The interaction of in-band and in-gap lattice soliton trains in optically induced two-dimensional photonic lattices*

Liu Sheng(刘 圣), Zhang Peng(张 鹏), Xiao Fa-Jun(肖发俊),
Gan Xue-Tao(甘雪涛), and Zhao Jian-Lin(赵建林)†

Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University, Xi'an 710072, *China*
The Key Laboratory of Space Applied Physics and Chemistry, Ministry of Education, Xi'an 710072, *China*

(Received 16 October 2009; revised manuscript received 7 November 2009)

We demonstrate the coherent interactions of lattice soliton trains, including in-band solitons (IBSs) and gap soliton trains (GSTs), in optically induced two-dimensional photonic lattices with self-defocusing nonlinearity. It is revealed that the π-staggered phase structures of the lattice soliton trains will lead to anomalous interactions. Solely by changing their initial separations, the transition between attractive and repulsive interaction forces or reversion of the energy transfer can be obtained. The 'negative refraction' effect of the soliton trains on the interaction is also discussed. Moreover, two interacting IBSs can merge into one GST when attraction or energy transfer happens.

Keywords: soliton interaction, in-band soliton, gap soliton train, photonic lattice
PACC: 5235M, 4265S, 4270Q

1. Introduction

The solitary wave, as a ubiquitous phenomenon in nature, has recently attracted much interest in many periodic systems, such as Bose–Einstein condensates in optical lattices,[1] photonic crystals[2] and photonic lattices (waveguide arrays).[3−23] In periodic optical media with either self-focusing or self-defocusing nonlinearity, spatial optical solitons arise from a balance between discrete diffraction and nonlinearity.[8−18] Especially in two-dimensional (2D) photonic lattices, discrete solitons[12−14] (with their propagation constant residing in the semi-infinite band gap) and gap solitons[14−16] (in the Bragg reflection gap) have been widely studied, both in theory and experiment. Furthermore, a new type of soliton named 'in-band soliton' (IBS), whose propagation constant resides in the transmission band, is also confirmed to exist in optically induced photonic lattices.[17,18] Such IBSs possess the characteristic structure of soliton trains, similar to discrete[13] and gap soliton trains (GSTs).[16] One of the most intriguing phenomena revealed in solitons is soliton interactions,[19−27] which have been very well understood, both in bulk media and discrete systems, with the motivation of manipulating light by light itself. It is believed that two mutually coherent solitons attract (or repulse) each other when they are in phase (or out of phase), whereas incoherent soliton pairs are always attractive. Thus a soliton can be controlled by changing the phase of another soliton. In this paper, we demonstrate for the first time the anomalous coherent interactions of lattice soliton trains (including IBSs and GSTs) in an optically induced photonic lattice with self-defocusing nonlinearity.

2. Lattice soliton trains

The nonlinear propagation of the light wave with slowly varying amplitude in optically induced photonic lattices in a photorefractive crystal can be described by the dimensionless model[28]

$$\left(\frac{\partial}{\partial z}-\frac{\mathrm{i}}{2}\nabla_{\perp}^{2}\right)B=\mathrm{i}E_{0}\left(\frac{I_{\mathrm{l}}+|B|^{2}}{1+I_{\mathrm{l}}+|B|^{2}}\right)B, \quad (1)$$

where the light wave propagates along the z axis, E_0 is the normalized externally biased field, B is

*Project supported by the Northwestern Polytechnical University (NPU) Foundation for Fundamental Research and the Doctorate Foundation of NPU (Grant No. CX200914).

†Corresponding author. E-mail: jlzhao@nwpu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

the envelope function of the optical field of the extraordinarily polarized probe beam, and $I_{\rm l} = 0.5\cos^2(x/d)\cos^2(\pi y/d)$ is the intensity distribution of the ordinarily polarized lattice beam with period d. In this dimensionless equation, the normalized parameters for the dimensionless coordinates (x, z) and E_0 are $x_0 = 4$ m, $z_0 = 0.45$ mm and $E_{00} = 100$ V/mm, respectively.

Under the condition of self-defocusing nonlinearity with bias field $E_0 = -2$, a backbone lattice structure is established by the lattice beam shown in the insert of Fig. 1(a). In this case, lattice soliton trains bifurcating from the first band of the transmission spectrum [shown in Fig. 1(a)] originate from the high symmetric points X_1 and M_1. By selecting the period $d = 8$, the exact solutions of the soliton trains can be obtained in the form of $B = b(x, y)\exp(\mathrm{i}\beta z)$, where β is the propagation constant, and then the real envelope $b(x, y)$ satisfies the following equation:

$$\beta b - \frac{1}{2}\nabla_\perp^2 b = E_0 \frac{I_{\rm l} + |b|^2}{1 + I_{\rm l} + |b|^2} b. \tag{2}$$

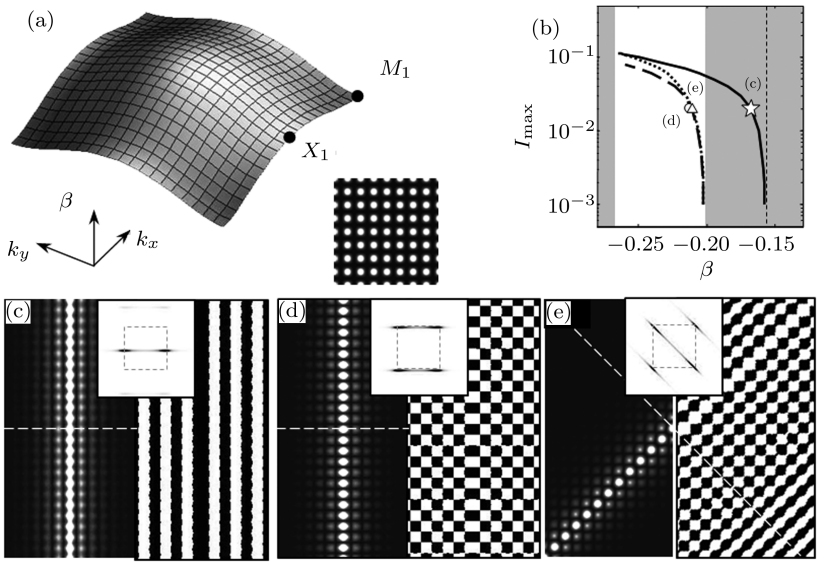


Fig. 1. Self-defocusing lattice soliton trains in an optically induced photonic lattice, showing (a) the lattice beam (insert) and the first transmission band; (b) peak intensity curves of the lattice soliton trains versus the propagation constant, where the transmission bands (shaded) and the position of point X_1 are marked by a vertical line; (c) in-band soliton (IBS) at point X_1; (d) and (e) gap soliton trains (GSTs) at point M_1.

Utilizing the self-consistent iteration method,[29] we find three types of lattice soliton trains as shown in Figs. 1(c)–1(e), in each of which the left, right and inserts depict the intensity profiles, phase structures and spectra of the solitons, respectively. The families of these soliton trains are shown in Fig. 1(b) which depicts the plots of peak intensity $I_{\max}$ versus the propagation constant with the transmission bands shaded. The vertical dash line marks the location of point X_1 in the first band, and the solid, dash and the dot curves depict the solitons shown in Figs. 1(c)–1(e), respectively. Figure 1(c) represents a typical IBS marked with an open star in Fig. 1(b) bifurcating from the interior point X_1, whose phase structure is uniform along the vertical direction but is staggered along the horizontal direction, resembling the character of the Bloch mode of point X_1. Two other soliton trains shown in Figs. 1(d) and 1(e) correspond to GSTs [marked with an open triangle and open circle in Fig. 1(b)] bifurcating from point M_1, with the strip orientations along vertical and diagonal directions, respectively. It should be noted that the phase structures of these two soliton trains are both out-of-phase along the vertical and horizontal directions, matching the Bloch mode of point M_1. In the following, we will demonstrate that the staggered

phase structures play nontrivial roles in the anomalous interactions of IBSs and GSTs.

3. Interactions of lattice soliton trains

For coherent IBS interactions, two identical solutions B_1 and B_2 as shown in Fig. 1(c) are utilized as input and perpendicularly launched into different lattice sites. In order to observe stronger interactions, we select the IBSs with wider distributions at a lower power level ($I_{\max} = 0.02$). The evolutions of B_1 and B_2 are numerically simulated in a normalized propagation distance $z = 300$, and the transverse patterns of the interaction processes along the white dash line in Fig. 1(c) are depicted in Fig. 2, where the separations between B_1 and B_2 (denoted as D) in Figs. 2(a)–2(d) are 8, 16, 24, and 32, respectively, and the phase differences between B_1 and B_2 in the panels from top to bottom are $\Delta\phi = 0$, $\pi/2$, and π, respectively. It can be seen that the two IBSs interact as the discrete solitons with uniform phases do[19−21] when their separation has an even number of lattice spaces [see Figs. 2(b) and 2(d)]. However, if the separation has an odd number of lattice spaces [see Figs. 2(a) and 2(c)], the in-phase (out-of-phase) IBSs will repel (attract) each other, representing anomalous interactions. Thus, a transition between attractive and repulsive interaction forces can be obtained solely by changing the initial separation as well as the phase difference. Furthermore, when the phase difference $\Delta\phi = \pi/2$, energy transfer accompanies the interactions, and the transfer direction is alternatively reversed as the value of D changes. Such an anomalous interaction can be explained by the staggered phase structure of IBS. For instance, an in-phase IBS pair separated by an odd number of lattice spaces will exhibit out-of-phase property in their overlapping region, leading to destructive superposition and the reduction of self-defocusing. Therefore, the IBS pair repulse each other. As is well known, for two discrete solitons B_1 and B_2, when the phase of B_2 is $\pi/2$ higher than that of B_1, the energy will transfer from B_1 to B_2.[19,21,22] However, in the central row of Figs. 2(b) and 2(d) corresponding to normal interactions, their initial energy transfer directions are opposite. This surprising outcome should be attributed to the 'negative refraction' effect in the photonic lattice.[6,7] When the phase structure of a tilted input beam matches with the Bloch modes of the minimal value points [such as points X_1 and M_1 in Fig. 1(b)] in the first band, the beam will deflect in the opposite direction with respect to the normal in the incident plane.[7]

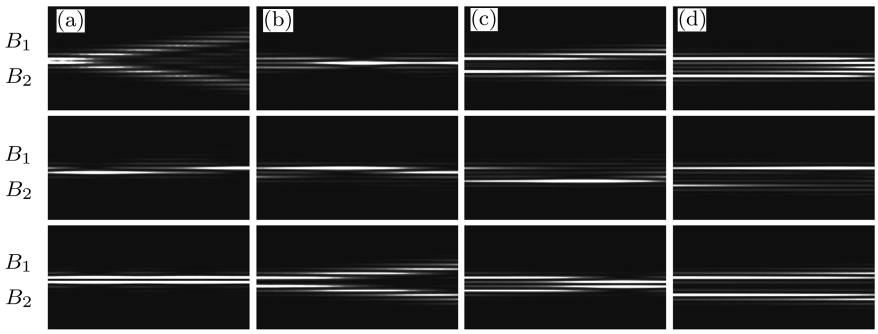


Fig. 2. Coherent interactions between IBSs. In panels (a)–(d) the separations between B_1 and B_2 are 8, 16, 24 and 32, respectively, and phase differences from top to bottom are 0, $\pi/2$ and π.

Figure 3(a) illustrates a confirmation of 'negative refraction' by launching a tilted IBS into a photonic lattice. As a result, the initial propagation direction of IBS in Fig. 2 is opposite to the driving (attractive or repulsive) force. Nevertheless, the beam will be pulled back to the 'correct direction' by the driving force after some propagation distance [see the central row of Fig. 2]. Actually, 'negative refraction' is always existent during the IBS interactions, as demonstrated in Figs. 3(b)–3(d), where an in-phase IBS pair with a peak intensity of $I_{\max} = 0.05$ and a separation of $D = 8$ have been used. Due to the anomalous interaction, the two IBSs repel and depart from each other in opposite directions [Fig. 3(b)]. However, in the interaction process the two components B_1 and B_2 of the IBS pair exhibit counter-propagation (just like attraction), approach first, and then depart after passing through each other. This process clearly demonstrates the 'negative refraction' effect.

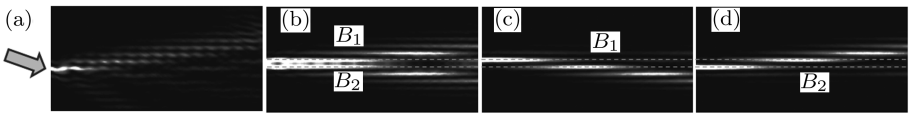


Fig. 3. (a) 'Negative refraction' of a tilted IBS, (b) repulsion of two in-phase IBSs, (c) and (d) propagations of the components of the IBS pair during the interaction in (b).

As demonstrated in Ref. [17], the lattice soliton bifurcating from point X_1 can also form a GST, whose propagation constant resides in the full forbidden gap of the photonic lattice, when its power is high enough. This type of GST possesses the same intensity and phase distributions as those of IBS. However, the difference between the GST and the IBS can be distinguished from their Fourier spectra. For the GST spectrum, the two spots at point X tend to be more concentrated to the centre of the Brillouin zone than that for the IBS. During the IBS interactions, when attractions or energy transfers occur, two IBSs will merge into one soliton, of which the power is higher than that of a single IBS. Thus, the IBS pair may turn into a GST. Figure 4 illustrates the intensities (left) and spectra (right) of the input [Figs. 4(a) and 4(c)] and output solitons [Figs. 4(b) and 4(d)] during the IBS interaction, where the top and the bottom of Fig. 4 correspond to the middle of Fig. 2(a) and the top of Fig. 2(b), respectively. According to the spectrum displayed in the right of Figs. 4(b) and 4(d), it is clear that the IBS pair in the middle of Fig. 2(a) experiences energy transfer and finally evolves into a GST, and the IBSs in the top of Fig. 2(b) will also fuse into a GST because of the attractive force.

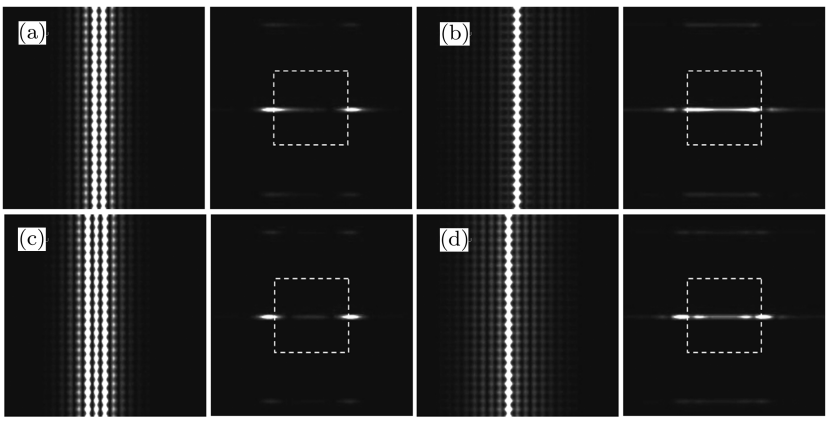


Fig. 4. Transition from IBS pair to GST. Figures (a) and (b) are the input and output for the middle of Fig. 2(a). Figures (c) and (d) are the input and output for the top of Fig. 2(b). The left panel is the intensity pattern, and the right panel is the spectrum.

Our simulation demonstrates that the coherent interactions of GSTs are similar to those of IBSs, which can be easily understood from the analogous phase structures. As an example, the in-phase GST interactions are demonstrated in Fig. 5, which depicts the transverse propagation patterns of the GST pairs ($I_{\max} = 0.02$) shown in Fig. 1(d) [see the top of Fig. 5] and 1(e) [see the bottom of Fig. 5]. And the separations between B_1 and B_2 in the top (bottom) parts of Figs. 5(a)–5(d) are $D = 8$ ($8\sqrt{2}$), 16 ($16\sqrt{2}$), 24 ($24\sqrt{2}$), and 32 ($32\sqrt{2}$), respectively. It is obvious that the two in-phase GSTs repel (attract) each other when they are separated by an odd (even) number of lattice periods, and represent anomalous (normal) interactions. Meanwhile, 'negative refraction' also exists during the GST interactions.

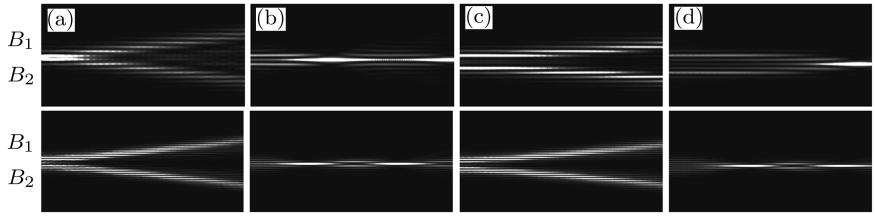


Fig. 5. Coherent interactions between in-phase GSTs. The top panel is the interactions between GSTs in Fig. 1(d), where figures (a)–(d) correspond to $D = 8$, 16, 24, and 32, respectively. The bottom panel is the interactions between GST in Fig. 1(e), where figures (a)–(d) correspond to $D = 8\sqrt{2}, 16\sqrt{2}$, $24\sqrt{2}$, and $32\sqrt{2}$, respectively.

4. Summary

The coherent interactions of the lattice soliton trains (including IBSs and GSTs) in optically induced 2D photonic lattices with self-defocusing nonlinearity are numerically studied. It is demonstrated that due to the staggered phase structure, the interaction processes of lattice soliton trains will experience anomalous dynamics when the soliton trains are separated by an odd number of lattice periods. Thus, transition between attraction and repulsion, or reversion of the energy transfer, can be realized merely by changing the initial separation as well as the phase difference. The directions of the energy transfer during the IBSs or GSTs with $\pi/2$ phase difference are also opposite to the 'normal' ones due to the 'negative refraction' of the lattice solitons in the self-defocusing photonic lattice. Moreover, two IBSs can merge into a GST bifurcating from point X_1 in the presence of attraction or energy transfer. We expect similar phenomena for lattice solitons with staggered phase structures in other discrete systems.

References

[1] Orzel C, Tuchman A K, Fenselau M L, Yasuda M and Kasevich M A 2001 *Science* **291** 2386

[2] Yablonovich E 1987 *Phys. Rev. Lett.* **58** 2059

[3] Christodoulides D N, Lederer F and Silberberg Y 2003 *Nature* **424** 817

[4] Zhang P, Lou C, Liu S, Xiao F, Zhao J, Xu J and Chen Z 2008 *Opt. Photonics News* **19** 25

[5] Eisenberg H S, Silberberg Y, Morandotti R, Boyd A R and Aitchison J S 1998 *Phys. Rev. Lett.* **81** 3383

[6] Rosberg C R, Neshev D N, Sukhorukov A A and Kivshar Y S 2005 *Opt. Lett.* **30** 2293

[7] Liu S, Zhang P, Xiao F, Yang D and Zhao J 2009 *Sci. Chin.* Ser. G **52** 747

[8] Liu S, Zhang P, Lou C, Xiao F and Zhao J 2009 *J. Mod. Opt.* **56** 677

[9] Neshev D, Ostrovskaya E, Kivshar Y and Królikowski W 2003 *Opt. Lett.* **28** 710

[10] Zhang P, Liu S, Zhao J, Lou C, Xu J and Chen Z 2008 *Opt. Lett.* **33** 878

[11] Fleischer J W, Carmon T, Segev M, Efremidis N K and Christodoulides D N 2003 *Phys. Rev. Lett.* **90** 023902

[12] Zhang P, Zhao J, Xiao F, Lou C, Xu J and Chen Z 2008 *Opt. Experss* **16** 3865

[13] Martin H, Eugenieva E D, Chen Z and Christodoulides D N 2004 *Phys. Rev. Lett.* **92** 123902

[14] Fleischer J W, Segev M, Efremidis N K and Christodoulides D N 2003 *Nature* **422** 147

[15] Fischer R, Träger D, Neshev D N, Sukhorukov A A, Krolikowski W, Denz C and Kivshar Y S 2006 *Phys. Rev. Lett.* **96** 023905

[16] Lou C, Wang X, Xu J, Chen Z and Yang J 2007 *Phys. Rev. Lett.* **98** 213903

[17] Wang X, Chen Z, Wang J and Yang J 2007 *Phys. Rev. Lett.* **99** 243901

[18] Hu Y, Lou C, Liu S, Zhang P, Zhao J, Xu J and Chen Z 2009 *Opt. Lett.* **34** 1114

[19] Meier J, Stegeman G I, Silberberg Y, Morandotti R and Aitchison J S 2004 *Phys. Rev. Lett.* **93** 093903

[20] StepićM, Smirnov E, Rüter C E, Prönneke L, Kip D and Shandarov V 2006 *Phys. Rev.* E **74** 046614

[21] Xiao F, Zhang P, Liu S and Zhao J 2007 *Chin. Phys. Lett.* **24** 3435

[22] Xiao F J, Zhang P, Liu S and Zhao J L 2008 *Acta Phys. Sin.* **57** 2529 (in Chinese)

[23] Xiao F J, Zhang P, Liu S, Gan X T and Zhao J L 2010 *Chin. Phys.* B **19** 044208

[24] Shih M, Segev M and Salamo G 1997 *Phys. Rev. Lett.* **78** 2551

[25] Królikowski W, Saffman M, Davies B L and Denz C 1998 *Phys. Rev. Lett.* **80** 3240

[26] Yang P B, Cao L G, Hu W, Guo Q, Zhu Y Q and Yang X B 2008 *Acta Phys. Sin.* **57** 285 (in Chinese)

[27] Jiang D S and She W L 2007 *Acta Phys. Sin.* **56** 245 (in Chinese)

[28] Zozulya A and Anderson D Z 1995 *Phys. Rev.* A **51** 1520

[29] Mitchell M, Segev M, Coskun T H and Christodoulides D N 1997 *Phys. Rev. Lett.* **79** 4990