb/Al–$AlO_x$/Nb Josephson junction are measured in a temperature range from 25 mK to 800 mK. We analyse the phase escape properties by using the theory of Larkin and Ovchinnikov (LO) which takes discrete energy levels into account. Our results show that the phase escape can be well described by the LO approach for temperatures near and below the crossover from thermal activation to macroscopic quantum tunneling. These results are helpful for further study of macroscopic quantum phenomena in Josephson junctions where discrete energy levels need to be considered.

**Keywords:** Nb junctions, phase escapes, macroscopic quantum phenomena, discrete energy levels
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## 1. Introduction

Macroscopic quantum phenomena in Josephson junctions and circuits have attracted much attention for nearly three decades,[1−4] and are becoming more interesting due to the possible superconducting qubit applications.[5] The simplest device in these studies is the single Josephson junction biased at certain current $I$. Such a device can be modeled by phase particle $\varphi$ with mass $M = C(\Phi_0/2\pi)^2$ and dissipative viscous force $1/RC$, which moves in a washboard potential $U(i,\varphi) = -E_J(\mathrm{i}\,\varphi + \cos\varphi)$. Here $\Phi_0 = h/2e$ is flux quantum, $E_J = I_\mathrm{c}\Phi_0/2\pi$, and $i \equiv I/I_\mathrm{c}$. $C$ and $R$ are the capacitance and effective resistance of the junction, respectively. For $i < 1$, the phase particle may escape (phase escape) from the potential well via thermal activation (TA) at high temperature $T$ with the rate[6]

$$\Gamma_\mathrm{t} = \frac{\omega_\mathrm{p}}{2\pi} a_\mathrm{t} \exp\left(-\frac{\Delta U}{k_\mathrm{B}T}\right), \tag{1}$$

where $a_\mathrm{t} = 4/(\sqrt{1 + Qk_\mathrm{B}T/1.8\Delta U}+1)^2$ is a damping-dependent factor, and via macroscopic quantum tunneling (MQT) at low $T$ with the rate[3]

$$\Gamma_\mathrm{q} = \frac{\omega_\mathrm{p}}{2\pi} a_\mathrm{q} \exp\left[-\frac{7.2\Delta U}{\hbar\omega_\mathrm{p}}\left(1 + \frac{0.87}{Q}\right)\right], \tag{2}$$

where $a_\mathrm{q} \simeq [120\pi(7.2\Delta U/\hbar\omega_\mathrm{p})]^{1/2}$. In the above expressions, $\omega_\mathrm{p} = \sqrt{2\pi I_\mathrm{c}/C\Phi_0}(1 - i^2)^{1/4}$, $Q = \omega_\mathrm{p}RC$, and $\Delta U(i) = 2E_J[\sqrt{1 - i^2} - i\arccos(i)]$ are the plasma frequency, the quality factor, and the barrier height, respectively.

Phase escape plays a central role in many of the fundamental studies and of the superconducting qubit applications. When dissipation is low, it leads directly to junctions switching from the zero-voltage state to the finite-voltage state that results from the particle running in the washboard potential. With the increase of dissipation, the running particle may be retrapped into one of the potential wells so that a process called phase diffusion occurs. The rich physics involved herein has made the current-biased junctions an interesting system for the study of phenomena such as quantum dissipation.[3,4] On the other hand, an understanding of the phase escape process, especially at low temperatures where MQT dominates, is important in some qubit applications. For example, the macroscopic quantum states of a flux qubit are usually detected from the switching of a dc-SQUID, which often has a small loop inductance and is therefore identical to a current biased junction.

Equation (2), used widely for fitting the experimental results, describes the MQT rate of the phase particle in the ground state and at $T = 0$, by also taking into account the influence of dissipation.[3] Consideration of the excited states for $T \geq 0$ was discussed by Larkin and Ovchinnikov (LO)[7] and other authors.[8,9]

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The LO theory with considering discrete energy levels is useful in analysing resonant activation phenomena, etc.[9] and is sometimes necessary to explain phase escape results when retrapping is involved.[10] In this work, we present experimental data measured from an Nb/Al–$AlO_x$/Nb Josephson junction in a temperature range from 25 mK to 800 mK. We discuss the phase escape rate by the LO theory[7] and compare the results with those obtained from other theories.[3,4]

## 2. Experimental results

For the junctions with low or moderate damping, phase escape leads directly to their switching from zero to the finite voltage state. In our experiment, we measured switching current distribution $P(I)$ by using the time-of-flight technique as described elsewhere.[11−13] The bias current was ramped up at a constant rate of $\mathrm{d}I/\mathrm{d}t = 0.4$ mA/s with a repetition rate of 97 Hz. In Fig. 1, we show the sample's $I$–$V$ curve at 25 mK and the current profile for the measurement. The results of $P(I)$ at some typical temperatures between 25 mK and 800 mK are shown in the inset of Fig. 2.

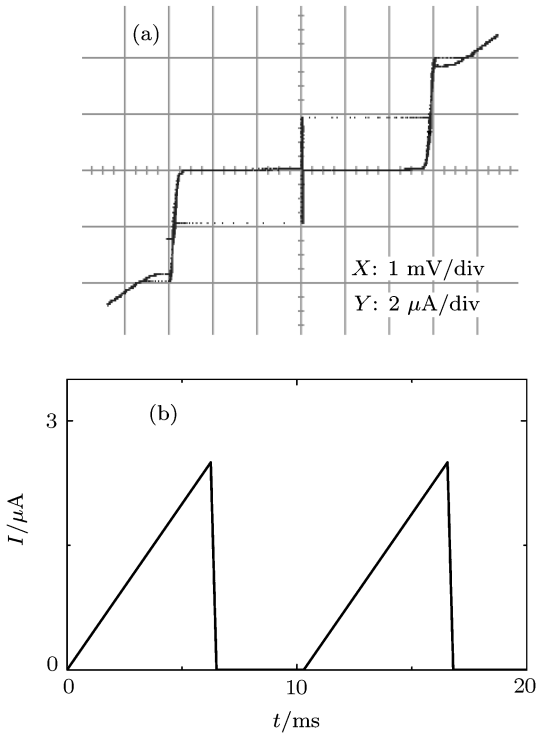



**Fig. 1.** (a) The $I$–$V$ curve of an Nb/Al–$AlO_x$/Nb junction at 25 mK. (b) Sweeping current profile used for the $P(I)$ measurement.

The open symbols in Figs. 2(a) and 2(b) show the temperature dependences of mean ($\langle I_{\rm sw}\rangle$) and width ($\sigma$) of the measured $P(I)$ respectively. The dotted lines and the solid lines represent the results calculated from the TA [Eq. (1)] and MQT [Eq. (2)] theories, respectively, by using the standard conversion between $\Gamma_{\rm t,q}$ and $P(I)$.[14] In fitting to the experimental data, the sample's parameters $I_{\rm c} = 1.957$ $\mu$A, $C = 620$ fF, and $R = 300$ $\Omega$ are determined. Taking the $P(I)$ peak position at 25 mK in the inset of Fig. 2 ($i \simeq 0.953$), we find that the TA-to-MQT crossover temperature $T_{\rm cr} = \hbar\omega_{\rm p}[(1 + 1/4Q^2)^{1/2} - 1/2Q]/2\pi k_{\rm B}$ calculated from these parameters is 62.4 mK, which is in fair agreement with the experimental value of about 65 mK as indicated in Fig. 2(b).

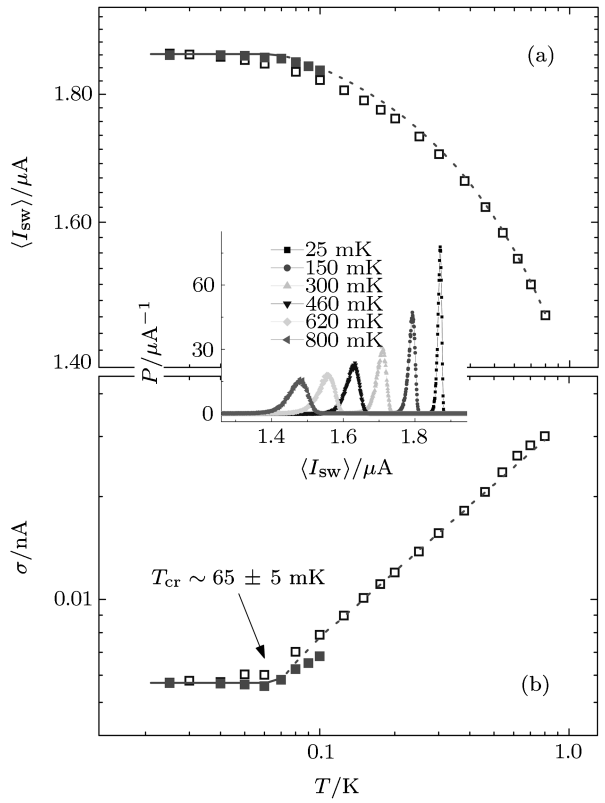



**Fig. 2.** Experimental results (open symbols) of the mean (a) and width (b) of $P(I)$ (inset). Calculated results are shown as dotted (TA theory) and solid (MQT theory) lines and solid symbols (LO theory) using the parameters discussed in the text. Experimental $T_{\rm cr}$ is indicated in panel (b).

## 3. Phase escape in LO theory

In the LO theory, discrete energy levels $E_j$ ($j = 0, 1, \ldots, n$) are considered and the level occupation $\rho_j$ satisfies the master equation[7−9]

$$\frac{\mathrm{d}\rho_j}{\mathrm{d}t} = \sum_{i=0}^{n}[\gamma_{i,j}\rho_i - \gamma_{j,i}\rho_j] - \Gamma_j\rho_j, \qquad (3)$$

in which $\Gamma_j$ is the level tunneling rate and $\gamma_{i,j}$ is the transition rate between two different levels. When

only the transition between the neighbouring levels is considered, the above equation can be written in the matrix form as follows:

$$\frac{d\boldsymbol{\rho}}{\mathrm{d}t} = \boldsymbol{M}\boldsymbol{\rho}, \tag{4}$$

where

$$\boldsymbol{\rho} = (\rho_0 \ \rho_1 \ \cdots \ \rho_n)^{\mathrm{T}} \tag{5}$$

and

$$\boldsymbol{M} = \begin{pmatrix} -d_0 & \gamma_{10} & & & \\ \gamma_{01} & -d_1 & & & \\ & \ddots & & & \\ & & & -d_{n-1} & \gamma_{n,n-1} \\ & & & \gamma_{n-1,n} & -d_n \end{pmatrix}, \tag{6}$$

in which

$$\begin{cases} d_0 = \gamma_{01} + \Gamma_0, \\ d_j = \gamma_{j,j-1} + \gamma_{j,j+1} + \Gamma_j, \\ d_n = \gamma_{n,n-1} + \Gamma_n. \end{cases} \tag{7}$$

The stationary solutions of Eq. (4) can be easily obtained by using known parameters $E_j$, $\Gamma_j$, and $\gamma_{i,j}$, which can be calculated as described in Ref. [8]. If the temperature is not too high, we have $\rho_j/\rho_0 \ll 1$. In this case, the total escape rate of the phase particle is

$$\Gamma_{\mathrm{LO}} = \frac{\Gamma_0 + \sum_{j=1}^{n} \Gamma_j(\rho_j/\rho_0)}{1 + \sum_{i=1}^{n}(\rho_j/\rho_0)}. \tag{8}$$

The solid symbols in Fig. 2 represent the calculated results by using parameters $I_{\mathrm{c}} = 1.98\ \mu$A, $C = 574$ fF, and $R = 300\ \Omega$. The results indicate that the LO theory gives a satisfactory fit to the experimental data for temperatures near and below $T_{\mathrm{cr}}$.

The slight changes in $I_{\mathrm{c}}$ and $C$ compared with those used in Eq. (2) arise from the fact that dissipation is not considered for $\Gamma_j$ in the LO theory.[7,8] Dissipation is known to have subtle influence on $\Gamma_j$ as can be seen in Eq. (2).[3,8] But analogous results considering the dissipation for the excited states are presently not available although equation (2) was generalized for the ground state to $T > 0$.[4]

One advantage of the LO treatment is that it provides an interesting quantum picture for particle escape near the crossover temperature $T_{\mathrm{cr}}$. In Fig. 3(a), we plot the energy levels at $T = 65$ mK from our calculation. The dependences of the product of occupation $\rho_j$ and tunneling rate $\Gamma_j$ on level for three different temperatures near $T = 65$ mK are shown in Fig. 3(b). It can be seen that at 50 mK, the majority of the particles still escape from the ground state at $E_0$, whereas at 80 mK, particle escape at excited levels becomes dominant. At $T_{\mathrm{cr}} \sim 65$ mK, particle escapes at different energy levels are nearly equal.

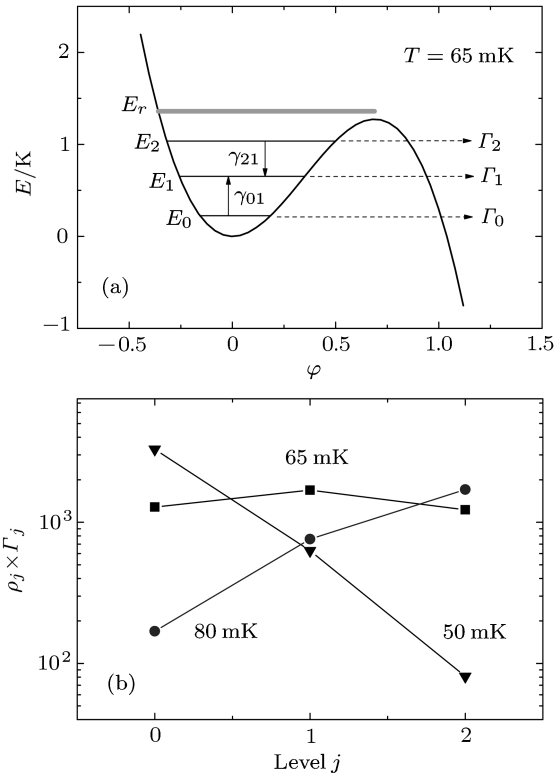



**Fig. 3.** (a) Calculated energy levels by using bias $I$ corresponding to the peak position of $P(I)$ at $T = 65$ mK and (b) dependences of product of level occupation $\rho_j$ and tunneling rate $\Gamma_j$ on energy-level $j$ for three different temperatures near $T_{\mathrm{cr}}$.

This gives an example illustrating how the particle escapes cross from the MQT regime to the TA regime, where the escapes occur almost entirely at levels near the barrier top. From Fig. 2(b), we can see that there is a slight deviation in $\sigma$ from the experimental data and from the TA prediction at the two highest temperatures, which is caused by increasingly unsatisfied condition $\rho_j/\rho_0 \ll 1$ required in the calculation when the temperature is far above $T_{\mathrm{cr}}$.

# 4. Conclusion

Switching current distributions of an Nb junction are measured from 25 mK to 800 mK. Phase escape properties are analysed by using LO theory that takes discrete energy levels into account. Our results show that phase escape can be well described by the LO approach for temperatures near and below $T_{\mathrm{cr}}$. These results are useful for future study of macroscopic quantum phenomena in Josephson junctions where discrete energy levels need to be considered.

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