Cross-phase modulation instability in optical fibres with exponential saturable nonlinearity and high-order dispersion*

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Utilizing the linear-stability analysis, this paper analytically investigates and calculates the condition and gain spectra of cross-phase modulation instability in optical fibres in the case of exponential saturable nonlinearity and high-order dispersion. The results show that, the modulation instability characteristics here are similar to those of conventional saturable nonlinearity and Kerr nonlinearity. That is to say, when the fourth-order dispersion has the same sign as that of the second-order one, a new gain spectral region called the second one which is far away from the zero point may appear. The existence of the exponential saturable nonlinearity will make the spectral width as well as the peak gain of every spectral region increase with the input powers before decrease. Namely, for every spectral regime, this may lead to a unique value of peak gain and spectral width for two different input powers. In comparison with the case of conventional saturable nonlinearity, however, when the other parameters are the same, the variations of the spectral width and the peak gain with the input powers will be faster in case of exponential saturable nonlinearity.

Keywords: nonlinear optics, cross-phase modulation instability, exponential saturable nonlinearity, high-order dispersion

PACC: 4265, 4265T, 4281D

1. Introduction

As an important phenomenon, modulation instability (MI) can be found in many nonlinear systems such as fluid mechanics, plasma physics, Bose–Einstein condensates,[1,2] optics,[3−22] and so on. Among them, MI in nonlinear optics has been investigated extensively whether in the temporal domain,[1−17] spatial domain,[18−21] or spatiotemporal domain,[22] for its significant theoretical and practical importance. The temporal domain MI resulted from the interplay between the dispersion and the nonlinear effects of an optical fibre, has attracted special attention for its important applications in producing high-repetition-rate soliton-like pulse trains,[3] generation of super-continuum spectra,[4] etc.

At present, MI in Kerr nonlinear fibres has been extensively investigated whether for lower- or higher-order dispersions. However, previous studies indicated that, when the incident optical field is intense enough or when the optical field propagates in an optical fibre with relatively high values of nonlinear optical coefficients, the higher-order nonlinearities or saturable nonlinearity will take effect and must be taken into account even at a moderate optical intensity level.[23] Thus, the optical solitary solutions[24,25] and MI[23,26−33] for several kinds of saturable nonlinearities have been investigated in recent years. And the typical saturable nonlinearities include the negative quintic nonlinearity,[26−29] the conventional saturable nonlinearity (CSN),[30−33] and exponential saturable nonlinearity (ESN) as involved in Ref. [23]. Up to now, studies on self-phase[23,26,27,30,31] and cross-phase MI (XPMMI)[17,28,29,32,33] characteristics in both the quintic nonlinear and the CSN fibres have already been reported whether for lower-[23,26,29−31,33] or higher-order[27,28,31,32] dispersions. The self-phase MI in the ESN fiber has been studied for second-order dispersion.[23] To our knowledge, however, no reports involved the combined effects of higher-order dispersions and ESN on XPMMI. Thus, in this paper, taking both the high-order dispersion and ESN into account, we analytically investigated and calculated the condition and gain spectra of MI of two optical waves in detail.

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2. Theoretical analysis

Taking both the high-order dispersion and ESN into consideration, we find that the slowly varying envelopes A_1 and A_2 of the two optical fields should satisfy the following extended coupling nonlinear Schrödinger equations:[23,34]

$$\frac{\partial A_j}{\partial z}+\frac{1}{V_{gj}}\frac{\partial A_j}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2j}\frac{\partial^2 A_j}{\partial t^2}-\frac{\beta_{3j}}{6}\frac{\partial^3 A_j}{\partial t^3}+\frac{\mathrm{i}}{24}\beta_{4j}\frac{\partial^4 A_j}{\partial t^4}=\mathrm{i}\,\gamma_j\frac{1-\exp\left[-\Gamma\,\left(|A_j|^2+2\,|A_{3-j}|^2\right)\right]}{\Gamma}A_j, \tag{1}$$

where, V_{gj} ($j=1$, 2) is the group velocity, β_{kj} ($k=2$, 3, 4) is the k-order group velocity dispersion coefficient, γ_j is the nonlinear coefficient, $\Gamma=P_{\mathrm{s}}$ is the saturable parameter, P_{s} is the saturable optical power of the fibre, z is the propagating distance, t is the time coordinate. To simplify the discussion, here we have omitted the loss of the optical fibre.

One can readily obtain the continuous-wave steady-state solution of Eq. (1). And its corresponding nonlinear phase shift ϕ_j can also be deduced as

$$\phi_j=\gamma_j\frac{1-\exp\{-\Gamma\,(P_j+2P_{3-j})\}}{\Gamma}z, \tag{2}$$

where P_j is the incident optical power.

Adopting the familiar linear stability analysis, one can obtain the following linearized coupled nonlinear Schrödinger equations for the weak perturbation a_j ($|a_j|^2\ll P_j$):

$$\begin{aligned}&\frac{\partial a_j}{\partial z}+\frac{1}{V_{gj}}\frac{\partial a_j}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2j}\frac{\partial^2 a_j}{\partial t^2}-\frac{\beta_{3j}}{6}\frac{\partial^3 a_j}{\partial t^3}+\frac{\mathrm{i}}{24}\beta_{4j}\frac{\partial^4 a_j}{\partial t^4}\\&=\mathrm{i}\,\gamma_j\exp\left[-\Gamma\,(P_j+2P_{3-j})\right]\,\left[P_j\left(a_j+a_j^*\right)+2\sqrt{P_jP_{3-j}}\left(a_{3-j}+a_{3-j}^*\right)\right].\end{aligned} \tag{3}$$

Following the procedure in many references and adopting the following approximate relation:[32,34] $V_{\mathrm{g1}}\approx V_{\mathrm{g2}}=V_{\mathrm{g}}$, $\beta_{21}\approx\beta_{22}=\beta_2$, $\beta_{31}\approx\beta_{32}=\beta_3$, $\beta_{41}\approx\beta_{42}=\beta_4$, $\gamma_1\approx\gamma_2=\gamma$, one can get the following dispersion relation

$$k=\frac{\Omega}{V_g}+\frac{1}{6}\beta_3\Omega^3\pm\left\{\frac{f_1+f_2}{2}\pm\left[\left(\frac{f_1+f_2}{2}\right)^2+(C_{\mathrm{XPM}}-f_1f_2)\right]^{1/2}\right\}^{1/2}, \tag{4}$$

where, k and Ω are the wave number and angular frequency of the modulation wave, respectively. And the parameters f_1, f_2 and C_{XPM} respectively satisfy

$$f_j=\left(\frac{1}{2}\beta_{2j}\Omega^2-\frac{1}{24}\beta_{4j}\Omega^4\right)\,\left[\frac{1}{2}\beta_{2j}\Omega^2-\frac{1}{24}\beta_{4j}\Omega^4+2\gamma_j'P_j\right], \tag{5}$$

$$C_{\mathrm{XPM}}=16\gamma_1'\gamma_2'P_1P_2\,\left(\frac{1}{2}\beta_{21}\Omega^2-\frac{1}{24}\beta_{41}\Omega^4\right)\,\left(\frac{1}{2}\beta_{22}\Omega^2-\frac{1}{24}\beta_{42}\Omega^4\right), \tag{6}$$

$$\gamma_j'=\gamma_j\exp\{-\Gamma\,(P_j+2P_{3-j})\}. \tag{7}$$

For some angular frequencies, if $f_1f_2<C_{\mathrm{XPM}}$ holds, k becomes complex and MI occurs. Then the corresponding power gain $g(\Omega)$ equals $2\mathrm{Im}(k)$, where Im stands for the imaginary part. In comparison with the previous cases,[14,27,28,31–33] the condition $f_1f_2<C_{\mathrm{XPM}}$ and gain coefficient $g(\Omega)$ are formally all the same. Owing to the different definitions of the parameters f_j and C_{XPM}, however, XPMMI will take on different features.

3. Calculations and discussions

The XPMMI will be calculated and discussed for different dispersion regions in the following. To simplify the discussion, we set the incident optical powers of the two optical waves as $P_2=P_1$.

3.1. In the normal dispersion region $\beta_2 > 0$ or $\beta_4 > 0$

Variations of gain spectra of MI with the input powers of the two optical waves for three kinds of normal dispersion regions have been shown in Fig. 1. The common parameters are set as: $\gamma = 15$ W^{-1}·km^{-1}, $P_s = 10$ W. The other parameters are respectively shown in every small figure. It can be seen that, some similar features to the cases of Kerr nonlinearity,[34] the CSN,[32] and quintic nonlinearity,[28] appear here, and therefore need not be described in detail once more. To observe more clearly, we have divided Fig. 1(a) into two small figures as shown in Fig. 1(a1) and Fig. 1(a2). For in case of $\beta_2 > 0$ and $\beta_4 > 0$, depending on different parameters, we find that two separated spectral regions may appear. In the other dispersion regions, the gain spectra consist of only one region as shown in Figs. 1(b) and 1(c). The existence of ESN will make the spectral width as well as the peak gain of every spectral regime increase with the input powers before decrease. Thus, for every spectral regime, this may lead to a unique value of peak gain and spectral width for two different input powers. This feature is of significant importance for the study of bistable optical solitons. To see this feature more clearly, we have shown the variations of the peak gain coefficients in Fig. 2(a) and spectral widths in Fig. 2(b) of gain spectra with the incident powers, where the solid and dashed lines stand for the cases of ESN and CSN, respectively. The parameters used correspond to those in every small figure in Fig. 1. Obviously, in comparison, when the other parameters are the same, the variations of the spectral width and the peak gain with the input powers will be faster in the case of ESN. While in the case of Kerr nonlinearity, the peak gains and spectral widths will increase monotonously with the incident powers. It can still be seen from Fig. 2 that, the peak gain and spectral width of the first spectral region in the case of $\beta_2 > 0$ and $\beta_4 > 0$ has hardly any differences from the case of $\beta_2 > 0$ and $\beta_4 < 0$. In the case of $\beta_2 > 0$ and $\beta_4 > 0$, the spectral width of the first spectral region is wider than that of the second one, but its peak gain is smaller than that of the second one. In addition, larger spectral width and peak gain can be seen for $\beta_2 = 0$ and $\beta_4 > 0$. And further investigation indicates that, with increase of parameter β_4, the spectral width increases considerably while the spectral peak varies slightly.

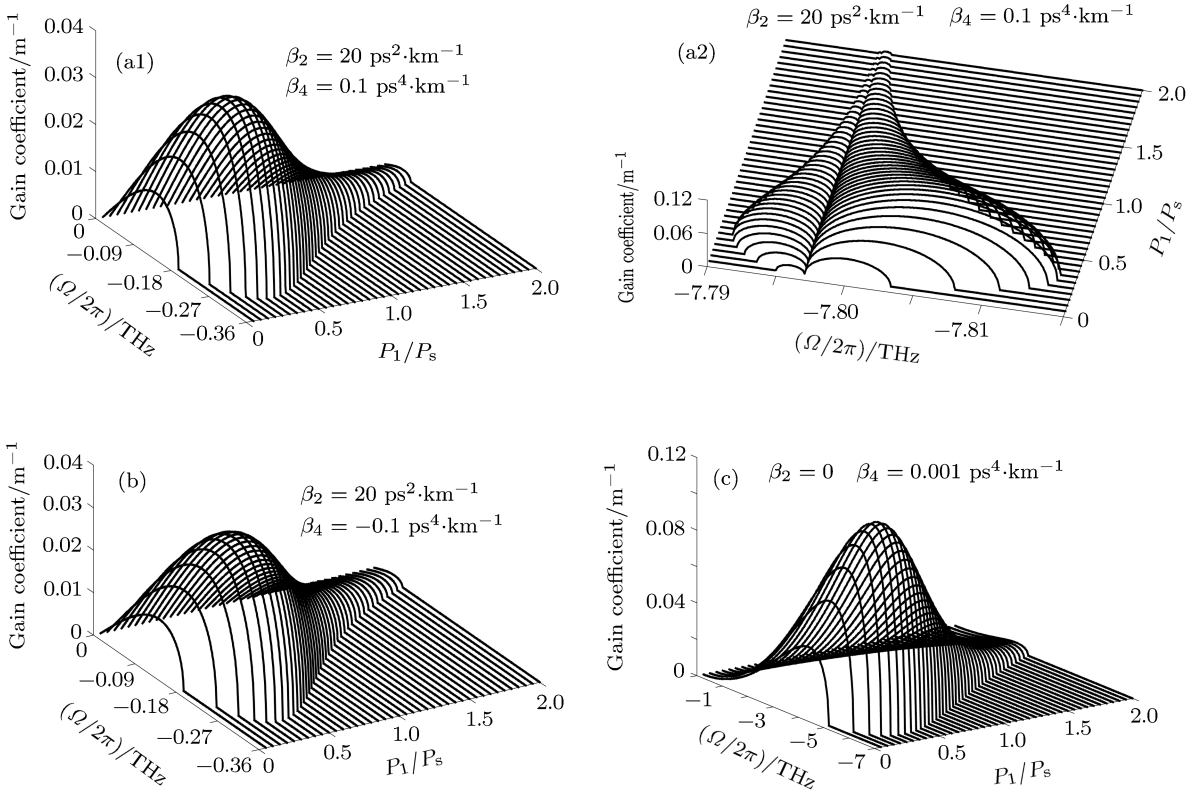


Fig. 1. Variations of gain spectra of modulation instability with the input powers of the two optical waves in the normal dispersion region.

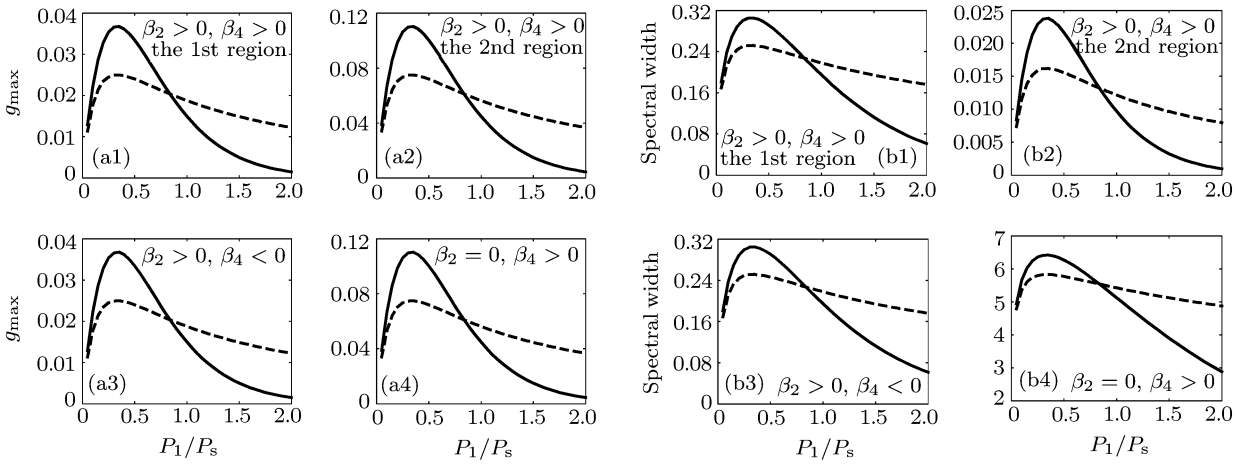


Fig. 2. Variations of the peak gain coefficients (a1–a4) and spectral widths (b1–b4) of gain spectra of modulation instability with the input powers of the two optical waves in the normal dispersion region.

3.2. In the anomalous dispersion region $\beta_2 < 0$ or $\beta_4 < 0$

Variations of gain spectra with the input powers for three kinds of anomalous dispersion regions have been shown in Fig. 3. The common parameters here are set as: $\gamma = 10$ W^{-1}·km^{-1}, $P_s = 20$ W. Corresponding to Fig. 3, we show the variations of the peak gain coefficient in Fig. 4(a) and spectral width in Fig. 4(b) of gain spectra with the input powers. Similarly, the dashed lines also stand for the cases of CSN.

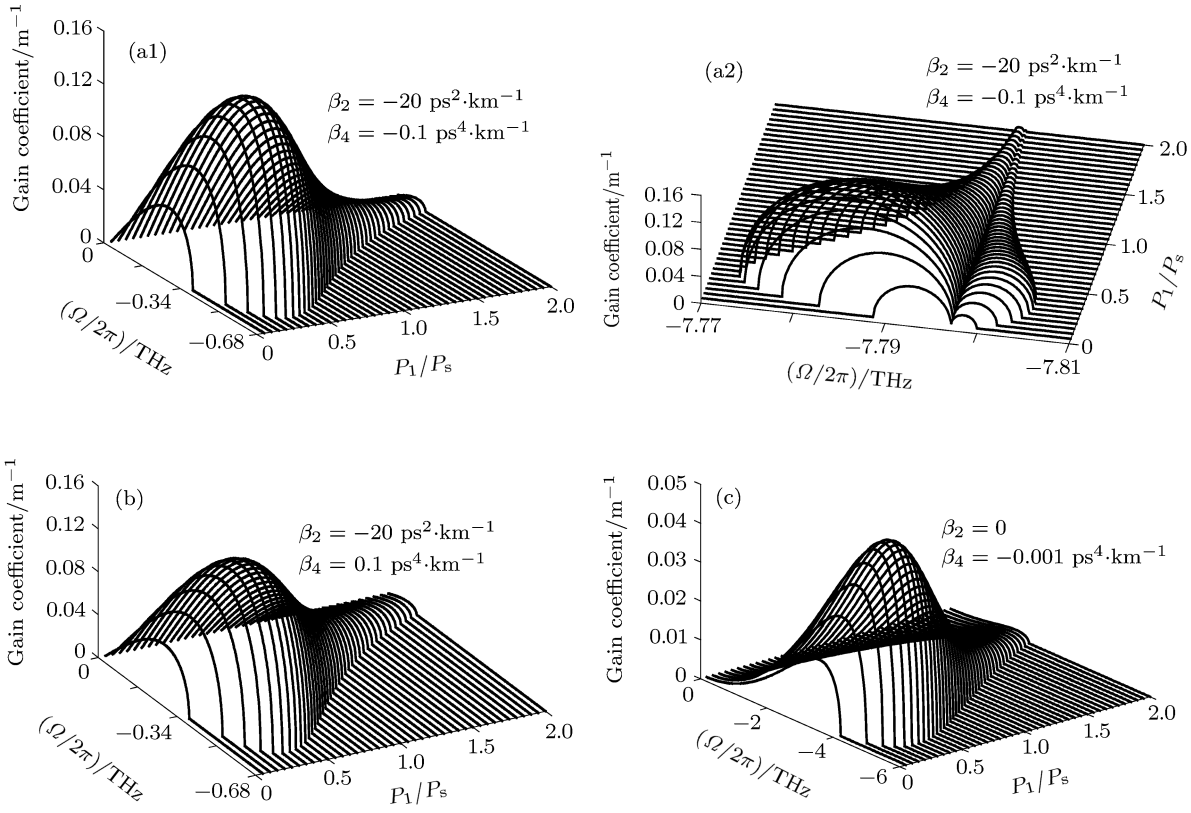


Fig. 3. Variations of gain spectra of modulation instability with the input powers of the two optical waves in the anomalous dispersion region.

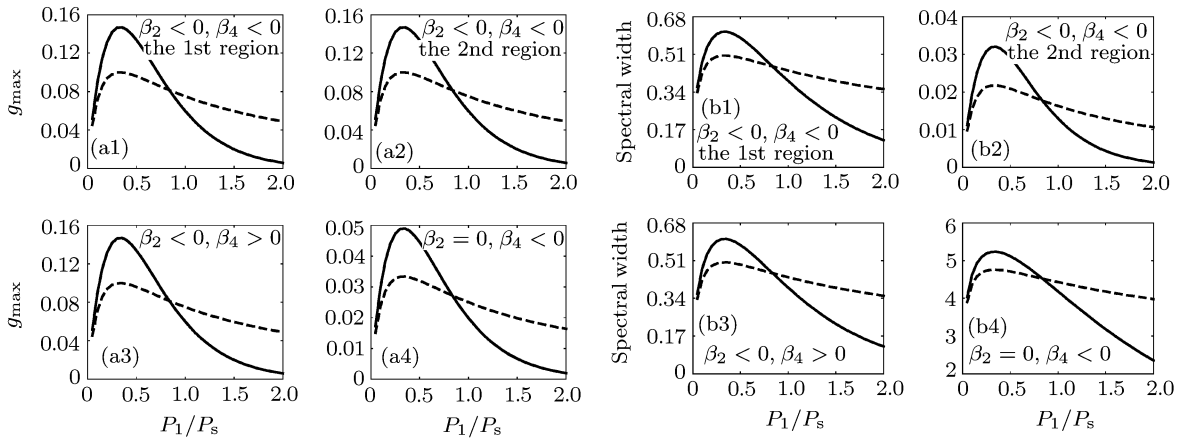


Fig. 4. Variations of the peak gain coefficient (a1–a4) and spectral width (b1–b4) of gain spectra of modulation instability with the input powers of the two optical waves in the anomalous dispersion region.

One can see from Figs. 3 and 4 that, the variations of gain spectra with the input powers in the anomalous dispersion regions are completely similar to those in the normal ones. The difference is that, the peak gains of the two spectral regions in the case of $\beta_2 < 0$ and $\beta_4 < 0$ have hardly any differences from the case of $\beta_2 < 0$ and $\beta_4 > 0$. Furthermore, for the second spectral region, in the case of $\beta_2 > 0$ and $\beta_4 > 0$, the small spectral region which is far away from the centre has wider and larger gain spectrum, while in the case of $\beta_2 > 0$ and $\beta_4 > 0$, the small spectral region which is close to the centre has wider and larger gain spectrum. In addition, for $\beta_2 = 0$, when the other parameters are the same, the spectral width is wider in the case of $\beta_4 > 0$ than that in case of $\beta_4 < 0$.

The MI characteristics above can be partly interpreted according to the condition $f_1 f_2 < C_{\rm XPM}$. Corresponding to the previous calculation, we also assume $P_2 = P_1$. According to Eqs. (5)–(7), we can rewrite the condition of MI as

$$\left(\Omega^2 - \Omega_1^2\right)\left(\Omega^2 - \Omega_2^2\right)\left(\Omega^2 - \Omega_3^2\right)\left(\Omega^2 - \Omega_4^2\right) < 0, \tag{8}$$

where the parameters are defined as

$$\Omega_{1,2,3,4}^2 = \left[6\beta_2 \pm 2\sqrt{9\beta_2^2 + 12\beta_4\,\gamma\,P_1\,\exp\{-3\Gamma\,P_1\} \pm 24\beta_4\,\gamma\,P_1\,\exp\{-3\Gamma\,P_1\}}\,\right] \Big/ \beta_4. \tag{9}$$

In Eq. (9), when both the former and the latter take "+", it stands for Ω_1^2. While when the former takes "+" and the latter takes "−", it stands for Ω_2^2. When the former takes "−" and the latter takes "+", it stands for Ω_3^2. While both the former and the latter take "−", it stands for Ω_4^2. For simplicity, two other parameters are defined as

$$A = 9\beta_2^2 - \{\,12\beta_4\,\gamma\,P_1 \exp\,[-3\,\Gamma\,P_1]\,\}, \tag{10}$$

$$B = \beta_2^2 - \{\,4\,|\beta_4|\,\gamma\,P_1\,\exp\,[-3\,\Gamma\,P_1\,]\,\}. \tag{11}$$

One can deduce from Eqs. (8)–(11) that, for $\beta_2 > 0$ and $\beta_4 > 0$, the conditions $\Omega_1^2 > 0$ and $\Omega_3^2 < 0$ must hold. Then the number of spectral regions depends on the parameter A. If $A > 0$, the conditions $\Omega_2^2 > 0$, $\Omega_4^2 > 0$ and $\Omega_4^2 < \Omega_2^2 < \Omega_1^2$ must hold. Under these circumstances, two spectral regions $0 < |\,\Omega| < |\,\Omega_4|$ and $|\,\Omega_2| < |\,\Omega| < |\,\Omega_1|$ will form as shown in Figs. 1(a1) and 1(a2). Otherwise, if $A \leq 0$, the condition $\left(\Omega^2 - \Omega_2^2\right)\left(\Omega^2 - \Omega_4^2\right) > 0$ must hold and only one spectral region $0 < |\,\Omega| < |\,\Omega_1|$ will form. Similarly, for $\beta_2 < 0$ and $\beta_4 < 0$, the conditions $\Omega_4^2 > 0$ and $\Omega_2^2 < 0$ must hold. Then the number of spectral regions depends on the parameter B. If $B > 0$, the conditions $\Omega_1^2 > 0$, $\Omega_3^2 > 0$ and $\Omega_1^2 < \Omega_3^2 < \Omega_4^2$ must hold. Under these circumstances, two spectral regions $0 < |\,\Omega| < |\,\Omega_1|$ and $|\,\Omega_3| < |\,\Omega| < |\,\Omega_4|$ will form as shown in Figs. 3(a1) and 3(a2). Otherwise, if $B \leq 0$, the condition $\left(\Omega^2 - \Omega_1^2\right)\left(\Omega^2 - \Omega_3^2\right) > 0$ must hold and only one spectral region $0 < |\,\Omega| < |\,\Omega_4|$ will form. For $\beta_2 > 0$ and $\beta_4 < 0$, the conditions $\Omega_4^2 > 0$ and $\Omega_2^2 < 0$ must hold. Further, if

$B > 0$, the conditions $\Omega_1^2 < 0$ and $\Omega_3^2 < 0$ must hold and only one spectral region $0 < |\Omega| < |\Omega_4|$ will form as shown in Fig. 1(b). Otherwise, if $B \leq 0$, the condition $\left(\Omega^2 - \Omega_1^2\right)\left(\Omega^2 - \Omega_3^2\right) > 0$ must hold and still only one spectral region $0 < |\Omega| < |\Omega_4|$ will form. For $\beta_2 < 0$ and $\beta_4 > 0$, the conditions $\Omega_1^2 > 0$ and $\Omega_3^2 < 0$ must hold. Further, if $A > 0$, the conditions $\Omega_2^2 < 0$ and $\Omega_4^2 < 0$ must hold and only one spectral region $0 < |\Omega| < |\Omega_1|$ will form as shown in Fig. 3(b). Otherwise, if $A \leq 0$, the condition $\left(\Omega^2 - \Omega_2^2\right)\left(\Omega^2 - \Omega_4^2\right) > 0$ must hold and still only one spectral region $0 < |\Omega| < |\Omega_1|$ will form. For $\beta_2 = 0$, the condition $f_1 f_2 < C_{\rm XPM}$ becomes as $\Omega^2 < \Omega_c^2 = \sqrt{2 + \mathrm{sgn}\,(\beta_4)}\,\Omega_{\rm c0}^2$, where $\Omega_{\rm c0}^2 = 4\sqrt{3\,\gamma\,P\,\exp\{-3\,\Gamma\,P_1\}/|\,\beta_4\,|}$ and sgn stands for the symbol function. Thus, there is only one spectral region as shown in Figs. 1(c) and 3(c). The spectral regions are $0 < |\Omega| < \left|\sqrt[4]{3}\Omega_{\rm c0}\right|$ and $0 < |\Omega| < |\Omega_{\rm c0}|$ for $\beta_4 > 0$ and $\beta_4 < 0$, respectively. Moreover, it can be easily seen that, when the other parameters are the same, the spectral width in the case of $\beta_4 > 0$ will be wider than that in the case of $\beta_4 < 0$. It needs to be mentioned that for the peak gain, it cannot be analysed so intuitively as that of the spectral region.

4. Conclusions

Starting from the extended coupled nonlinear Schrödinger equations including ESN and high-order dispersion in optical fibres, and adopting the linear-stability analysis, the condition and gain spectra of XPMMI are analysed and calculated in detail. The results show that, the MI characteristics here are similar to those of CSN and Kerr nonlinearity. In comparison with the case of Kerr nonlinearity, the existence of ESN will make the spectral width as well as the peak gain of every spectral region increase with the input powers before decrease. Thus, for every spectral region, this may lead to a unique value of peak gain and spectral width for two different input powers. On the other hand, however, when the other parameters are the same, the variations of the spectral width and the peak gain with the input powers in the case of ESN will be faster than those in the case of CSN.

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