Relative carrier–envelope phase dependence of resonant propagation of two-colour femtosecond pulses in V-type atomic medium*

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Using numerical solution of the full Maxwell–Bloch equations, which is obtained by the finite-difference time-domain method and the iterative predictor–corrector method, we investigate the modulation effect of relative carrier–envelope phase (hereinafter referred to as the relative phase) on resonant propagation of two-colour femtosecond ultrashort laser pulses in a V-type three-level atomic medium. It is found that the pulse splitting occurs for a smaller value of relative phase; when the value of relative phase increases to a certain value, only the variation of pulse shape is present and the pulse splitting does not occur any more; moreover, when the value of relative phase is smaller, the pulse group velocity is larger. The relative phase also has an obvious effect on population and spectral property. Different population transfers can be realized by adjusting the value of relative phase. Generally speaking, for the pulses with smaller areas their spectral strengths and frequency ranges decrease obviously with the value of relative phase increasing; for the pulses with larger areas, with value of the relative phase increasing, their spectral strengths decrease remarkably but the relative strengths of the higher frequency components increase significantly, while the spectral frequency range is not varied evidently.

Keywords: two-colour ultrashort pulses, relative carrier–envelope phase, resonant propagation, V-type three-level atomic medium

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1. Introduction

Recently, the study on the interaction of ultrashort laser pulse with atom and molecular has aroused tremendous interest[1−10] and also the study of using a relative carrier–envelope phase (RCEP) between two ultrashort laser pulses to realize coherent control under extreme nonlinear optics condition has received considerable attention.[11−16] The overlap of two-colour laser pulses in the propagation process will lead to an interference effect. In addition, the RCEP of two laser pulses adds an additional freedom degree. And this has an important effect on the propagation and the spectral property of the ultrashort laser pulse in an atomic medium. Song *et al.*[17] investigated the spectra of two-colour ultrashort laser pulses (ω–2ω) propagating in a two-level atomic medium. Under the condition of two-colour laser pulses with different frequencies, high-order harmonic generation was widely investigated.[18−20] Serrat[21,22] reported on the RCEP-related coherent control technique for four-wave mixing nonlinear interaction with two-colour femtosecond laser pulses (ω–3ω) in a two-level atomic medium. Loiko and Serrat[23] discussed phase-sensitive phenomena of two-colour ultrashort laser pulses (ω–3ω) propagating in a three-level Λ-type system. However, the study on the effect of RCEP on propagation behaviour of two-colour ultrashort laser pulses, especially resonant propagation behaviour of two-colour pulses, i.e., two frequencies of the two-colour pulses are respectively equal to the two transition frequencies between levels, in a three-level atomic medium is still lacking. In this paper, the modulation effect of RCEP on resonant propagating behaviour of two-colour femtosecond ultrashort laser pulses in a V-type three-level atomic medium is studied by numerical solution of the full Maxwell–Bloch equations, which is obtained by using the finite-difference time-domain method[24] and the predictor–

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corrector method[25] neither in the rotating wave approximation (RWA) nor in the slowly varying envelope approximation (SVEA).

2. Theoretical model

We consider two-colour femtosecond ultrashort laser pulses propagating in a V-type three-level atomic medium as shown in Fig. 1. Let levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ correspond to levels $5s^2S_{1/2}$, $5p^2P_{1/2}$ and $5p^2P_{3/2}$ of the Rb atom, respectively. Assume that the electric field of the laser pulses is linear polarization along the x-axis and the magnetic field is along the y-axis then the Maxwell equations will take the following form:

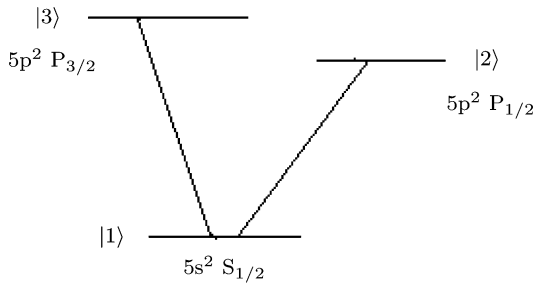


Fig. 1. V-type three-level atomic system.

$$\partial_t H_y = -\frac{1}{\mu_0}\partial_z E_x, \tag{1a}$$

$$\partial_t E_x = -\frac{1}{\varepsilon_0}\partial_z H_y - \frac{1}{\varepsilon_0}\partial_t P_x, \tag{1b}$$

where ε_0 and μ_0 are the electric permittivity and the magnetic permeability in vacuum respectively; P_x is the macroscopic nonlinear polarization. Taking into account the dissipative effect, the Bloch equations of the three-level system can be written as

$$\dot{u}_1 = -\omega_1 u_4 - \gamma\Omega_x u_5 - \frac{1}{T_1}(u_1 - u_{10}), \tag{2a}$$

$$\dot{u}_2 = (\omega_1 - \omega_2)u_5 + \gamma\Omega_x u_4 + \Omega_x u_6 - \frac{1}{T_2}(u_2 - u_{20}), \tag{2b}$$

$$\dot{u}_3 = -\omega_2 u_6 + \Omega_x u_5 - \frac{1}{T_3}(u_3 - u_{30}), \tag{2c}$$

$$\dot{u}_4 = \omega_1 u_1 - 2\Omega_x u_7 - \gamma\Omega_x u_2 - \frac{1}{T_4}(u_4 - u_{40}), \tag{2d}$$

$$\dot{u}_5 = u_2(\omega_2 - \omega_1) + \gamma\Omega_x u_1 - \Omega_x u_3 - \frac{1}{T_5}(u_5 - u_{50}), \tag{2e}$$

$$\dot{u}_6 = \omega_2 u_3 - \Omega_x u_2 - 2\gamma\Omega_x u_8 - \frac{1}{T_6}(u_6 - u_{60}), \tag{2f}$$

$$\dot{u}_7 = 2\Omega_x u_4 + \gamma\Omega_x u_6 - \frac{1}{T_7}(u_7 - u_{70}), \tag{2g}$$

$$\dot{u}_8 = 2\gamma\Omega_x u_6 + \Omega_x u_4 - \frac{1}{T_8}(u_8 - u_{80}), \tag{2h}$$

where ω_1 and ω_2 represent the transition frequencies from $|1\rangle$ to $|2\rangle$ and from $|1\rangle$ to $|3\rangle$ respectively; μ_{ij} is the dipole moment between levels i and j; γ is the ratio between two transition dipole moments μ_{13} and μ_{12}, i.e. $\gamma = \mu_{13}/\mu_{12}$; $\Omega_x(= \mu_{12}E_x/\hbar)$ is the Rabi frequency of the pulse laser field. $P_x = -N\langle\boldsymbol{\mu}\rangle = N\mu_{12}(u_1 + \gamma u_3)$, where N is the density of the medium. The initial input field can be written as

$$\begin{aligned} E_x(t=0,z) = {} & E_{x1}\,\mathrm{sech}\,[1.76(z/c + z_0/c)/\tau_{\mathrm{p1}}] \\ & \times \cos\,[\omega_{\mathrm{p1}}(z + z_0)/c)] \\ & + E_{x2}\,\mathrm{sech}\,[1.76(z/c + z_0/c)/\tau_{\mathrm{p2}}] \\ & \times \cos\,[\omega_{\mathrm{p2}}(z + z_0)/c) + \varphi]\,, \end{aligned}$$

where τ_{p1} and τ_{p2} are the full widths at half-maximum of the pulse intensity envelopes of the two laser pulses, respectively; φ is the initial relative carrier–envelope phase (RCEP) of the two pulses. The areas of the two pulses are $A_1 = \Omega_{m1}\tau_{\mathrm{p1}}\pi/1.76\hbar$ and $A_2 = \Omega_{m2}\tau_{\mathrm{p2}}\pi/1.76\hbar$, respectively. $\Omega_{m1} = \mu_{12}E_{x1}/\hbar$ and $\Omega_{m2} = \mu_{12}E_{x2}/\hbar$ are the maximum Rabi frequencies of the two laser pulses, respectively. In this paper, we define the effective area as $A_i' = \sqrt{A_i^2 + \gamma^2 A_i^2}$ $(i = 1, 2)$. In the following numerical analysis, the initial conditions are $u_i(0) = 0\,(i = 1, \ldots, 6)$, $u_7(0) = -1$, $u_8(0) = -1$ and this means that at the initial time all atoms are at level $|1\rangle$. The values of the material parameters and the pulse parameters are $\tau_{\mathrm{p1}} = \tau_{\mathrm{p2}} = 5$ fs, $\omega_{\mathrm{p1}} = \omega_1 = 2.37$ fs^{-1}, $\omega_{\mathrm{p2}} = \omega_2 = 2.42$ fs^{-1}, $\mu_{12} = 1.48 \times 10^{-29}$ A·s·m, $\gamma = \sqrt{2}$, $T_1 = \cdots = T_8 = 1 \times 10^{-10}$ s.

3. Numerical results and analysis

First we consider the resonant propagation of the two-colour femtosecond pulses (ω_{p1}, ω_{p2}) with effective areas $A_1' = A_2' = 2\pi$ for different values of φ as shown in Fig. 2.

From Fig. 2 we can see that RCEP has a great influence on the variation of the two-colour pulses in the propagation process. When $\varphi = 0\pi$ (see Fig. 2(a)), with propagation distance increasing, the pulse splitting occurs at $z = 96$ μm. When $\varphi = 0.25\pi$ (see Fig. 2(b)), the pulse splittings occur at $z = 60$ μm and $z = 96$ μm; the sub-pulse amplitude is smaller, the sub-pulse width is wider, and the interval between the main pulse and sub-pulse is larger than those in the case of $\varphi = 0$. When φ increases to a certain value, the

pulse splitting does not rise any more when $\varphi = 0.5\pi$ and $\varphi = 2\pi/3$, and approximate self-induced transparency (SIT)[25,26] appears when $\varphi = 2\pi/3$. Figure 2(c) shows that when $\varphi = 0.5\pi$ many obvious oscillations are present on the pulse trailing edge in the propagation process and this is just due to self-phase modulation.[26,27] Now two questions arise: why does the pulse splitting occur when the value of φ is smaller ($\varphi = 0$ and $\varphi = 0.25\pi$), and why does the pulse splitting not appear when the value of φ is larger ($\varphi = 0.5\pi$ and $\varphi = 2\pi/3$)? This is due to the fact that when the value of φ is smaller, the dispersion is larger (see analysis bellow), and a larger dispersion will be favourable for pulse splitting. Because the different frequency components of the pulse correspond to different group velocities due to the group velocity dispersion (GVD) effect.[28] For the pulse with larger dispersion, e.g. the pulse corresponding to $\varphi = 0$ (Fig. 2(a)) with propagation distance increasing, the temporal interval between higher- and lower-frequency components reaching the same positions increases. When the propagation distance is long enough, hence the temporal interval becomes large enough correspondingly, then the higher-frequency part and lower-frequency part of the pulse will both split into two pulses: main-pulse and sub-pulse; for example, when $\varphi = 0$ (Fig. 2(a)), the pulse splitting occurs at $z = 96$ μm. Why is the width of the sub-pulse much larger than that of the mainpulse in Fig. 2(a)? This is because the sub-pulse includes a much wider frequency range than the main pulse.

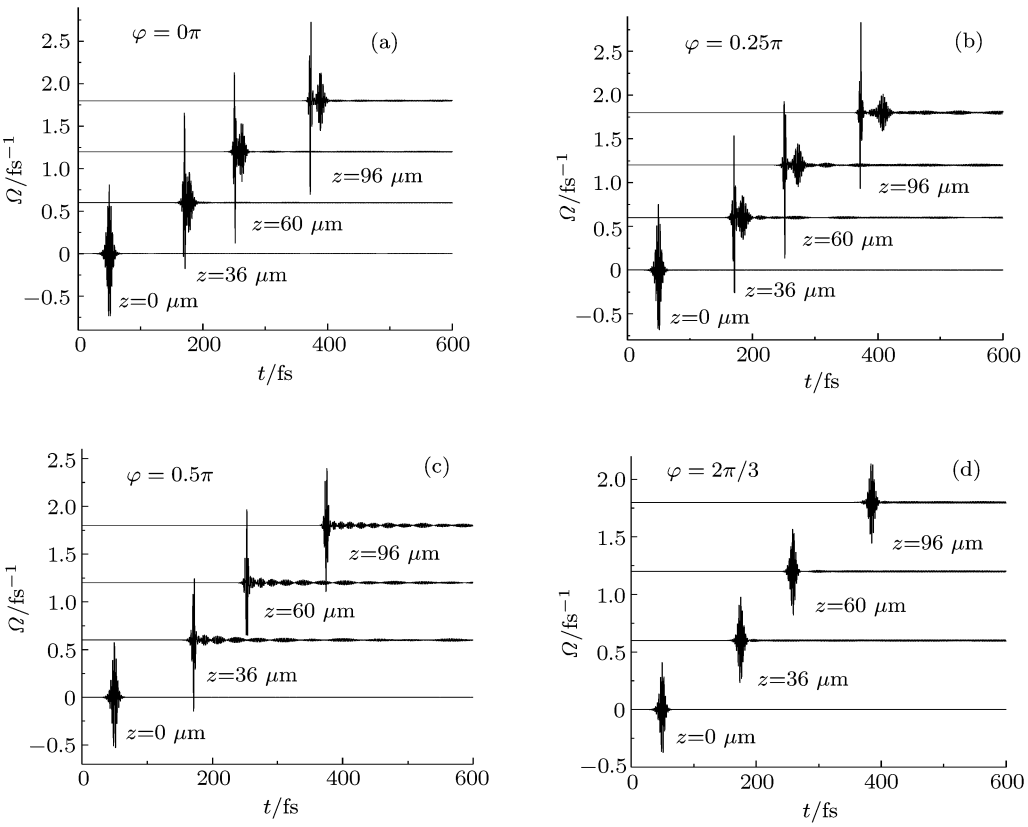


Fig. 2. Temporal evolutions of the pulse at distances $z = 0$, 36, 60, 96 μm with $N = 6 \times 10^{24}$ m^{-3} and $A_1' = A_2' = 2\pi$ for different values of φ.

Comparing carefully the pulse propagations corresponding to different values of φ, we find that at the same propagation distance, the time moment at which the pulse appears when the value of φ is smaller is earlier than that when the value of φ is larger as shown in Fig. 3, which means that when the value of φ is smaller, the pulse group velocity is larger. And this is due to the fact that the group velocity is proportional to the square of pulse amplitude[29] and the pulse with a smaller value of φ has a larger amplitude.

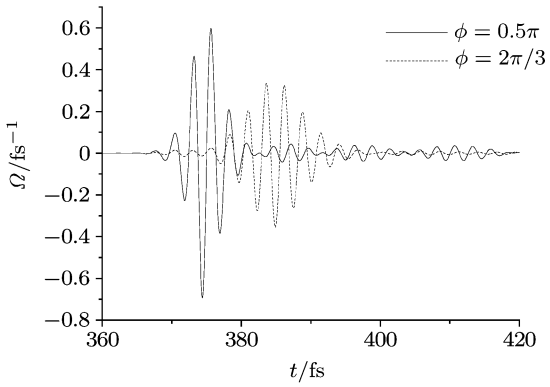


Fig. 3. Comparison between temporal evolutions of the pulse at $z = 96$ μm for $\varphi = 0.5\pi$ and $\varphi = 2\pi/3$ with the same parameter values as those in Fig. 2.

Figure 4 depicts temporal evolutions of populations corresponding to those in Fig. 2 at the input position ($z = 0$ μm). When $\varphi = 0$ and $\varphi = 0.25\pi$, for each level of the three levels twice Rabi oscillations of populations are complete and not complete, respectively; when $\varphi = 0$, populations of the three levels are the same before and after the interaction of the two-colour pulses with medium has occurred, i.e. $\rho_{11} = 1$, $\rho_{22} = \rho_{33} = 0$; when $\varphi = 0.25\pi$, there is about 30% of the particles in the two excited level states after the interaction has occurred. When $\varphi = 0.5\pi$ and $\varphi = 2\pi/3$ for each level of the three levels only one irregular oscillation and one approximate Rabi oscillation of populations are present, respectively. When $\varphi = 0.5\pi$, there is about nearly 90% of the particles in the two excited states after the interaction has occurred; when $\varphi = 2\pi/3$, populations of the three levels are approximately the same before and after the interaction has occurred. Our numerical result shows that at other propagation distances, different values of RCEP will lead to different populations. And this shows that by adjusting the value of RCEP, different population evolutions and transformations can be obtained.

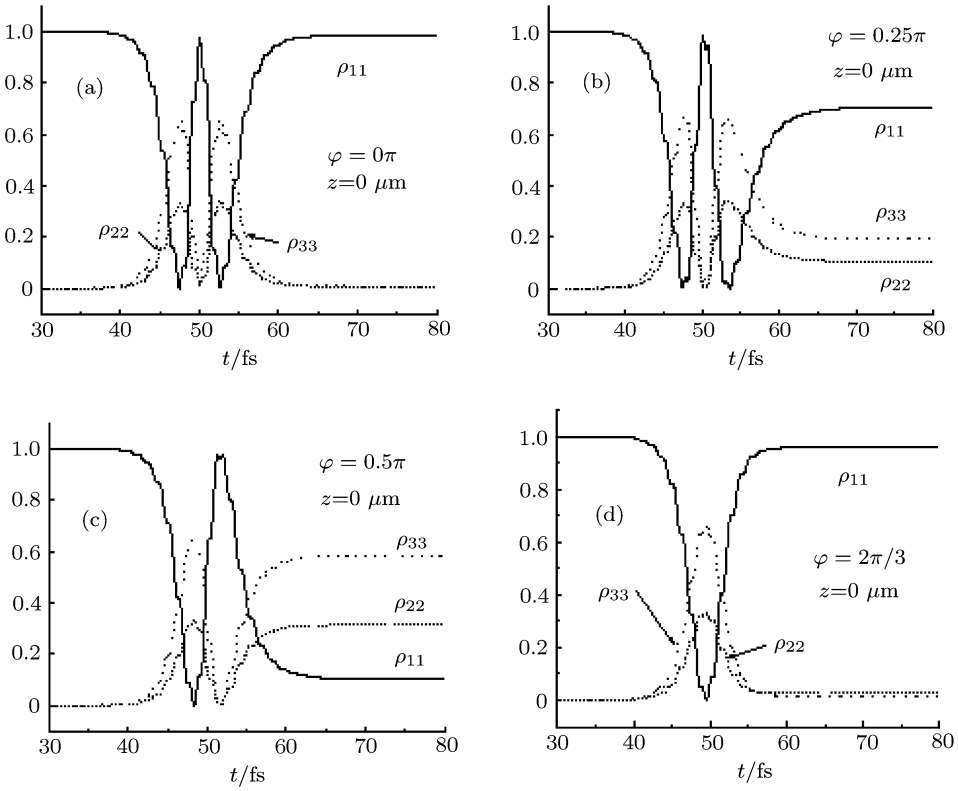


Fig. 4. Temporal evolutions of populations corresponding to those in Fig. 2 at $z = 0$ μm.

From our numerical results similar conclusions about effect of RCEP on pulse shape, group velocity and populations can be obtained for the case of larger pulse area.

Figure 5 shows the spectral evolutions corresponding to those in Fig. 2(a) for $\varphi = 0$. In the propagation process, the pulse spectra are slightly broadened but there are a few higher components. Oscillatory features can be found around the resonance frequency and they are consistent with those in a two-level medium.[17] The oscillatory features become more obvious with propagation distance increasing.

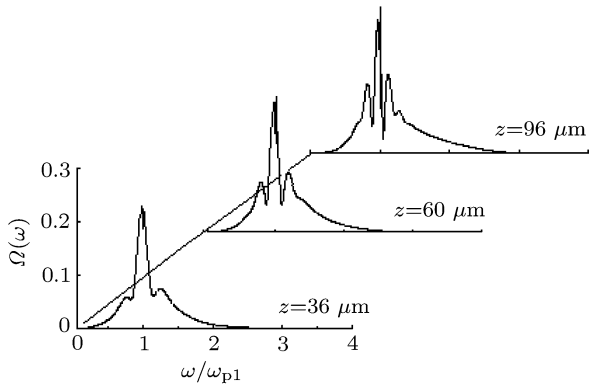


Fig. 5. Temporal evolutions of spectra at distances $z = 36$, 60, 96 μm corresponding to those in Fig. 2(a) for $\varphi = 0$.

The RCEP also has an obvious influence on the pulse spectral properties. Figure 6 presents the spectra corresponding to those in Fig. 2 at $z = 96$ μm for different values of φ. From Fig. 6 we can see that around the resonance frequency ($\omega = \omega_{p1}$), each spectral strength exhibits an oscillation feature, the oscillation number increases rapidly but amplitude of the spectral component decreases with the value of φ increasing; in addition, the frequency blueshift in the region $\omega > \omega_{p1}$ and the frequency redshift in the region $\omega < \omega_{p1}$ both decrease with the value of φ increasing, so spectral frequency range decreases considerably. This means that the dispersion decreases with the value of φ increasing and this is due to the fact that an increase in value of φ leads to a reduce in the total electric field intensity, thereby reducing the coherent interaction of the electric field with the medium. A similar phenomenon also appears at other propagation distances.

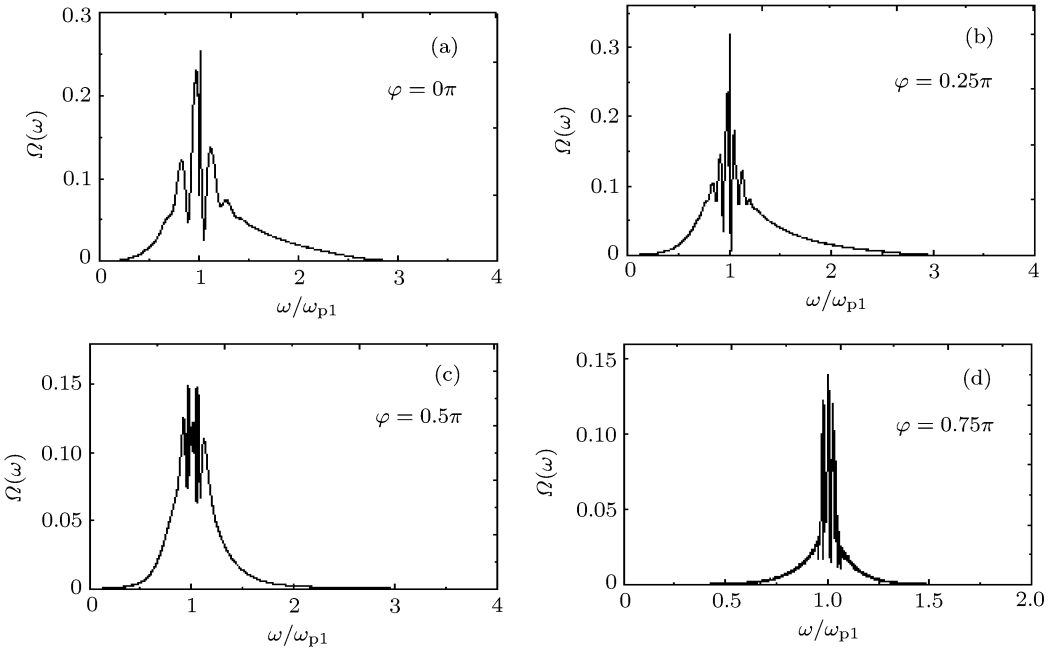


Fig. 6. Temporal evolutions of spectra corresponding to those in Fig. 2 at $z = 96$ μm.

Our numerical results show that with the pulse areas increasing, spectral strength, especially the strength of the higher frequency component, increase significantly, but the largest frequency range has just a little increase. For the case with a larger area, except the case for $\varphi = 0.25\pi$, with the value of φ increasing, spectral strength decreases remarkably but the relative strength of the higher frequency component increases significantly, while the spectral frequency range is not varied evidently. As an example, figure 7 presents the temporal evolutions of spectra at $z = 96$ μm with $N = 6 \times 10^{24}$ m^{-3} and effective areas $A_1' = A_2' = 8\pi$.

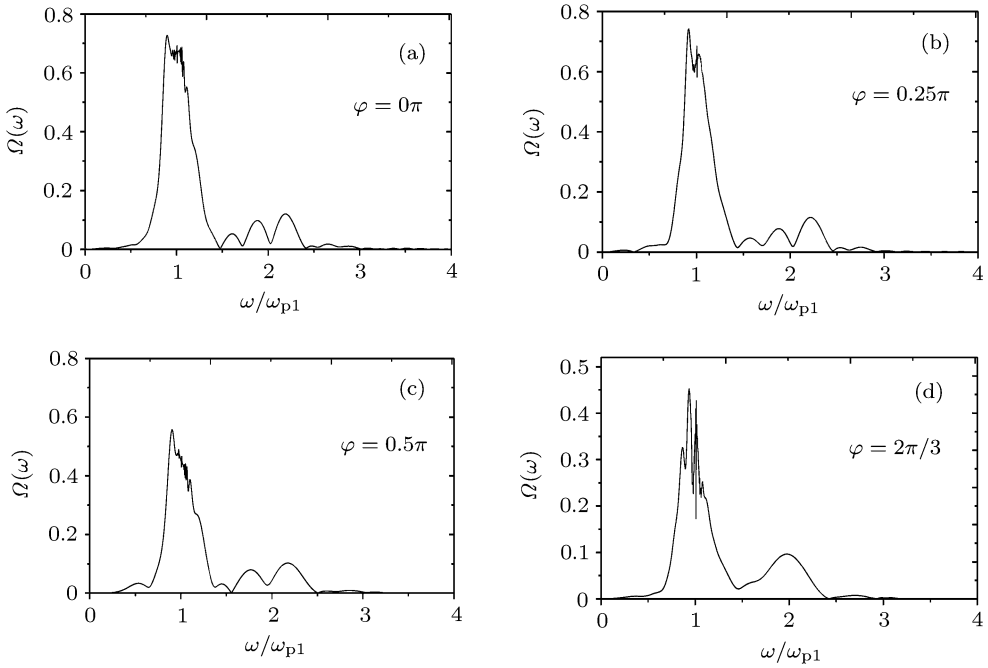


Fig. 7. Temporal evolutions of spectra at $z = 96$ μm with $N = 6 \times 10^{24}$ m^{-3} and $A_1' = A_2' = 8\pi$.

4. Conclusions

Using the numerical solution of the full Maxwell–Bloch equations, which is obtained by finite-difference time-domain method and the predictor–corrector method, we investigated the modulation effect of the relative carrier–envelope phase (RCEP) of two femtosecond ultrashort laser pulses on resonant propagation behavior in a V-type three-level atomic medium. It is found that the pulse splitting occurs for a smaller value of RCEP; when the value of RCEP increases to a certain value, only pulse shape varying is present and pulse splitting does not occur any more; moreover, when the value of RCEP is smaller, the pulse group velocity is larger. The RCEP also has an obvious effect on the populations and the spectral property. Different population transfers can be realized by adjusting the value of RCEP. Generally speaking, for the two pulses with smaller areas their spectral strengths and frequency ranges decrease obviously with the value of RCEP increasing; for the two pulses with larger areas, with the value of RCEP increasing, their spectral strengths decrease remarkably but the relative strengths of the higher frequency components increase significantly, while the spectral frequency range is not evidently varied.

References

[1] Brabec T and Krausz F 2000 *Rev. Mod. Phys.* **72** 545

[2] Agostini P and DiMauro L 2004 *Rep. Prog. Phys.* **67** 813

[3] Winterfeldt C, Spielmann C and Gerber G 2008 *Rev. Mod. Phys.* **80** 117

[4] Feng Z H, Fu X Q, Zhang L F, Xu H W and Wen S C 2008 *Acta Phys. Sin.* **57** 2253 (in Chinese)

[5] Hu Q L, Liu S B and Li W 2008 *Chin. Phys.* B **17** 1050

[6] Peng Y J, Liu Y Q, Wang Y H, Zhang S P and Yang Y Q 2009 *Acta Phys. Sin.* **58** 655 (in Chinese)

[7] Lu D Q, Hu W, Qian L J and Fan D Y 2009 *Acta Phys. Sin.* **58** 1655 (in Chinese)

[8] Sun Y P, Liu J C and Wang C K 2009 *Acta Phys. Sin.* **58** 3934 (in Chinese)

[9] Quan W, Lin Z, Wu M, Kang H, Liu H, Liu X, Chen J, Liu J, He X T, Chen S G, Xiong H, Guo L, Xu H, Fu Y, Cheng Y and Xu Z Z 2009 *Phys. Rev. Lett.* **103** 093001

[10] Li L S, Feng G Y, Yang J, Zhou G R, Zhou H, Zhu Q H, Wang J J and Zhou S H 2009 *Acta Phys. Sin.* **58** 7005 (in Chinese)

[11] Yang W F, Song X H, Gong S Q, Cheng Y and Xu Z Z 2007 *Phys. Rev. Lett.* **99** 133602

[12] Zuo T and Bandrauk A D 1996 *Phys. Rev.* A **54** 3254

[13] Song X H, Gong S Q, Jin S Q and Xu Z Z 2003 *Phys. Lett.* A **319** 150

[14] Brown A and Meath W J 2002 *Phys. Rev.* A **65** 063401
[15] Avetissian H K, Avchyan B R and Mkrtchian G F 2006 *Phys. Rev.* A **74** 063413
[16] Xia K Y, Niu Y P, Li C F and Gong S Q 2007 *Phys. Lett.* A **361** 173
[17] Song X H, Gong S Q, Jin S Q and Xu Z Z 2004 *Phys. Rev.* A **69** 015801
[18] Zhang C L, Qi Y Y and Liu X S 2007 *Acta Phys. Sin.* **56** 774 (in Chinese)
[19] Liu C P, Gong S Q, Li R X and Xu Z Z 2004 *Phys. Rev.* A **69** 023406
[20] Zuo T and Bandrauk A D 1995 *Phys. Rev.* A **51** 3991
[21] Serrat C 2005 *Phys. Rev.* A **72** 023808
[22] Serrat C 2005 *Phys. Rev.* A **72** 051803
[23] Loiko Y and Serrat C 2006 *Phys. Rev.* A **73** 063809
[24] Yee K S 1966 *IEEE Trans. Antennas Propag.* **14** 302
[25] Ziolkowski R W, Arnold J M and Gogny D M 1995 *Phys. Rev.* A **52** 3082
[26] McCall S L and Hahn E L 1967 *Phys. Rev. Lett.* **18** 908
[27] Lamb Jr G L 1971 *Rev. Mod. Phys.* **43** 99
[28] Engelen R J P, Sugimot Y, Watanabe Y, Korterik J P, Ikeda N, van Hulst N F, Asakawa K and Kuipers L 2006 *Opt. Express* **14** 1658
[29] Tarasishin A V, Magnitskii S A, Shuaev V A and Zheltikov A 2001 *Opt. Express* **8** 452