Quantum communication and entanglement between two distant atoms via vacuum fields*

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This paper proposes an efficient scheme for quantum communication between two atoms trapped in distant cavities which are connected by an optical fibre. During the operation, all the atomic system, the cavity modes and the fibre are not excited. The quantum state is mediated by the vacuum fields. The idea can be used to realize quantum entanglement between two distant atoms via vacuum.

Keywords: quantum communication, entanglement, vacuum field

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Quantum communication, the transmission of quantum information between different sites, is a building block for the construction of a quantum network. It is also crucial for linking separate quantum information processors. The power of a quantum computer increases with the number of qubits. If one needs to solve a complex problem on a quantum computer, then a large number of qubits should be assembled. However, due to technological limitations the number of qubits in a single quantum register cannot be increased arbitrarily. In order to get around this problem one should build a quantum computer involving several local quantum information processors, which are connected via quantum channels, such as optical fibres. Such quantum computational networks are referred to as distributed quantum computers.[1] The cavity QED system is a qualified candidate for quantum communication as atoms with long-lived internal states are suitable for storage of quantum information and photons are ideal for transportation of information. Local two-qubit phase gates[2] and entanglement[3] have been experimentally implemented in a high-Q superconducting microwave cavity. Quantum state transfer between two atoms in a cavity has also been demonstrated via resonant interaction.[4]

Proposals have been suggested for quantum communication between two atoms trapped in distant optical cavities via an optical fibre.[5,6] The atomic state in an optical cavity is mapped to a photon wave packet by using a suitably designed laser pulse. After the photon wave packet carrying the information enters the other optical cavity through the optical fibre, it is absorbed by the other atom. Serafini *et al.* have proposed a scheme for realizing deterministic swap and phase gates between two two-level atoms trapped in separate optical cavities via coherent resonant coupling mediated by an optical fibre.[7] Yin and Li generalized the idea to implement quantum state transfer and logic gates with multiple atoms trapped in each cavity.[8] Schemes based on adiabatic evolution of dark states have been proposed in such a system.[9−12] In the previous schemes the quantum state is mediated by photons carried by the optical fibre. Therefore the schemes are extremely sensitive to decoherence and put a very stringent requirement on the decay rates, which is experimentally challenging. We have presented a quantum gate between two atoms trapped in distant cavities connected by an optical fibre.[13] In the scheme the bosonic modes are never populated and the atoms undergo no transitions. Therefore the gate operation is insensitive to the decoherence effect. However, the scheme cannot be used to transfer quantum states between two distant atoms.

In this paper we present an alternative scheme for quantum communication and entanglement for two atoms trapped in separate cavities linked by an optical fibre. In the scheme two ground states of each

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atom are coupled via the respective cavity mode and a classical field in the Raman manner. Under the large detuning condition, the atoms do not undergo real Raman transitions and the Hamiltonian for the atomic system is decoupled from the cavity modes and fibre mode. The quantum state is mediated by the vacuum fields. The scheme is insensitive to the atomic spontaneous emission, cavity decay, and fibre decay.

Two identical atoms are trapped in distant cavities connected by an optical fibre. The number of modes of the fibre that significantly interact with the corresponding cavity modes is on the order of $l\bar{\nu}/2\pi c$, where l is the length of the fibre and $\bar{\nu}$ is the decay rate of the cavities' fields into the continuum of the fibre modes. Under the condition $l\bar{\nu}/2\pi c \leq 1$, only one fibre mode essentially interacts with the cavity modes and the cavity–fibre coupling is described by the Hamiltonian[7]

$$H_0 = \nu b(a_1^\dagger + a_2^\dagger) + \text{H.c.}, \tag{1}$$

where b is the annihilation operator for the fibre mode, $a_j^\dagger$ is the creation operator for the j-th cavity mode, and ν is the cavity–fibre coupling strength.

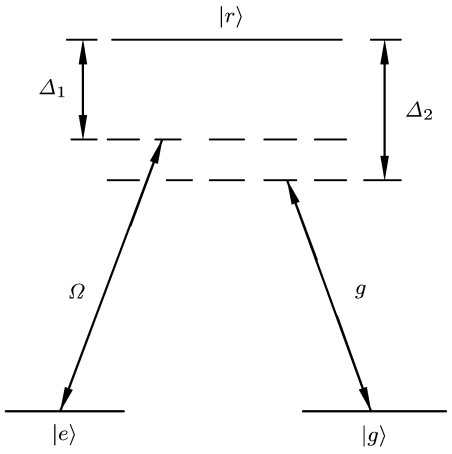


Fig. 1. The atomic level configuration and transitions. The transition $|e\rangle \to |r\rangle$ is driven by a classical laser field with the Rabi frequency Ω and detuning Δ_1, and the transition $|g\rangle \to |r\rangle$ is coupled to the cavity mode with the coupling constant g and detuning Δ_2.

The atomic level configuration is shown in Fig. 1. Each atom has one excited state $|r\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transition $|e\rangle \to |r\rangle$ is driven by a classical laser field with a Rabi frequency Ω, while the transition $|g\rangle \to |r\rangle$ is driven by the cavity mode with the coupling constant g. Assume that the classical field and cavity mode are detuned from the respective transitions by Δ_1 and Δ_2. In the interaction picture, the Hamiltonian describing the atom–field interaction is

$$\begin{aligned} H_{\text{int.}} =& \sum_{j=1}^{2}[\Omega \mathrm{e}^{\mathrm{i}\,\Delta_1 t}\,|r_j\rangle\langle e_j| \\ &+ g a_j \mathrm{e}^{\mathrm{i}\,\Delta_2 t}\,|r_j\rangle\langle g_j|] + \text{H.c.} \end{aligned} \tag{2}$$

In the case Δ_1, $\Delta_2 \gg \Omega, g$ the upper level $|r\rangle$ can be adiabatically eliminated, leading to

$$\begin{aligned} H_{\text{int.}} =& -\sum_{j=1}^{2}[\eta\,|e_j\rangle\langle e_j| + \xi a_j^\dagger a_j\,|g_j\rangle\langle g_j| \\ &+ \lambda(a_j S_j^+ \mathrm{e}^{\mathrm{i}\,\delta t} + a_j^\dagger S_j^- \mathrm{e}^{-\mathrm{i}\,\delta t})], \end{aligned} \tag{3}$$

where $\eta = \Omega^2/\Delta_1$, $\xi = g^2/\Delta_2$,

$$\lambda = \frac{\Omega g}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right),$$

$\delta = \Delta_2 - \Delta_1$, $S_j^+ = |e_j\rangle\langle g_j|$, and $S_j^- = |g_j\rangle\langle e_j|$. The first and second terms describe the Stark shifts for the states $|e_j\rangle$ and $|g_j\rangle$, induced by the classical field and cavity fields, respectively. The third term describes the Raman coupling of the two ground states.

Define

$$\begin{aligned} c &= \frac{1}{\sqrt{2}}(a_1 - a_2), \\ c_+ &= \frac{1}{2}(a_1 + a_2 + \sqrt{2}b), \\ c_- &= \frac{1}{2}(a_1 + a_2 - \sqrt{2}b), \end{aligned} \tag{4}$$

then we can rewrite the Hamiltonians H_0 and $H_{\text{int.}}$ as

$$H_0 = \sqrt{2}\nu c_+^\dagger c_+ - \sqrt{2}\nu c_-^\dagger c_-, \tag{5}$$

$$\begin{aligned} H_{\text{int.}} =& -\frac{\xi}{4}(c_+^\dagger c_+ + c_-^\dagger c_- + 2c^\dagger c)(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|) \\ &- \frac{1}{2}\Big[\lambda \mathrm{e}^{\mathrm{i}\,\delta t}(c_+ + c_- + \sqrt{2}c)S_1^+ \\ &+ \lambda \mathrm{e}^{\mathrm{i}\,\delta t}(c_+ + c_- - \sqrt{2}c)S_2^+ \\ &+ \frac{\xi}{2}(c_+^\dagger c_- + \sqrt{2}c_+^\dagger c + \sqrt{2}c_-^\dagger c)\,|g_1\rangle\langle g_1| \\ &+ \frac{\xi}{2}(c_+^\dagger c_- - \sqrt{2}c_+^\dagger c - \sqrt{2}c_-^\dagger c)\,|g_2\rangle\langle g_2| + \text{H.c.}\Big] \\ &- \sum_{j=1}^{2}\eta\,|e_j\rangle\langle e_j|\,. \end{aligned} \tag{6}$$

The time evolution of the whole system is decided by Schrödinger's equation:

$$\mathrm{i}\,\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = (H_0 + H_{\text{int.}})|\psi(t)\rangle. \tag{7}$$

Take H_0 as the "free Hamiltonian" and perform the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\,H_0 t}|\psi'(t)\rangle, \tag{8}$$

then we obtain the evolution of the system in the "interaction picture" with respect to the "free Hamiltonian" H_0

$$\mathrm{i}\,\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H'_{\mathrm{int.}}|\psi'(t)\rangle, \tag{9}$$

where

$$\begin{aligned} H'_{\mathrm{int.}} =& \,\mathrm{e}^{\mathrm{i}\,H_0 t}H_{\mathrm{int.}}\mathrm{e}^{-\mathrm{i}\,H_0 t} = \frac{\xi}{4}(c_+^\dagger c_+ + c_-^\dagger c_- + 2c^\dagger c)(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|) \\ &- \sum_{j=1}^{2}\eta\,|e_j\rangle\langle e_j| - \Big\{\frac{1}{2}\lambda[\mathrm{e}^{\mathrm{i}(\delta-\sqrt{2}\nu)t}c_+ + \mathrm{e}^{\mathrm{i}(\delta+\sqrt{2}\nu)t}c_- + \mathrm{e}^{\mathrm{i}\,\delta t}\sqrt{2}c]S_1^+ \\ &+ \frac{1}{2}\lambda[\mathrm{e}^{\mathrm{i}(\delta-\sqrt{2}\nu)t}c_+ + \mathrm{e}^{\mathrm{i}(\delta+\sqrt{2}\nu)t}c_- - \mathrm{e}^{\mathrm{i}\,\delta t}\sqrt{2}c]S_2^+ \\ &+ \frac{\xi}{4}(\mathrm{e}^{\mathrm{i}\,2\sqrt{2}\nu t}c_+^\dagger c_- + \sqrt{2}\mathrm{e}^{\mathrm{i}\,\sqrt{2}\nu t}c_+^\dagger c + \sqrt{2}\mathrm{e}^{-\mathrm{i}\,\sqrt{2}\nu t}c_-^\dagger c)\,|g_1\rangle\langle g_1| \\ &+ \frac{\xi}{4}(\mathrm{e}^{2\,\mathrm{i}\,\sqrt{2}\nu t}c_+^\dagger c_- - \sqrt{2}\mathrm{e}^{\mathrm{i}\,\sqrt{2}\nu t}c_+^\dagger c - \sqrt{2}\mathrm{e}^{-\mathrm{i}\,\sqrt{2}\nu t}c_-^\dagger c)\,|g_2\rangle\langle g_2| + \mathrm{H.c.}\Big\}. \end{aligned} \tag{10}$$

The Hamiltonian $H'_{\mathrm{int.}}$ describes multiple off-resonant Raman couplings for each atom induced by the classical field and the bosonic modes c_+, c_-, and c. Under the condition that $\delta \gg \lambda$, $\left|\delta \pm \sqrt{2}\nu\right| \gg \lambda/2$, and $\sqrt{2}\nu \gg \lambda/2, \xi/4$, the bosonic modes not only do not exchange quantum numbers with the atomic system, but also do not exchange quantum numbers with each other. The off-resonant Raman coupling leads to the Stark shifts and Heisenberg XY coupling between the atoms.[14,15] Then we have the effective Hamiltonian

$$\begin{aligned} H_{\mathrm{e}} =& -\frac{\xi}{4}(c_+^\dagger c_+ + c_-^\dagger c_- + 2c^\dagger c)(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|) - \sum_{j=1}^{2}\eta\,|e_j\rangle\langle e_j| \\ &+ \sum_{j=1}^{2}\frac{\lambda^2}{4}\Big\{\Big[\frac{2}{\delta}cc^\dagger + \frac{1}{\delta-\sqrt{2}\nu}c_+c_+^\dagger + \frac{c_-c_-^\dagger}{\delta+\sqrt{2}\nu}\Big]|e_j\rangle\langle e_j| \\ &- \Big[\frac{2}{\delta}c^\dagger c + \frac{1}{\delta-\sqrt{2}\nu}c_+^\dagger c_+ + \frac{1}{\delta+\sqrt{2}\nu}c_-^\dagger c_-\Big]|g_j\rangle\langle g_j|\Big\} \\ &+ \frac{\xi^2}{32\sqrt{2}\nu}[(c_+^\dagger c_+ - c_-^\dagger c_-)(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|)^2 \\ &+ 4(c_+^\dagger c_+ - c_-^\dagger c_-)(|g_1\rangle\langle g_1| - |g_2\rangle\langle g_2|)^2] - \chi(S_1^+S_2^- + S_1^-S_2^+), \end{aligned} \tag{11}$$

where

$$\chi = \frac{\lambda^2}{4}\Big[\frac{2}{\delta} - \frac{1}{\delta-\sqrt{2}\nu} - \frac{1}{\delta+\sqrt{2}\nu}\Big]. \tag{12}$$

Under the condition that the bosonic modes are initially in the vacuum state, they will remain in the vacuum state since $c_+^\dagger c_+$, $c_-^\dagger c_-$, $2c^\dagger c$ commute with the Hamiltonian H_{e}. Then H_{e} reduces to

$$H_{\mathrm{e}} = \sum_{j=1}^{2}\mu\,|e_j\rangle\langle e_j| - \chi(S_1^+S_2^- + S_1^-S_2^+), \tag{13}$$

where

$$\mu = \frac{\lambda^2}{4}\Big[\frac{2}{\delta} + \frac{1}{\delta-\sqrt{2}\nu} + \frac{1}{\delta+\sqrt{2}\nu}\Big] - \eta. \tag{14}$$

Suppose that the two atoms are initially in the state $(\alpha\,|e_1\rangle + \beta\,|g_1\rangle)\,|g_2\rangle$. After an interaction time t the system evolves to

$$\begin{aligned} &\alpha\,\mathrm{e}^{-\mathrm{i}\,\mu t}[\cos(\chi t)\,|e_1\rangle\,|g_2\rangle \\ &+\mathrm{i}\sin(\chi t)\,|g_1\rangle\,|e_2\rangle] + \beta\,|g_1\rangle\,|g_2\rangle\,. \end{aligned} \tag{15}$$

With the choice of $\chi t = \pi/2$, we obtain

$$|g_1\rangle\,(\mathrm{i}\,\alpha\,\mathrm{e}^{-\mathrm{i}\,\mu t}\,|e_2\rangle + \beta\,|g_2\rangle). \tag{16}$$

After the single-qubit operation $|e_2\rangle \to -\mathrm{i}\;\mathrm{e}^{\mathrm{i}\,\mu t}\,|e_2\rangle$, the state of atom 1 is transferred to atom 2. During the quantum communication, the quantum state is mediated by the virtual excitation and transportation of photons, which is in contrast with previous schemes.[5–8]

The idea can also be used to entangle the two atoms. Suppose that the two atoms are initially in

the state $|e_1\rangle\,|g_2\rangle$. After an interaction time t we obtain

$$|\psi(t)\rangle = \cos(\chi t)\,|e_1\rangle\,|g_2\rangle + \mathrm{i}\sin(\chi t)\,|g_1\rangle\,|e_2\rangle\,, \quad (17)$$

where we have discarded the common phase factor $\mathrm{e}^{-\mathrm{i}\,\mu t}$. With the choice of $\chi t = \pi/4$, we obtain the maximally entangled two-atom state

$$\frac{1}{\sqrt{2}}(|e_1\rangle\,|g_2\rangle + \mathrm{i}\,|g_1\rangle\,|e_2\rangle). \quad (18)$$

It is necessary to give a brief discussion on the experimental feasibility of the proposed scheme. Set $\Omega = g$, $\Delta_1 = 10g$, $\delta = g$, $\nu = \sqrt{2}g$, and $\Gamma = \kappa = \gamma = 0.01g$, where Γ, κ, and γ are the decay rates for the atomic excited state, the cavity modes, and the fibre mode, respectively. Then we have $\lambda \simeq g/10$ and $\chi \simeq 0.67 \times 10^{-2}g$. The probability that the atoms undergo a transition to the excited state due to the off-resonant interaction with the classical fields is $P_1 \simeq \Omega^2/\Delta_1^2 = 10^{-2}$. Meanwhile, the probability that the three modes c, c_+, and c_- are excited due to nonresonant coupling with the classical modes is

$$P_2 \simeq \frac{\lambda^2}{4}\left[\frac{2}{\delta^2} + \frac{1}{(\delta-\sqrt{2}\nu)^2} + \frac{1}{(\delta+\sqrt{2}\nu)^2}\right] \simeq 0.825\times 10^{-2}.$$

Thus the effective Hamiltonian H_{e} is valid. The effective decoherence rates due to the atomic spontaneous emission and the decay of the bosonic modes are $\Gamma' = P_1\Gamma \simeq 10^{-4}g$ and $\kappa' = P_2\kappa \simeq 0.825\times 10^{-4}g$, respectively. The time needed to realize the two-atom maximal entanglement is $t = \pi/4\chi \simeq 1.17\times 10^2/g$ and the infidelity induced by the decoherence is on the order of $(\Gamma' + \kappa')t \simeq 2.135\times 10^{-2}$. We note that for the case $\Gamma = \kappa = \gamma = 0.01g$ the scheme of Ref. [7] cannot be used to produce the most optimal entangling gates as the fidelity is completely spoiled by decoherence. The scheme works in the Lamb–Dicke regime. In a recent experiment,[16] the localization to the Lamb–Dicke limit of the axial motion was demonstrated for a single atom trapped in an optical cavity.

In conclusion, we have proposed a scheme for quantum communication and entanglement for two atoms trapped in separate cavities mediated by an optical fibre. The scheme is based on the long-range Heisenberg XY interaction induced by multiple off-resonant Raman couplings. The quantum state is mediated by virtual excitation and transportation of photons. During the procedure neither subsystem is excited and the scheme is insensitive to atomic spontaneous emission, cavity decay, and fibre loss.

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