Nonclassicality of photon-added squeezed vacuum states*

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By applying the bosonic creation operator to squeezed vacuum states, this paper introduces a new kind of quantum states: photon-added squeezed vacuum states. It also presents an experimental approach to prepare these states, and investigates their quantum statistical properties by the numerical method. The results indicate that these states reveal some interesting non-classical properties, such as anti-bunching effects, squeezing effects and negativities of the relevant Wigner functions.

Keywords: photon-added squeezed vacuum states, anti-bunching effect, squeezing effect, Wigner function

PACC: 4250, 0365

1. Introduction

In recent decades, non-classical optical states generated from either certain classical states or vacuum state have been extensively studied both theoretically and experimentally.[1−4] For example, Parigi *et al.* experimentally demonstrated how to generate a nonclassical state by applying alternating sequences of bosonic operators to the thermal field state,[5] Zavatta *et al.* experimentally prepared another nonclassical state, namely, single-photon-added coherent states, from the usual coherent states.[6] The nonclassical properties of these optical states have been verified by means of the novel ultrafast and quantum homodyne tomography technique.[7] Also, Kentaro *et al.* have demonstrated so-called photon-subtracted squeezed states.[8] In addition, nonclassical states could be generated via photon counting on the subsystem of an entangled state produced in parametric down-conversion.[9,10] Here we present a scheme to prepare the photon-added squeezed vacuum states by using an experimental setup shown in Fig. 1.

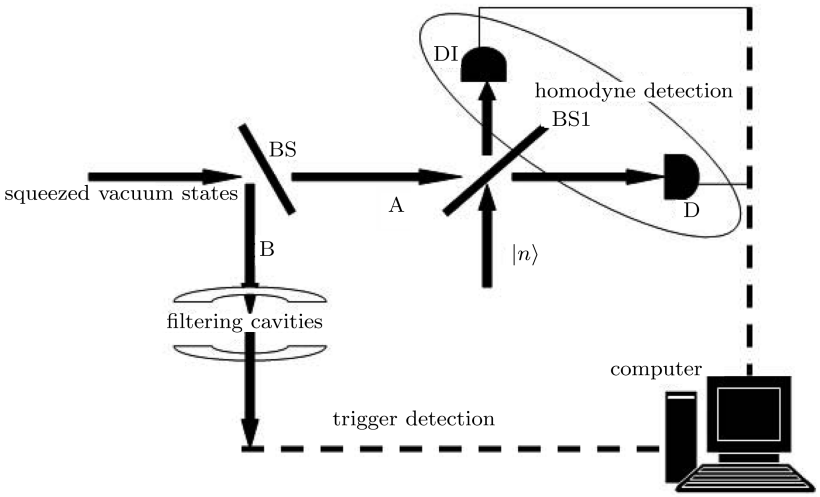


Fig. 1. Simplified experimental setup for generating photon-added vacuum states. BS and BS1: beam splitters (variable splitting ratio), D and D1: detectors, $|n\rangle$: Fock state.

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The non-classical properties of the optical state are discussed by using the anti-bunching effects, sub-Poisson distribution, the squeezed effects and the negativity of the Wigner function, etc. In the single-mode optical field, the sub-Poisson distribution and anti-bunching effects are equivalence.[11] The anti-bunching character is a core criterion[12−14] of the desirable single-photon source, which plays an important role in quantum secure communications,[5,15] while the squeezed effect reflects the quantum fluctuation nature of two orthogonal phase amplitudes.[16−19] Also, the quasi-probability distribution function, i.e., the so-called Wigner function, is usually utilized to determine a quantum state which is either nonclassical or classical. Basically, if the Wigner function of a quantum state is negative, then such a quantum state should be nonclassical and thus certain nonclassical effects could be revealed. Indeed, a series of nonclassical quantum states had been verified by showing the negativities of their Wigner functions.[20−30]

In this paper, we introduce a new kind of quantum states by applying the creation operator to the squeezed vacuum state and discuss their non-classical properties by the numerical method. The paper is organized as follows. In Section 2, we define the photon-added squeezed vacuum states, and discuss how to experimentally prepared them. Their second-order correlation functions, squeezed effects and the negativities of Wigner functions are discussed in Sections 3, 4 and 5, respectively. Finally, Section 6 presents the conclusions.

2. Photon-added squeezed vacuum states

It is well known that the squeezed vacuum state is defined by

$$|\psi_0\rangle = S(\xi)|0\rangle, \tag{1}$$

where $S(\xi) = \exp(\xi^* \hat{a}^2/2 - \xi \hat{a}^{\dagger 2}/2)$ is the squeezing operator, and $\xi = r\,\mathrm{e}^{\mathrm{i}\theta}$ is the complex squeezing parameter, $\hat{a}$ and $\hat{a}^\dagger$ represent the bosonic creation and annihilation operators, respectively. Therefore, the m-photon-added squeezed states can be written as

$$|\psi_m\rangle = \hat{a}^{\dagger m} S(\xi)|0\rangle = \hat{a}^{\dagger m} \exp\left(\frac{\xi}{2}\hat{a}^{\dagger 2}\right)|0\rangle \tag{2}$$

with m being positive integer. In Fock space these states can be rewritten as

$$|\psi_m\rangle = N_m \cdot \sum_{n=0}^{\infty} B_m |2n+m\rangle \tag{3}$$

with $B_m = (\xi/2)^2\sqrt{(2n+m)!}/n!$, $N_m^2 = 1/\sum_{n=0}^{\infty} B_m^2$ being the normalization coefficient. In this paper we focus on the simplest photon-added squeezed vacuum states, namely $m = 1$ in Eq. (3):

$$|\psi_1\rangle = N_1 \cdot \sum_{n=0}^{\infty} B_1 |2n+1\rangle \tag{4}$$

with

$$B_1 = \frac{(\xi/2)^2\sqrt{(2n+1)!}}{n!}. \tag{5}$$

The above quantum states could be prepared by using an experimental setup[8,31] displayed in Fig. 1. Firstly, the squeezed vacuum beam is split at the beam splitter (BS). A part of the squeezed vacuum beam is guided into the path A after the BS, and the other is guided into the path B. The beam of path B passes the optical filtering cavity, and is used as a trigger signal for conditional photon-added squeezed vacuum states. In fact, desirable photon-added squeezed vacuum states are generated by mixing the signal mode along the path A and another input mode of Fock state $|n\rangle$ at the beam splitter (BS1). After the BS1, in general, the output modes are prepared in a highly entangled quantum state. If the photon number of the output channel is measured and m photons are detected by the detector D1, then the quantum state of the other output channel is found to be "collapsed" to a photon-added ($n > m$) squeezed vacuum state, which can be detected by the detector D. The detector is a balanced homodyne detector with Si photodiodes.

3. Anti-bunching property of the photon-added squeezed vacuum states

The second-order correlation function

$$g^{(2)} = \frac{\langle\psi|\hat{a}^{\dagger 2}\hat{a}^2|\psi\rangle}{(\langle\psi|\hat{a}^\dagger\hat{a}|\psi\rangle)^2} \tag{6}$$

reflects the bunching or anti-bunching effects of the photons in the quantum optical state $|\psi\rangle$. The relation $g^{(2)} < 1$ indicates that the state is non-classical, wherein photons of optical field present anti-bunching effects, while $g^{(2)} \geq 1$ suggests that the optical field is classical (or random) and the relevant photons are bunching.

For the squeezed vacuum state Eq. (1), the second-order correlation function can be easily calculated as

$$g_0^{(2)} = 3 + \frac{1}{\sinh^2 r} > 1. \tag{7}$$

Obviously, photons in the squeezed vacuum state are always bunching. However, for the single photon-added squeezed vacuum states, we have

$$\langle\psi_1|\hat{a}^\dagger\hat{a}|\psi_1\rangle = N_1^2 \sum_{n=0}^{\infty} B_1^2(2n+1), \tag{8}$$

and

$$\langle\psi_1|\hat{a}^{\dagger 2}\hat{a}^2|\psi_1\rangle = N_1^2 \sum_{n=0}^{\infty} B_1^2(2n+1)(2n), \tag{9}$$

and thus the second-order correlation function reads

$$g_1^{(2)} = \frac{N_1^2 \sum_{n=0}^{\infty} B_1^2(2n+1)(2n)}{[N_1^2 \sum_{n=0}^{\infty} B_1^2(2n+1)]^2}. \tag{10}$$

Figure 2 shows that, for the relatively small values of $|\xi|$ (about $|\xi| < 0.57$) the second-order correlation function of the single photon-added squeezed vacuum state is less than one, e.g., $g_1^{(2)} < 1$ (see more clearly in Fig. 2(b), $\theta = 0$), while for larger values of $|\xi|$ (about $|\xi| > 0.57$) $g_1^{(2)} \geq 1$. This implies that, depending on the different values of the squeezed parameters, the photons of the single photon-added squeezed vacuum state $|\psi_1\rangle$ can be bunching or anti-bunching.

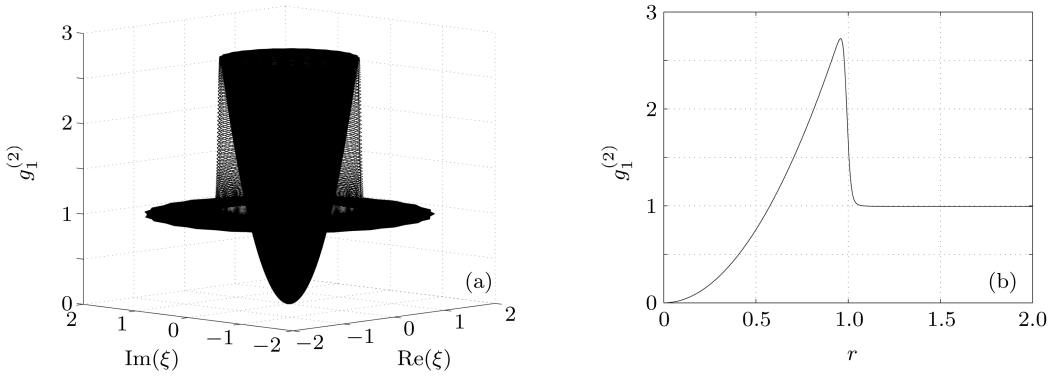


Fig. 2. The second-order correlation function of the single photon-added squeezed state $|\psi_1\rangle$ versus the squeezed parameter ξ.

4. Squeezing effects of the photon-added squeezed vacuum states

Squeezing is also one of the most important nonclassical properties of the optical field, and could be widely applied in both quantum communication and gravitational wave detection.[32−36] Indeed, squeezed optical states have been successfully prepared in experiments.[37−40] The two quadrature operators of optical field,

$$X_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger), \quad X_2 = \frac{1}{2\mathrm{i}}(\hat{a} - \hat{a}^\dagger), \tag{11}$$

satisfy the commutation relation $[X_1, X_2] = 1/2$. The squeezing of X_i ($i = 1$ or 2) requires

$$(\Delta X_i)^2 = \langle X_i^2\rangle - \langle X_i\rangle^2 < \frac{1}{4}. \tag{12}$$

Using the unitary transformation of $S(\xi)$ and $\hat{a}^\dagger$, the single photon-added squeezed vacuum states can be written as

$$\begin{aligned} |\psi_1\rangle &= a^\dagger S(\xi)|0\rangle \\ &= S(\xi)\, S^+(\xi)\, a^\dagger S(\xi)|0\rangle \\ &= S(\xi)|1\rangle. \end{aligned} \tag{13}$$

So one can easily obtain

$$\langle\hat{a}\rangle = \langle\hat{a}^\dagger\rangle = 0, \tag{14}$$

$$\langle\hat{a}^2\rangle = 3\cosh(r)\sinh(r)\,\mathrm{e}^{\mathrm{i}\theta}, \tag{15}$$

$$\langle\hat{a}^{\dagger 2}\rangle = 3\cosh(r)\sinh(r)\,\mathrm{e}^{-\mathrm{i}\theta}, \tag{16}$$

and

$$\langle\hat{a}^\dagger\hat{a}\rangle = 1 + 3\sinh^2(r). \tag{17}$$

Therefore,

$$(\Delta X_1)^2 = \frac{1}{4}\langle\hat{a}^2 + 1 + 2\hat{a}^\dagger\hat{a} + \hat{a}^{\dagger 2}\rangle$$

$$-\frac{1}{4}\langle\hat{a}+\hat{a}^{\dagger}\rangle^2 \tag{18}$$

and

$$(\Delta X_2)^2 = -\frac{1}{4}\langle\hat{a}^2-1-2\hat{a}^{\dagger}\hat{a}+\hat{a}^{\dagger 2}\rangle + \frac{1}{4}\langle\hat{a}+\hat{a}^{\dagger}\rangle^2. \tag{19}$$

As a consequence, the quantum fluctuations of the operators X_1 and X_2 read

$$\langle(\Delta X_1)^2\rangle = \frac{1}{4}S_1 \tag{20}$$

with $S_1 = 6\cosh(r)\sinh(r)\cos(\theta)+3+6\sinh^2(r)$, and

$$\langle(\Delta X_2)^2\rangle = \frac{1}{4}S_2 \tag{21}$$

with $S_2 = 6\cosh(r)\sinh(r)\cos(\theta) - 3 - 6\sinh^2(r)$. Thus, $S_i < 1$, ($i = 1$ or 2), means that the quantum fluctuation of orthogonal component X_1 or X_2 is squeezed.

Figure 3 shows how S_i changes for different parameters θ and r. Obviously, the single-photon squeezed vacuum states reveal significant quantum squeezings for certain suitably-selected parameters.

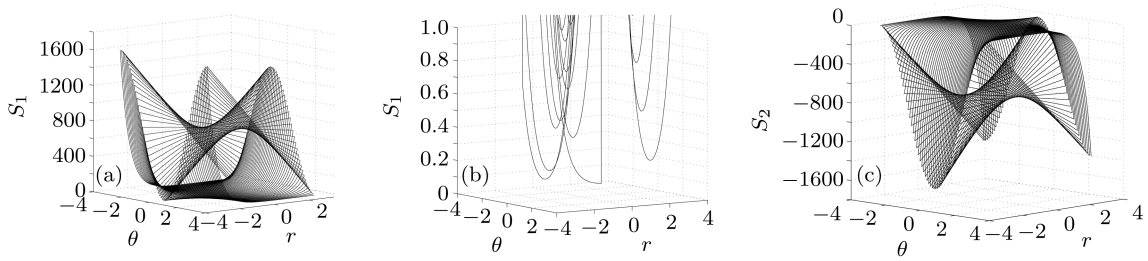


Fig. 3. (a) The quantum fluctuation of X_1 versus the parameters θ and r. (b) Quantum squeezing on X_1 for different parameters θ and r. (c) The quantum fluctuation of X_2 versus the parameter θ and r.

5. Negativity of the Wigner function

The basic property of the Wigner function is that it may take negative values for a specific quantum state. This is another obvious nonclassical feature of the quantum state. Basically, the Wigner function of the Fock states $|n\rangle$ reads[41,42]

$$W_{\rm Fn}(\alpha,\alpha^*) = \frac{2}{\pi}(-1)^n\exp(-2|\alpha|^2)\mathrm{L}_n^{(0)}(4|\alpha|^2), \tag{22}$$

where the Laguerre polynomial

$$\mathrm{L}_n^{(j)}(x) = \sum_{k=0}^{n}(-1)^k\frac{(n+j)!}{(n-k)!(j+k)!}\frac{x^k}{k!}. \tag{23}$$

Specifically, the Wigner function of the vacuum state $|0\rangle$ is

$$W_{\rm V0}(\alpha,\alpha^*) = \frac{2}{\pi}\exp(-2|\alpha|^2). \tag{24}$$

It is seen from Fig. 4(a) that the Wigner function of the vacuum state clearly displays the Gaussian wave-packet, and its value is always positive. Following Biswas and Agarwal,[20] the Wigner function of the squeezed vacuum state $|\psi_0\rangle$ can be written as

$$W_0(\alpha,\alpha^*) = \frac{2}{\pi}\exp(-2|\tilde{\alpha}|^2), \tag{25}$$

where $\tilde{\alpha} = \alpha\cosh(r) - \alpha^*\,\mathrm{e}^{\mathrm{i}\theta}\sinh(r)$. The Wigner wave-packets of the squeezed vacuum state is similar to that of the vacuum state and shown in Figs. 4(b) and 4(c). Similarly, for the single photon-added squeezed vacuum states $|\psi_1\rangle$ we have verified that $|\psi_1\rangle = S(\xi)|1\rangle$. Thus, the Wigner function of the present single photon-added squeezed vacuum states is the same as that of the single photon-subtracted squeezed vacuum states shown in Ref. [20], i.e.,

$$W_1(\alpha,\alpha^*) = -\frac{2}{\pi}\exp(-2|\tilde{\alpha}|^2)(1-4|\tilde{\alpha}|^2). \tag{26}$$

Figure 5(a) shows the Wigner function of the Fock state $|1\rangle$ and Figs. 5(b) and 5(c) display the Wigner function of the single photon-added squeezed vacuum states for the different squeezed parameters. Obviously, the Wigner function of the Fock state $|1\rangle$ clearly displays non-Gaussian and negative values for a certain region in the phase space. Also, negative values are also displayed for the Wigner function of the state $|\psi_1\rangle$, whose region is also "squeezed" relating to the different squeezed parameters.

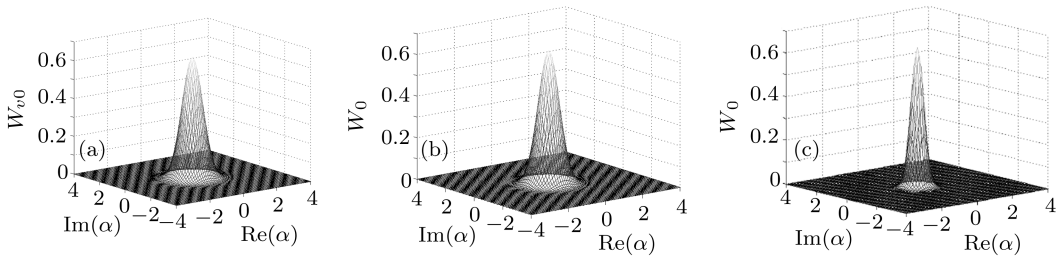


Fig. 4. (a) The Wigner function of vacuum state $|0\rangle$ versus the parameter α. The Wigner function of squeezed vacuum states versus the parameter α: (b) for $r = 0.3$, $\theta = 0$; (c) for $r = 0.9$, $\theta = 0$.

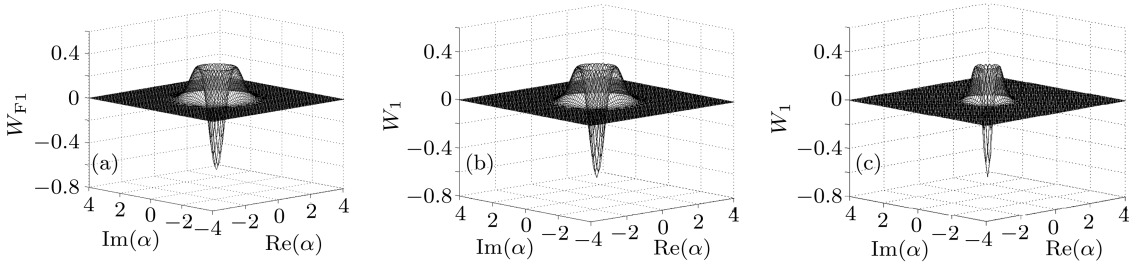


Fig. 5. (a) The Wigner function W_{F1} of Fock state $|1\rangle$ versus the parameter α. The Wigner function of single-photon added squeezed vacuum states versus the parameter α: (b) for $r = 0.3$ and $\theta = 0$; (c) for $r = 0.9$, $\theta = 0$.

6. Conclusion

In this paper, we introduced a new kind of quantum states, i.e. photon-added squeezed vacuum states, by applying the creation operator to the squeezed vacuum state. We also presented an experimental scheme, via conditional output measurement at a BS, to generate these quantum states. By using the numerical method we found that these quantum states could show some typical nonclassical effects, such as anti-bunching of photons, squeezing on two quadrature components and negativities of their relevant Wigner functions, for certain parameters.

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