Semi-analytical modeling of tokamak density evolution

Shi Bing-Ren(石秉仁)

Southwestern Institute of Physics, Chengdu 610041, *China*

(Received 30 July 2009; revised manuscript received 23 November 2009)

Tokamak plasma density evolution is generally modeled by a diffusion–convection equation in cylindrical geometry. By using a semi-analytical approach, we solve such an equation for a given diffusion coefficient and inward convection velocity as an arbitrary function of the radial position. Through variable separation, a Sturm–Liouville-type eigenvalue problem is solved, thereby constructing a complete set of orthogonal eigenfunctions. Based on the decomposition of the solution, the initial function, and the source function in these eigenfunctions, several problems of practical interest about the density evolution are analyzed. They include the density evolution, with boundary density not being zero; the density profile with internal transport barrier; the damping profile during particle source being shut-down. Results are found to be qualitatively consistent with the tokamak experiments.

Keywords: tokamak density evolution, semi-analytical approach, particle transport barrier

PACC: 5225L, 5225F, 5255G

1. Introduction

Particle transport plays an important role in studying tokamak confinement. Basically, an initial plasma density profile is established through a fast discharge to ionize the neutral working gas (usually hydrogen or deuterium) in the vacuum chamber, then through external fueling to increase or maintain a required density profile. The evolution of the density profile is modeled by a macroscopic conservation equation of the form $\partial n_{\rm e}/\partial t + \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma} = S$ where $n_{\rm e}$, $\boldsymbol{\Gamma}$ and S represent the electron density, the particle flow density and the particle source, respectively. Comparison of the particle transport process and plasma heating process that is also modeled by a temperature evolution equation of the same type shows that the particle source term is more complicated than the power density source term because of the existence of wall recycling. In this paper, we do not investigate concrete fueling processes but assume them to be some known functions. It is also worth noting that the equation we will discuss has wider applications, such as in impurity transport,[1] however, we restrict ourselves to investigation of particle transport because the direct motivation for our present study originates from some spectacular phenomena about density evolution from HL-1M and HL-2A tokamaks and superconducting tokamak HT-7 experiments.[2−6] The present work is also an extension of our previous study[7] where some important aspects were already investigated but not related to concrete experiments. Since then, several similar studies were developed[8−10] to deal with diffusion with damping, some simultaneous diffusion and convection, and heat pulse propagation. These studies were devoted to obtaining exact analytical solutions. In Ref. [10], the authors obtained solutions in various functional forms of the diffusion coefficient and convection velocity, however, their expressions involve very complicated functions. In the present paper, to relate our study to recent HL-2A experiments, we use a semi-analytic approach to solve some problems of practical interests by assuming an internal transport barrier to be used for both the diffusion and the inward convection and assuming the boundary density to be unequal to zero, which was routinely found. The remainder of the present paper is arranged as follows. In Section 2, general characteristics of the density evolution equation are revealed and two cases are compared: one is the case for the continuous diffusion and convection model (referred to as case I, hereafter), the other is the case for the discontinuous diffusion and convection model (named case II, hereafter) due to the occurrence of an internal transport barrier (ITB). In Section 3, we consider the case where the particle source is time-independent, which makes the solution procedure relatively simple. In Section 4, density damping when the source is shut down is discussed. In Section 5 a more realistic case, i.e. the boundary density is not zero, is studied. Finally, conclusions are drawn in Section 6.

© 2010 Chinese Physical Society and IOP Publishing Ltd

2. General analysis of the density evolution equation

We assume the density evolution equation to be in the following form:

$$\frac{\partial n_{\rm e}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[D(r)\frac{\partial n_{\rm e}}{\partial r} - V(r)n_{\rm e}\right]\right\} + S(r,t), \quad (1)$$

under the initial and the boundary conditions:

$$\begin{aligned} &n_{\rm e}(r, t=0) = \varphi(r), \\ &n_{\rm e}(r=a, t) = 0, \\ &\left(\frac{\partial n_{\rm e}(r,t)}{\partial r}\right)_{r=0} = 0. \end{aligned} \quad (2)$$

According to tokamak experiments, it is generally assumed that diffusion coefficient $D(r)$ and convection velocity $V(r)$ are dominated by some anomalous processes so that if such a process is suppressed to some extent, both the diffusion and the convection will be reduced simultaneously. In Eq. (1), $n_{\rm e}(r,t)$ is the electron density and the ion density is determined by the charge neutrality condition, $S(r,t)$ is the total particle deposition density, including the particles from the wall recycling and the external fueling. The source term is generally a very complicated function depending on the fuelling method and wall condition. In the present paper, we do not identify the concrete mechanisms for how neutral atoms or molecules penetrate into the plasma interior, but we only assume that this source is a known function of the radial variable and time. Besides, the diffusion coefficient $D(r)$ and convection velocity are assumed to have the following forms: for case I

$$D(r) = D_0(1+\alpha x^2), \quad (3)$$

$$V(r) = -V_0 x, \quad (4)$$

where $x = r/a$ and a is the minor radius of a circular tokamak plasma. The model diffusion coefficient (3) that is an increasing function of the minor radius, is similar to the heat diffusivity profile found in both L-mode and H-mode confinements and it is generally recognized that the anomalous heat conduction and the particle transport stem from the same turbulences; it is also assumed that the convection velocity mainly results from turbulence and its absolute value is an increasing function of the minor radius. The function form of expression (4), was used in the impurity transport study.[1] It is also worth noting that the method discussed in the following sections is applicable to diffusion and convection models that are arbitrary functions of the minor radius. Showing in (3) and (4) is then only, to some extent, a physically reasonable example. Though the principle method to solve Eqs. (1)–(4) was already presented in our previous paper,[7] we did not discuss concrete applications there. In addition, for case II, we investigate a case where both the diffusion and the convection are discontinuous in some regions due to the confinement mode transition found in Hl-2A recently.[3,4] To fit such a case, we assume

$$D(r) = \begin{cases} D_0(1+\alpha x^2) & \text{for } 0 < x < x_1 \text{ and } x_2 \le x \le 1, \\ D_0(1+\alpha x^2/8) & \text{for } x_1 \le x < x_2, \end{cases} \quad (5)$$

$$V(r) = \begin{cases} -V_0 x & \text{for } 0 < x < x_1 \text{ and } x_2 \le x \le 1, \\ V_0 x/8 & \text{for } x_1 \le x < x_2. \end{cases} \quad (6)$$

The above selections for the diffusion coefficient and the convection velocity are plotted in Figs. 1 and 2 for the case of $\alpha = 5, \beta = 3$ (β is defined below) and $x_1 = 0.5, x_2 = 0.7$.

Now we perform the following transformations to turn the relevant equations into dimensionless ones, i.e.

$$\begin{aligned} &y = n_{\rm e}/n_{e0}, \quad \tau_0 = a^2/D_0, \\ &\tau = t/\tau_0, \quad \beta = V_0 a/D_0, \end{aligned}$$

and

$$s = Sa^2/n_{\rm e0}D_0. \quad (7)$$

Thus equation (1) becomes

$$\frac{\partial y}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left\{x\left[(1+\alpha x^2)\frac{\partial y}{\partial x} + \beta x y\right]\right\} + s(x,\tau), \quad (8)$$

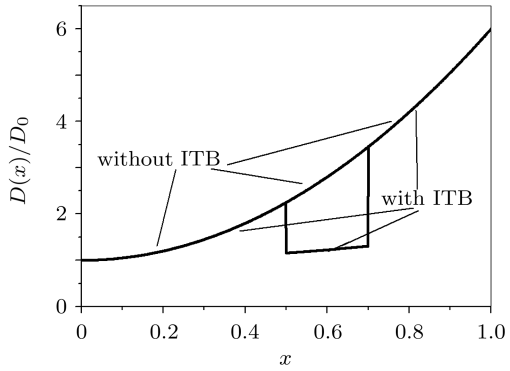


Fig. 1. Diffusion coefficient models for continuous and discontinuous cases.

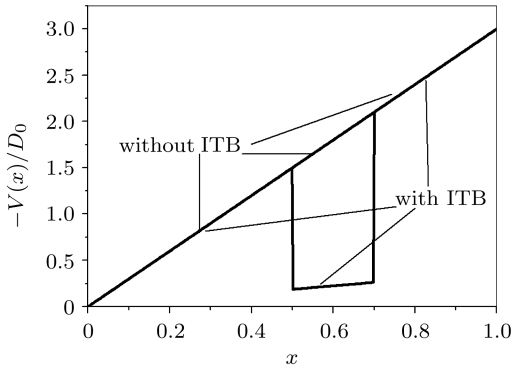


Fig. 2. Inward convection models for continuous and discontinuous cases.

the initial and boundary conditions are turned into

$$y(0,t)=1,$$
$$(\partial y/\partial t)_{x=0}=0, \tag{9}$$
$$y(1,t)=0, \tag{10}$$
$$y(x,0)=\varphi(x). \tag{11}$$

An analytical method to solve Eq. (8), under initial and boundary conditions (9)–(11), was suggested by Fussmann in his impurity transport problem study[1] for the constant diffusion coefficient case and extended by Shi *et al.*, to consider the radial-dependent diffusion coefficient case.[7] Afterwards, some new investigations were performed.[6−8] First, we briefly collect the main results.

Using the variable separation technique, we assume

$$y(x,\tau)=\sum_k U_k(x)\exp(-\Gamma_k\tau), \tag{12}$$

where $U_k(x)$ is the eigenfunction of the corresponding Sturm–Liouville equation:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{x\left[(1+\alpha x^2)\frac{\mathrm{d}U_k}{\mathrm{d}x}+\beta x U_k\right]\right\}+\Gamma_k x U_k=0, \tag{13}$$

and Γ_k is the positive eigenvalue. The eigenfunction $U_k(x)$ satisfies the boundary condition: $U_k(0)=1$, $U_k(1)=0$ and $\mathrm{d}U_k(x)/\mathrm{d}x=0$ at $x=0$ so that it is an even function of x. If we collect all these eigenfunctions according to the magnitude sequence of the values: $\Gamma_1<\Gamma_2<\cdots<\Gamma_N$, they constitute a complete set of the orthogonal function base. It is found that for Eq. (13), a transformation $U_k(x)=\bar{U}_k(x)\exp[-\beta\int_0^x \mathrm{d}x'x'/(1+\alpha x'^2)]$ yields a self-adjoint equation for $\bar{U}_k$, it implies the orthogonality condition

$$\int_0^1 \mathrm{d}x x h(x)U_i(x)U_j(x)=0 \quad (i\neq j) \tag{14}$$

with the weight function being

$$h(x)=\exp\left(\beta\int_0^x\frac{x'\mathrm{d}x'}{1+\alpha x'^2}\right)=(1+\alpha x^2)^{\beta/2\alpha}, \tag{15}$$

for the case of the continuous diffusion coefficient and convection velocity. For the discontinuous case,

$$h(x)=\exp\left[-\int_0^x \mathrm{d}x' V(x')/D(x')\right]. \tag{16}$$

For both cases, the orthogonality condition can be expressed as

$$\int_0^1 \mathrm{d}x x h(x)U_i(x)U_j(x)=C_i\delta_{ij}, \tag{17}$$
$$C_i=\int_0^1 \mathrm{d}x x h(x)U_i(x)U_i(x). \tag{18}$$

If we expand the initial density function of expression (11) as

$$\varphi(x)=\sum_k \lambda_k U_k(x),$$
$$\lambda_k=C_k^{-1}\int_0^1 \mathrm{d}x x h(x)U_k\varphi(x), \tag{19}$$

the general solution of Eq. (8) under conditions (9)–(11) can be expressed as

$$y(x,\tau)=\int_0^\tau \mathrm{d}\tau'\sum_k\int_0^1 \mathrm{d}x'\eta_k(\tau')U_k(x')\exp(-\Gamma_k(\tau-\tau'))+\sum_k\lambda_k U_k(x)\exp(-\Gamma_k\tau), \tag{20}$$

$$\eta_k(\tau')=\frac{1}{C_k}\int_0^1 \mathrm{d}x x h(x)U_k(x)s(x,\tau'). \tag{21}$$

A general demonstration of this Green function type of expression is shown in the appendix.

In the following, we will show some selected examples for eigenfunctions and eigenvalues. The relevant parameters are $\alpha = 5,\ \beta = 3,\ x_1 = 0.5$, and $x_2 = 0.7$. For case I, the first seven eigenfunctions are shown in Fig. 3 while for case II, the corresponding functions are shown in Fig. 4.

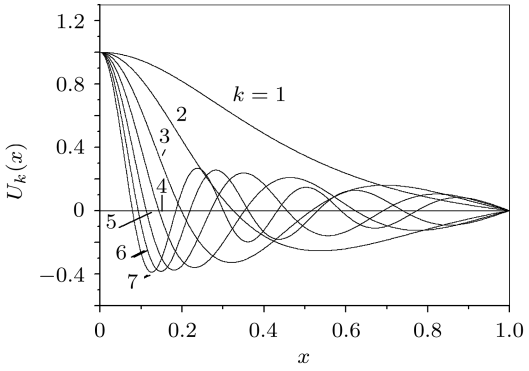


Fig. 3. The first 7 eigenfunctions for the case of continuous diffusion and convection.

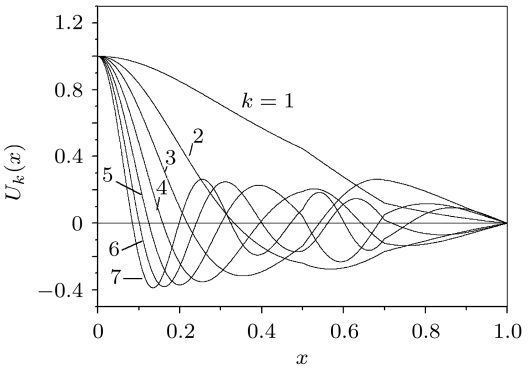


Fig. 4. The first 7 eigenfunctions for case II with ITB.

The corresponding eigenvalues are given below. For case I:

$$\Gamma_1 = 15.47656, \quad \Gamma_2 = 67.125,$$
$$\Gamma_3 = 160.172, \quad \Gamma_4 = 294.563,$$
$$\Gamma_5 = 470.25, \quad \Gamma_6 = 687.375,$$

and

$$\Gamma_7 = 945.813;$$

for case II:

$$\Gamma_1 = 11.4356, \quad \Gamma_2 = 64.4219,$$
$$\Gamma_3 = 134.3672, \quad \Gamma_4 = 243.875,$$
$$\Gamma_5 = 386.75, \quad \Gamma_6 = 579.875,$$

and

$$\Gamma_7 = 828.906.$$

It is found that these eigenvalues are largely separated from each other so that the eigenfunction with the larger eigenvalue damps very quickly. This also means that the eigenfunction with the smallest eigenvalue should dominate the evolution finally. This is spectacular behavior of the tokamak density profile when the source term is shut down.

3. Density evolution with steady state source

Now we turn to discuss some more experimentally relevant cases. First, we assume that there exists a steady state source $s(r)$, independent of time. Under such a condition, physically, the stationary solution is permitted to exist. Since $\partial y/\partial t = 0$, we have

$$\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}\left\{x\left[(1+\alpha x^2)\frac{\mathrm{d}y}{\mathrm{d}x} + \beta x y\right]\right\} + s(r) = 0. \quad (22)$$

Back to the original function, the corresponding solution is

$$n_{\mathrm{est}}(x) = \left[n_{\mathrm{e0}} - \frac{a^2}{D_0}\int_0^x \frac{h(x')}{x'(1+\alpha x'^2)}\int_0^{x'} \mathrm{d}z z s(z)\right]/h(z). \quad (23)$$

By using the boundary condition, $n_{\mathrm{e}}(1,t) = 0$, we obtain

$$n_{\mathrm{est}}(x) = \frac{a^2}{D_0}\int_x^1 \frac{h(x')}{x'(1+\alpha x'^2)}\int_0^{x'} \mathrm{d}t t s(t)/h(x). \quad (24)$$

Now we consider the following question:

$$\frac{\partial n_{\mathrm{e}}}{\partial t} = L(n_{\mathrm{e}}) + S(x) \quad (25)$$

with operator $L \equiv (1/x)\mathrm{d}\{x[D(x)\mathrm{d}/\mathrm{d}x - V(x)]\}/\mathrm{d}x$. The boundary condition is

$$n_{\mathrm{e}}(x, t=0) = \varphi(x). \quad (26)$$

Letting

$$n_{\mathrm{e}}(x,t) = n_{\mathrm{est}}(x) + n_{\mathrm{e1}}(x,t), \quad (27)$$

because $L(n_{\mathrm{st}}) + S = 0$, we have

$$\frac{\partial n_{\mathrm{e1}}}{\partial t} = L(n_{\mathrm{e1}}), \quad (28)$$
$$n_{\mathrm{e1}}(x, t=0) = \varphi(x) - n_{\mathrm{est}}(x) = \varphi_1(x). \quad (29)$$

As shown above, we can expand

$$\varphi_1(x) = \sum_{k=1} \lambda_k U_k(x), \quad (30)$$

where

$$\lambda_k = C_k^{-1} \int_0^1 \mathrm{d}x x h(t) \varphi_1(x) U_k(x). \quad (31)$$

Finally, we obtain

$$n_\mathrm{e}(x,t) = n_\mathrm{est}(x) + \sum_k \lambda_k U_k(x) \exp(-\Gamma_k t). \quad (32)$$

The characteristic time to arrive at a stationary state is determined dominantly by the smallest eigenvalue to be $\tau_1 = a^2/\Gamma_1 D_0$.

It is worth noting that solution (32) can be derived from the general time-dependent form of expression (20) if we assume the source term to be time-independent.

For a given source function, the steady-state solution (24) can be easily obtained. Figure 5 shows a comparison between two profiles for case I and case II and an assumed source function $s(x) \propto x^2$.

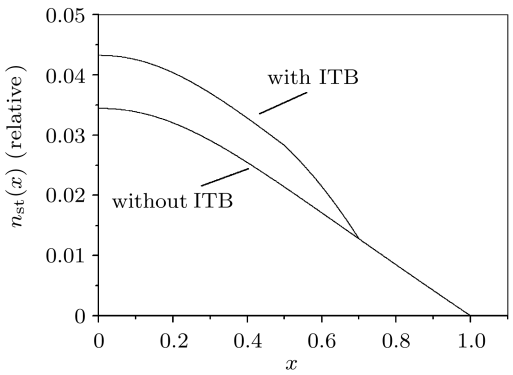


Fig. 5. Steady-state density profile (relative) for case I (without ITB) and case II (with ITB) and an assumed source function $s(x) \propto x^2$.

4. Density evolution with source shut down

When the particle source is shut down, the density evolution is described by the second term on the right-hand side of the general solution (20). Finally, the damped profile is dominated by the first eigenfunction $U_1(x)$ because the second eigenvalue is well separated from the first one: $\Gamma_2 \gg \Gamma_1$, the third one is much larger than the second one: $\Gamma_3 \gg \Gamma_1$, etc., which means that the initial profile may be rapidly turned into the shape described by function $U_1(x)$, then the shape can be kept unchanged for a relatively long time. Figure 6 shows the two profiles for case I and case II.

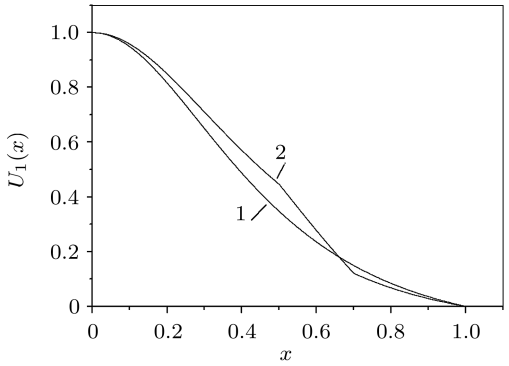


Fig. 6. The first eigenfunctions for two cases without (curve 1) and with ITB (curve 2).

It can be seen that for both cases, the damped profiles are much steeper than those of the steady-state profiles, which is consistent with the fact that the tokamak plasma density usually becomes steeper when external fueling is shut down and the wall recycling is well controlled. The well-known experiments for the improved Ohmic confinement (IOC)[11] on Asdex and the enhanced reversed shear confinement (ERS)[12] on TFTR are two typical examples. The recent HL-2A experimental results show that when gas puffing is shut down, a relatively steep density profile, referred to as a density barrier, can maintain 200–400 ms.[3,4] A quantitative comparison is still not possible because the adjustable parameters are more than 3: D_0, α, β, and the forms of diffusion and convection can be freely selected. However, if we recognize that, generally, expressions (3) and (4) for case I or expressions (5) and (6) for case II are suitable physically, we can find a reasonable width of the parameter window from the relationships of the first eigenvalue $\Gamma_1(\alpha, \beta)$ (see Figs. 7 and 8). Now, we assume that $a = 0.4m$, $D_0 = 0.1m^2/\mathrm{s}$, and $\tau_0 = 1.6$ s.

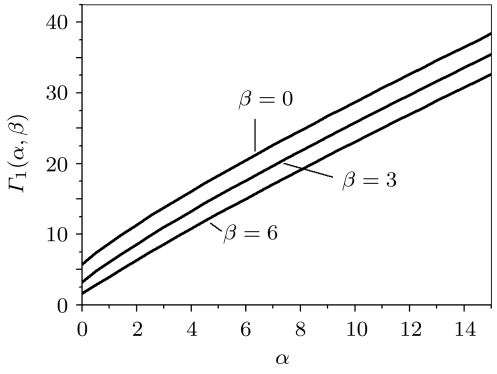


Fig. 7. Dependence of the first eigenvalue on parameters α, β for the continuous diffusion and convection model (case I).

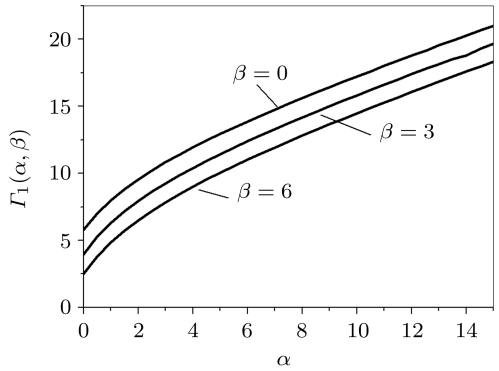


Fig. 8. Dependence of the first eigenvalue on parameters α, β for the discontinuous diffusion and convection model (case II).

For case II, we can select the parameter range as $0 < \alpha < 4$, and $3 < \beta < 6$, the characteristic damping time $\tau = \tau_0/\Gamma_1$ will be in the range 160 ms $< \tau <$ 640 ms. This should be qualitatively consistent with the experiments.

5. Method for nonzero boundary density

The boundary density in a tokamak plasma is usually nonzero, and its value can be one-fifth to one-tenth the value of the central density. The method illustrated above cannot be directly used to study this case. Now the governing equations are

$$\frac{\partial n_{\rm e}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[D(r)\frac{\partial n_{\rm e}}{\partial r} - V(r)n_{\rm e}\right]\right\} + S(r,t), \quad (33)$$

which is under the following initial and boundary conditions:

$$\begin{aligned} &n_{\rm e}(r, t=0) = \varphi(r), \\ &n_{\rm e}(r=a, t) = n_{\rm ea}, \\ &\left(\frac{\partial n_{\rm e}(r,t)}{\partial r}\right)_{r=0} = 0. \end{aligned} \quad (34)$$

Comparing Eqs. (33) and (34) with Eqs. (1) and (2), it can be found that the only difference is the boundary condition at $r = a$. To use the available results derived above, we make the following transform:

$$\bar{n} = n - n_{\rm b}, \quad (35)$$

then equation (33) turns into

$$\frac{\partial \bar{n}}{\partial t} = L(\bar{n}) + S - \frac{n_{\rm b}}{r}\frac{\rm d}{{\rm d}r}(rV(r)). \quad (36)$$

As far as new function $\bar{n}(x)$ is concerned, equation (36) means that there always exists an additional source term $S_{\rm ad} = -(n_{\rm b}/r)({\rm d}/{\rm d}r)(rV(r))$ that will yield

$$\begin{aligned} &\left(\frac{\bar{n}_1}{n_0}\right) \\ &= h^{-1}(x)\left[-\frac{a^2}{D_0}\int_0^x {\rm d}x'\frac{h(x')}{x'}\right. \\ &\left.\times \int_0^{x'} {\rm d}t t\frac{n_{\rm b}}{n_0}\frac{1}{t}\frac{\rm d}{{\rm d}t}(-tV(t))\right] \\ &= h^{-1}(x)\frac{n_{\rm b}a^2}{n_0 D_0}\int_0^x {\rm d}x' V(x')h(x'). \end{aligned} \quad (37)$$

If we shut down the real external source at $t = 0$, we have

$$\frac{\partial \bar{n}}{\partial t} = L(\bar{n}) - \frac{n_b}{r}\frac{\rm d}{{\rm d}r}(rV(r)). \quad (38)$$

The initial condition is

$$\bar{n}(r, t=0) = \varphi(x) - n_{\rm ea}, \quad (39)$$

while the boundary condition is

$$\bar{n}(a, t) = 0. \quad (40)$$

Now the additional source is equivalent to a steady state source. Note $V(x) = -V_0 x$ and $\beta = V_0 a/D_0$, then the additional contribution in Eq. (37) will become

$$(\bar{n}_1/n_0) = \beta(n_{\rm b}/n_0)\Delta n(x), \quad (41)$$

$$\Delta n(x) = h^{-1}(x)\int_x^1 {\rm d}x' x' h(x'). \quad (42)$$

For case II, this term is shown in Fig. 9.

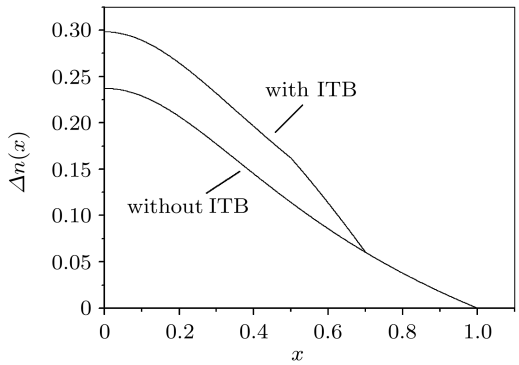


Fig. 9. Function $\Delta n(x)$ versus x.

The general solution is

$$\frac{n(x)}{n_0} = \left(\frac{n_b}{n_0}\right)(1 + \beta\Delta n(x))$$

$$+\sum_k \alpha_k U_k(x)\exp(-\Gamma_k t/\tau_0), \qquad (43)$$

where α_k is the expanding coefficient of function $(\varphi(x)-\Delta n(x))$ against the complete set $U_k(x)$.

6. Conclusions

In this paper, the density evolution of tokamak plasma is analyzed by a semi-analytic method. By separating the time and the radial variables, we obtained a generalized Sturm–Liouville-type eigenequation with diffusion and convection as functions of the radial variable. These equations can be easily solved numerically, and a complete set of orthogonal eigenfunctions can be constructed. The general solution of the time-dependent density evolution can be expressed as a sum of a source related integral of the Green function and a damping part of the initial density profile. Some cases of interest experimentally are discussed in detail. The relevant physics of some experiments on particle transport can be interpreted qualitatively. The method presented here has further applications in heat transport and impurity transport or heat and density modulation problems.

Acknowledgement

This work was partly motivated by the particle transport experiments carried out on the HL-2A tokamak. We are particularly grateful to professor Ding Xuan-Tong and Dr. Xiao Wei-Wen of the HL-2A tokamak team for showing us recent experimental results.

Appendix: Derivation of the general solution (20)

Now we consider the general solution of Eq. (8)

$$\frac{\partial y(x,\tau)}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left\{x\left[(1+\alpha x^2)\frac{\partial y}{\partial x}+\beta x y\right]\right\}+s(x,\tau) \qquad (A1)$$

with the initial condition given by

$$y(x,\tau=0)=0. \qquad (A2)$$

First, we assume that during a shot period ($\tau=\tau',\tau'+\mathrm{d}\tau'$), the source produces a response $w(x,\tau')\mathrm{d}\tau'$ such that the total response can be written as

$$y(x,\tau)=\int_0^\tau \mathrm{d}\tau' w(x,\tau'). \qquad (A3)$$

Evolution for function $w(x,\tau)$ is governed by

$$\frac{\partial w(x,\tau)}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left\{x\left[(1+\alpha x^2)\frac{\partial w}{\partial x}+\beta x w\right]\right\} + s(x,\tau)\delta(\tau-\tau'), \qquad (A4)$$

$$w(x,\tau=0)=0. \qquad (A5)$$

We can transform Eqs. (4) and (5) into an alternative form

$$\frac{\partial w(x,\tau)}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left\{x\left[(1+\alpha x^2)\frac{\partial w}{\partial x}+\beta x w\right]\right\}, \qquad (A6)$$

$$w(x,\tau=\tau')=s(x,\tau'). \qquad (A7)$$

We have already known that the solution of Eqs. (A6) and (A7) is

$$w(x,\tau')=\sum_k \eta_k(\tau')U_k(x)\exp[-\Gamma_k(\tau-\tau')]. \qquad (A8)$$

The expanded coefficient

$$\eta_k(\tau')=\frac{1}{C_k}\int_0^1 \mathrm{d}x' x' s(x',\tau')U_k(x')h(x') \qquad (A9)$$

implies that we can define the Green function as follows:

$$G(x,x',\tau-\tau') = \sum_k U_k(x)U_k(x')h(x')/C_k \times \exp[-\Gamma_k(\tau-\tau')]. \qquad (A10)$$

The general solution is then obtained as

$$y(x,\tau)=\int_0^\tau \mathrm{d}\tau' \sum_k \int_0^1 \mathrm{d}x' s(x',\tau')G(x,x',\tau-\tau').$$

It is worth noting that the correct form of the Green function (A10) is different from that in Refs. [1] and [7] by a weight function $h(x)$.

References

[1] Fussmann G 1986 *Nucl. Fusion* **26** 983

[2] Jiao Y M, Zhou Y, Deng Z C, Ding X T, Liu Y and Wang E Y 2005 *Chin. Phys.* **14** 1600

[3] Xiao W W, Ding X T, Liu Z T, Yao L H, Sun H J, Duan X R, Yan L W and Yang Q W 2006 *Plasma Science & Tech.* **8** 133

[4] Xiao W W, Ding X T , Liu Z T, Yao L H, Sun H J, Duan X R, Yan L W and Yang Q W 2008 *Plasma Science & Tech.* **10** 403

[5] Zhou Q, Wan B N, Wu Z W and Huang J 2005 *Chin. Phys.* **14** 2539

[6] Chen W, Cui Z Y, Sun P, Huang Y, Deng W, Pan Y D, Shi Z B, Zhou Y, Zheng Y Z and Yang Q W 2007 *Chin. Phys.* **16** 1150

[7] Shi B R, Long Y X and Li J Q 2000 *Plasma Science & Tech.* **3** 287

[8] Imbeaux F and Garbet X 2002 *Plasma Phys. Contr. Fusion* **44** 1425

[9] Clemencon A, Guivarch C, Eury S P, Zou X L and Giruzzi G 2004 *Phys. Plasmas* **11** 4998

[10] Eury S P, Harauchamps E, Zou X L and Giruzzi G 2005 *Phys. Plasma* **12** 102511

[11] The Asdex Team 1989 *Nucl. Fusion* **29** 1959

[12] Hawryluk R J, Anunasslam V, Barnes C W, Beer M, Bell M, Bell R, Biglari H, Bitter M, Boivin R, Bretz N L, Budny R, Bush C E, Cheng C Z, Chu T K, Cohen S A, Cowley S, Efthimion P C, Fonck R J, Fredrickson E, Furth P, Goldston R J, Greene G, Grek B, Grisham L R, Hammett G, Heidbrink W, Hill K W, Hosea J, Hulse H, Hsuan H, Janos A, Jassby D, Jobes C, Johson D W, Johson L C, Kesner J, Kieras-Philips C, Kilpatrick J, Kugel H, La Marche P H, LeBlance B, Manos D M, Mazzucato E, McCarthy M P, Mauer M, McCune C, McGuire K M, Meade D M, Medley S S, Mikkelson D R, Monticello D, Motley R, Mueller D, Nagayama Y, Navratil G A, Nazikian R, Owens K, Park H, Park W, Paul S, Perkins F, Pitcher S, Ramsey A T, Redi H, Rewoldt G, Roberts D, Roquemore A L, Rutherford P H, Sabbagh S, Schilling G, Schivell J, Schmidt G L, Scott S D, Snipes J, Stevens J, Stratton B C, Stodiek W, Synakowski E, Takase Y, Tang W, Taylor G, Terry J, Timberlake J R, Towner H H, Ulrickson M, Von Goeler S, Wielan R, Williams M, Wilson J R, Wong K L, Yamada M, Yoshikawa S, Young K M, Zarnstorff M C and Zweban S J 1991 *Plasma Phys. Contr. Fusion* **33** 1509