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Electron trajectory evaluation in laser-plasma interaction for effective output beam

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Using the ellipsoidal cavity model, the quasi-monoenergetic electron output beam in laser-plasma interaction is described. By the cavity regime the quality of electron beam is improved in comparison with those generated from other methods such as periodic plasma wave field, spheroidal cavity regime and plasma channel guided acceleration. Trajectory of electron motion is described as hyperbolic, parabolic or elliptic paths. We find that the self-generated electron bunch has a smaller energy width and more effective gain in energy spectrum. Initial condition for the ellipsoidal cavity is determined by laser-plasma parameters. The electron trajectory is influenced by its position, energy and cavity electrostatic potential.

Keywords: electron, intense laser, plasma accelerator, laser wake field, bubble regime, wave breaking

PACC: 4260H, 4260K, 5240, 5240D

1. Introduction

Generation of laser pulses with multi-terawatt (or even pettawatt) power has been possible with compact CPA systems in recent years. In this power rate, laser intensity increases up to $I = 10^{22}$ W/cm^2 in order to obtain the electric field strength more than 10^{14} V/m.[1,2] Accelerated particles in this high gradient field have various applications, including transmutation of cheap and hazardous materials with long-lived radioactive wastes into the valuable radioisotopes.[3] Large amplitude plasma waves are generated by the effect of ponderomotive force in the laser wake field accelerator (LWFA). This ponderomotive force is given by $F_{\rm p} \approx -\nabla a^2$, where $F_{\rm p}$ is ponderomotive force and a is the laser pulse envelope. The acceleration gradient resulting from the charge displacement is reported to be about 100 GV/cm when plasma density is 10^{18} cm^{-3}.[4–8] To obtain high efficiency of accelerated electron, it should be injected in an appropriate phase of plasma wave. Injection of the background plasma electrons is possible instead of an external electron injector (e.g. a linac) when the wavebreaking occurs.[4–12] By using the steepened density profile the wave-breaking injection can be made fast. A well collimated, ultra short MeV electron bunch is obtained due to the transverse wave breaking[13–15] through using a shock wave driven by the irradiation of laser prepulses.[16,17] The generation of quasi-monoenergetic electrons is reported in a recent experiment and PIC simulation.[18,19] A cavity free from cold plasma electrons (bubble) behind the laser pulse is observed.[20] The following features are absent in the ordinary regime of laser wake field acceleration:[18,21–25] (i) a cavity free from cold plasma electrons, instead of a periodic plasma wave, is formed behind the laser pulse; (ii) a dense bunch of relativistic electrons with a quasi-monoenergetic spectrum is self-generated; (iii) the laser pulse propagates many Rayleigh lengths in the homogeneous plasma without a significant diffraction. Cavity behind the laser pulse is shown by shadowgraphs[20] and PIC simulation.[26] In the present paper, we present a new ellipsoid model instead of the previous spheroidal cavity. In fact, the cavity shape is not exactly spheroidal, and this is a defect in previous studies.[21,27,28] Some differences among shadowgraphs, PIC simulation and analytical calculation results are reported because of the assumption of spheroidal cavity shape. Appropriate conditions of forming an ellipsoidal cavity are obtained. We evaluate fields inside this cavity and energy spectrum for relativistic trapped electrons. In this work, we obtain energy and electron gain when self-focusing is considered.

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2. Electron trajectory and cavity model

Considering electron dynamic equations, internal electromagnetic fields and ponderomotive force of the laser propagation, we obtain the following equation:

$$\begin{aligned}\ddot{r}\hat{r} &= (\ddot{r} - r\dot{\theta}^2)\hat{r} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})\hat{\theta} \\ &= \frac{1}{\mu}\left(\frac{-eQ_i}{4\pi\varepsilon_0 r^2}\hat{r} + E_P\hat{\theta}\right),\end{aligned} \tag{1}$$

where E_p, μ, r, $\dot{r}$, θ, $\dot{\theta}$, Q_i, e, and ε_0 are the ponderomotive force, the electron relativistic mass approximately with laser group velocity, the radial component, the derivative of radial component, the tangential component, the derivative of tangential component, the average ion charge in the cavity, the electron charge and the cavity dielectric constant, respectively. Equation (1) can be separated into radial and tangential components. We can assume that for electron motion in the cavity sheath, radius vector r sweeps out angle $\mathrm{d}\theta$ in short time $\mathrm{d}t$, if S is defined as the area swept then we have $\mathrm{d}S = \frac{1}{2}r^2\mathrm{d}\theta$ and $\mathrm{d}S/\mathrm{d}t = \frac{1}{2}r^2\dot{\theta}$. We usually define constant angular momentum (ℓ) of the system as $\ell = |E| = \mu r^2\dot{\theta}$. So, $\dot{\theta} = \ell/\mu r^2$, and $\mathrm{d}S/\mathrm{d}t = \ell/2\mu = \text{const.}$, then

$$\ddot{r} = r\dot{\theta}^2 - \left(\frac{e}{\mu}\frac{Q_i}{4\pi\varepsilon_0 r^2}\right) = \frac{\ell^2}{\mu^2 r^3} - \left(\frac{e}{\mu}\cdot\frac{Q_i}{4\pi\varepsilon_0 r^2}\right). \tag{2}$$

Assume that solution $r\,(t)$ can be written as $r = r(\theta)$, where $\theta = \theta(t)$, then we will seek a solution in the form of $u(r) = 1/r(\theta(t))$

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t},$$

recall $\ell = \mu r^2\dot{\theta}$ and $r = 1/u$, so

$$\dot{r} = -\frac{1}{u^2}\frac{\mathrm{d}u}{\mathrm{d}\theta}\frac{\ell}{\mu r^2} = -\frac{\ell}{\mu}\cdot\frac{\mathrm{d}u}{\mathrm{d}\theta} \tag{3}$$

and

$$\begin{aligned}\ddot{r} &= -\frac{\ell}{\mu}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}u}{\mathrm{d}\theta}\right) = -\frac{\ell}{\mu}\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2}\frac{\mathrm{d}\theta}{\mathrm{d}t} \\ &= -\frac{\ell^2}{\mu^2}u^2\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} = \frac{\ell^2}{\mu^2}u^3 - ku^2,\end{aligned}$$

where

$$k = -\frac{e}{\mu}\cdot\frac{Q_i}{4\pi\varepsilon_0},$$

$$\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = \frac{1}{\alpha}, \tag{4}$$

and

$$\frac{1}{\alpha} = \frac{k\mu^2}{\ell^2} = -\frac{e\mu}{\ell^2}\frac{Q_i}{4\pi\varepsilon_0},$$

$$u(\theta) = A\cos\theta + \frac{1}{\alpha}. \tag{5}$$

So

$$r(\theta) = \frac{1}{u} = \frac{1}{A\cos\theta + 1/\alpha}, \tag{6}$$

then if we set $A = \varepsilon/\alpha$

$$r(\theta) = \frac{\alpha}{1 + \varepsilon\,\cos\,\theta}, \tag{7}$$

where α and ε are constants

In Eq. (7) we find that the electron trajectory is hyperbolic if $\varepsilon > 1$, parabolic if $\varepsilon = 1$, elliptic if $0 < \varepsilon < 1$, and circular if $\varepsilon = 0$, which are shown in Fig. 1.

For the case of elliptic trajectory ($\varepsilon \neq 1$, $0 < \varepsilon < 1$), we define the length of the ellipse long axis $= 2a_\mathrm{e}$, then $2a_\mathrm{e} = r(0) + r(\pi) = 2\alpha/(1-\varepsilon^2)$ and $r(\theta) = a_\mathrm{e}(1-\varepsilon^2)/(1 + \varepsilon\,\cos\,\theta)$.

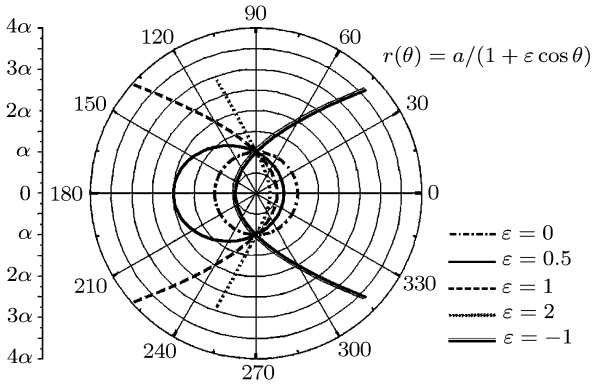


Fig. 1. Shapes of electron trajectory for different values of ε: hyperbola if $\varepsilon > 1$, parabola if $\varepsilon = 1$, ellipse if $0 < \varepsilon < 1$, and circle if $\varepsilon = 0$.

3. Discussion

Constant A in Eq. (6) can be determined by the initial condition of the electron. It means that the trajectory will depend strongly on the initial condition. For an electron far from the bubble $\theta = 0$, $r \to \infty$, $r = \alpha/(1 + \cos\theta)$ that shows a parabolic trajectory. For an electron at the margin of the bubble condition $\theta = 0$, $r = R$,

$$r = \frac{\alpha}{1 - (\alpha/R - 1)\cos\theta}, \quad \frac{\alpha}{R} = -\frac{l^2/\mu R^2}{eQ_i/4\pi\varepsilon_0 R},$$

where R is the radius of the bubble, and α/R is the ratio of electron energy to the electrostatic potential. So the electron trajectory will depend on α/R. The electron has a circular trajectory if $\alpha/R - 1 = 0$ or $\alpha/R = 1$. The electron has an elliptic trajectory if $0 < \alpha/R - 1 < 1$ or $1 < \alpha/R < 2$. The electron trajectory is influenced by its position, energy and cavity electrostatic potential.

4. Electromagnetic fields and electron energy inside the cavity

We suppose an electrically neutral ellipsoidal cavity in plasma, with axes $2a_{\rm e}$, $2b_{\rm e}$ and $2c_{\rm e}$. The electrostatic potential inside the cavity is calculated in volt unit in Ref. [29] to be

$$\psi(x,y,z) = (enV/c_{\rm cavity}) + \left\{3ena_{\rm e}b_{\rm e}c_{\rm e}V/2\left[(a_{\rm e}b_{\rm e})^2 + (a_{\rm e}c_{\rm e})^2 + (b_{\rm e}c_{\rm e})^2\right]\right\}\left[1-(x/a_{\rm e})^2-(y/b_{\rm e})^2-(z/c_{\rm e})^2\right], \quad (8)$$

where $V = 4\pi a_{\rm e}b_{\rm e}c_{\rm e}/3$ is the volume of the ellipsoid and n is the number of ions per unit volume of a sample. We normalize Eq. (8) and consider $\Phi_0 = 3a_{\rm e}b_{\rm e}c_{\rm e}c_{\rm cavity}/2\varepsilon$, where ε is the constant and $c_{\rm cavity}$ is the capacitance of the cavity in vacuum then we normalize the potential to unity at the ellipsoid boundary and obtain

$$\Phi(x,y,z) = 1 + \Phi_0\left[1-(x/a_{\rm e})^2-(y/b_{\rm e})^2-(z/c_{\rm e})^2\right]. \quad (9)$$

The ion dynamics is neglected so the electromagnetic fields inside the relativistic cavity are calculated as

$$E_\zeta = -2\zeta\Phi_0/a_{\rm e}^2, \quad E_y = -B_z = -2y\Phi_0/b_{\rm e}^2,$$
$$B_x = 0, \quad E_z = B_y = -2z\Phi_0/c_{\rm e}^2. \quad (10)$$

We have used all quantities dependent on $\zeta = x - v_0 t$ instead of x and t while the cavity runs with the relativistic velocity $v_0 \approx 1$.

The Lorentz forces (f_x, f_y and f_z) acting on a relativistic electron inside the cavity are

$$\frac{f_x}{-e} = -E_x = 2\zeta\Phi_0/a_{\rm e}^2,$$
$$\frac{f_y}{-e} = -E_y + B_z = 4y\Phi_0/b_{\rm e}^2,$$
$$\frac{f_z}{-e} = -E_z - B_y = 4z\Phi_0/c_{\rm e}^2. \quad (11)$$

The Lorentz force for the electron in the laser propagation direction is zero when its velocity reaches v_0, so the cavity can hold the isoenergetic electron bunch. The above equations show that the slopes of the fields are related to the reciprocals of squared axis-lengths, this can compensate the effect of beam loading during the cavity growing.

By using the Hamiltonian equation, we analyse the dynamics of electron trapping and acceleration in an ellipsoidal cavity as follows:[30]

$$H = \sqrt{(\boldsymbol{p}_c - q\boldsymbol{A})^2 c^2 + (m_0c^2)^2} + q\Phi_{\rm e}, \quad (12)$$

where $\boldsymbol{p}_c$ is the particle canonical momentum, q is the charge, and $\Phi_{\rm e}$ is the electrostatic scalar potential. The longitudinal field of laser can be written as

$$E_x = E_0\sin(\omega t - k_x x) = -\frac{\partial A_x}{\partial t},$$

where

$$A_x = \frac{E_0}{k_x v_\varphi}\cos(k_x(v_0 t - x)),$$

E_x is the electric field, E_0 is the peak electric field of the wave, A_x is the x component of the electromagnetic vector potential, ω is the laser frequency, t is the time, k_x is the laser wave number, x is the longitudinal position, and v_0 is the phase velocity of the wave. Ultimately, we describe the position of the particle by the phase it occupies in the forward wave. The logical choice for the new coordinate is $\zeta = x - v_0 t$, which is equivalent to a transformation of z into the frame of the forward traveling wave. We can write down the transformed Hamiltonian, $\overline{H}_{\parallel}$, so as to preserve the proper equations of motion, as follows:

$$\bar{H}_{||}(\zeta, p_{\zeta,c}) = \sqrt{\left(p_{\zeta,c} + e\frac{E_0}{K_x v_0}\cos(-k_x\zeta)\right)^2 c^2 + (m_0c^2)^2} - v_0 p_{\zeta,c} - e\Phi_0\left(1 - \frac{\zeta^2}{a_{\rm e}^2}\right). \quad (13)$$

We find

$$\frac{{\rm d}\zeta}{{\rm d}t} = \frac{\partial\bar{H}_{||}}{\partial p_{\zeta,c}} = \frac{p_c}{\gamma m_0} - v_0 = v_x - v_0, \quad (14)$$

and

$$\frac{{\rm d}p_{\zeta,c}}{{\rm d}t} = -\frac{\partial\bar{H}_{||}}{\partial\zeta} = \frac{-eE_0 v_x}{v_0}\sin(k_x\zeta) + \frac{2\zeta}{a_{\rm e}^2}v_0. \quad (15)$$

For an ion ellipsoidal cavity with semi-axial distance, equation (13) reduces into

$$\bar{H}_{||}\left(\zeta, p_{\zeta,c}\right) \cong p_{\zeta,c}c + \frac{e\Phi_0\zeta^2}{a_{\rm e}^2}. \tag{16}$$

The solution of the Hamilton equations is

$$\zeta \propto ct, \quad \dot{p}_x \propto \frac{-2e\Phi_0 ct}{a_{\rm e}^2}, \tag{17}$$

where $\Phi_0 = 3a_{\rm e}b_{\rm e}c_{\rm e}c_{\rm cavity}/2\varepsilon$, which has been described in the previous section. The maximum energy of the accelerated electrons peaks at the cavity centre

$$\gamma_{\max} \approx \frac{3b_{\rm e}c_{\rm e}c_{\rm cavity}e}{2\varepsilon m_0 c}. \tag{18}$$

We find that the cavity elongation in the laser propagation direction has no effect on electron energy, and the elongations in other directions are small and can be neglected. Initial condition for obtaining the ellipsoidal cavity is determined by laser-plasma parameters.

According to the discussion in Section 2 (Eq. (8)) the initial condition (ε) for obtaining the ellipsoidal regime is strongly related to laser and plasma parameters, but if the ellipsoidal cavity regime is obtained ($0 < \varepsilon < 1$), the electron energy in the ellipsoidal cavity has a low spread spectrum and its maximum value in the centre is not related to cavity elongation although the longitudinal elongation of the cavity during the laser pulse propagation is observed in experimental and PIC simulation results.

In summary, it can be considered that the electrons of the bunch have equal energies and the ellipsoidal cavity holds the electron bunch in a quasi-monoenergetic case, which is better than the case of the previous spheroidal model. In the spheroid model, the energy of the accelerated electrons peak is given by[21]

$$\gamma_{\max} \approx \frac{1}{2}\gamma_0^2 R^2, \tag{19}$$

where R is the radius of spheroidal bubble and γ_0 is determined by $\gamma_0 = (1 - v_0)^{-1/2}$, with v_0 being the laser pulse group velocity.[21] Equation (19) shows that the electron bunch energy is strongly related to the transverse radius of cavity and during the laser propagation because of transverse elongation of cavity the energy peak spectrum will spread. But in the ellipsoid model equation (18) shows that the longitudinal elongation ($a_{\rm e}$) has no effect on the energy spectrum of an electron bunch. The resulting electron beam in the ellipsoid model will be quasi-monoenergetic (see Fig. 2)

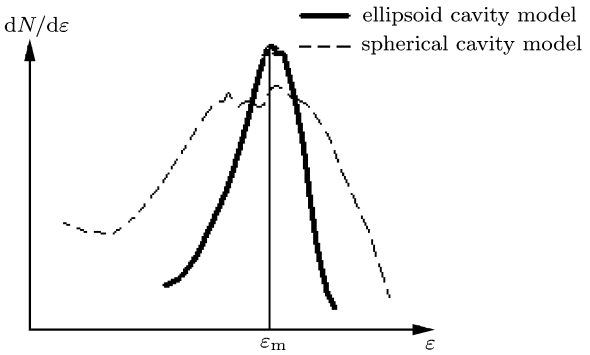


Fig. 2. Comparison between ellipsoidal and spheroidal cavity models.

5. Conclusions

Cavity, instead of periodic plasma wave, is possible to be generated behind the laser pulse. By increasing the intensity of laser pulse, the wave-breaking is observed. Wave-breaking can be a starting point for other acceleration regimes acting as cavities. The previous cavity model such as spheroid model has some differences in comparison with experimental and PIC simulation results. In this work we presented a new ellipsoid model for the first time. We showed that the electron trajectory can be shaped into hyperbola, parabola and ellipse and there are initial and final conditions for the formation of ellipsoidal cavity. A dense bunch of relativistic electrons with a monoenergetic spectrum is self-generated. The fields linearly depend on coordinate. It is shown that the cavity elongation has no effect on maximum electron energy, so the quality of electron beam is improved

References

[1] Umstadter D 2003 *J. Phys.* D: *Appl. Phys.* **36** R151

[2] Pukhov A and Meyer-ter Vehn J 2002 *Appl. Phys.* B: *Lasers and Optics* **74** 355

[3] Sadighi-Bonabi R and Kokabee O 2006 *Chin. Phys. Lett.* **23** 1434

[4] Bulanov S V, Kirsanov V I and Sakharov A S 1991 *JETP Lett.* **53** 565

[5] Hemker R G, Tzeng K C, Mori W B, Clayton C E and

Katsouleas T 1998 *Phys. Rev.* E **57** 5920

[6] Liseikina T V, Califano F, Vshivkov V A, Pegoraro F and Bulanov S V *Phys. Rev.* **60** 5991

[7] Hemker R G, Hafz N M and Uesaka M 2002 *Phys. Rev. ST Accel. Beams* **5** 041301

[8] Pukhov A and Meyer-Ter-Vehn J 2002 *Appl. Phys.* B: *Lasers Opt.* **74** 355

[9] Bulanov S, Naumova N, Pegoraro F and Sakai J 1998 *Phys. Rev.* E **58** R5257

[10] Suk H, Barov N, Rosenzweig J B and Esarey E 2001 *Phys. Rev. Lett.* **86** 1011

[11] Tomassini P, Galimberti M and Giulietti A 2004 *Laser Part. Beams* **22** 423

[12] Tomassini P, Galimberti M and Giulietti A 2003 *Phys. Rev. ST Accel. Beams* **6** 121301

[13] Ohkubo T, Zhidkov A, Hosokai T, Kinoshita K and Uesaka M *Phys. Plasmas* (to be published)

[14] Bulanov S V, Pegoraro F, Pukhov A M and Sakharov A S 1997 *Phys. Rev. Lett.* **78** 4205

[15] Bulanov S V, Califano F and Dudnikova G I 1999 *Plasma Phys. Rep.* **25** 468

[16] Hosokai T, Kinoshita K, Zhidkov A, Nakamura K, Watanabe T, Ueda T, Kotaki H, Kando M, Nakajima K and Uesaka M 2003 *Phys. Rev.* E **67** 036407

[17] Giulietti D and Galimberti M 2002 *Phys. Plasmas* **9** 3655

[18] Malka V, Lifschitz A, Faure J and Glinec Y 2006 *Phys. Rev. Special Topics—Accelerators and Beams* **9** 091301

[19] Malka V, Faure J, Glinec Y, Pukhov A and Rousseau J P 2005 *Phys. Plasmas* **12** 056702

[20] Mangles S P D, Thomas A G R, Kaluza M C, Lundh O, Lindau F, Persson A, Tsung F S, Najmudin Z, Mori W B, Wahlstroöm C G and Krushelnick K 2006 *Phys. Rev. Lett.* **96** 215001

[21] Kostyukov I, Pukhov A and Kiselev S 2004 *Phys. Plasma* **11** 5256

[22] Faure J, Glinec Y, Santos J J, Ewald F, Rousseau J P, Kiselev S, Pukkov A, Hosokai T and Malka V 2005 *Phys. Rev. Lett.* **95** 205003

[23] Faure J, Glinec Y, Gallot G and Malka V 2006 *Phys. Plasmas* **13** 056706

[24] Gordienko S and Pukhov A 2005 *Phys. Plasmas* **12** 043109

[25] Esarey E, Sprangle P, Krall J and Ting A 1996 *IEEE Trans. Plasma Sci.* **24** 252

[26] Pukhov A and Meyer-Ter Vehn J 2002 *Appl. Phys.* B: *Lasers and Optics* **74** 355

[27] Pukhov A, Gordienko S, Kiselev S and Kostyukov I 2004 *Plasma Phys. Control. Fusion* **46** B 179

[28] Zobdeh P, Sadighi-Bonabi R and Afarideh H 2008 *Plasma Device and Operation* (to be published)

[29] Amusia M Y and Kornyushin Y 2000 *Contemp. Phys.* **41** 219

[30] Landau L D and Lifshitz E M 1975 *The Classical Theory of Fields, Course of Theoretical Physics* Volume 2 4th ed. (Oxoford: Pergamon Press)