Study of the near-field modulation property of microwaviness on a KH_2PO_4 crystal surface*

Chen Ming-Jun(陈明君)†, Jiang Wei(姜 伟), Li Ming-Quan(李明全), and Chen Kuan-Neng(陈宽能)

Center for Precision Engineering, Harbin Institute of Technology, Harbin 150001, *China*

(Received 19 May 2009; revised manuscript received 25 December 2009)

The KH_2PO_4 crystal is a key component in optical systems of inertial confinement fusion (ICF). The microwaviness on a KH_2PO_4 crystal surface is strongly related to its damage threshold which is a key parameter for application. To study the laser induced damage mechanism caused by microwaviness, in this paper the near-field modulation properties of microwaviness to the incident wave are discussed by the Fourier modal method. Research results indicate that the microwaviness on the machined surface will distort the incident wave and thus lead to non-uniform distribution of the light intensity inside the crystal; in a common range of microwaviness amplitude, the light intensity modulation degree increases about 0.03 whenever the microwaviness amplitude increases 10 nm; 1 order diffraction efficiencies are the key factors responsible for light intensity modulation inside the crystal; the light intensity modulation is just around the microwaviness in the form of an evanescent wave, not inside the crystal when the microwaviness period is below 0.712 μm; light intensity modulation degree has two extreme points in microwaviness periods of 1.064 μm and 1.6 μm, remains unchanged between periods of 3 μm and 150 μm, and descends above the period of 150 μm to 920 μm.

Keywords: KH_2PO_4 crystal, laser induced damage threshold, microwaviness Fourier modal method, modulation degree

PACC: 4225F, 6110D, 7820B

1. Introduction

KH_2PO_4 (potassium dihydrogen phosphate, KDP for short) crystals are mainly used in the inertial confinement fusion (ICF) terminal optical system for frequency multiplication and Pockels cells. As an important optical index, the laser induced damage threshold (LIDT) has become a bottleneck that restricts the application and the development of ICF.[1−4] Recently, studies of the LIDT have focused on factors like stationary-state and transient-state deficiency inside the crystals, impurity content, multi-photon ionization and collision ionization inside the crystals, incident wavelength, etc.[5−7] However, machined micro-nano superficial deficiencies (microwaviness, steel pits, surface stress, etc.) have a great influence on the laser energy-flux density and operating life of KDP crystals. Using the single point diamond turning (SPDT) method to machine KDP crystals, periodic tool marks will be left on the machined surface along with the cutter path, called microwaviness. The existence of microwaviness will add periodic micro disturbance to the transmission wave front, and sometimes the microwaviness structures are even similar to the grating. The near-field modulation properties of microwaviness can lead to the nonlinear increase of the intense laser, and consequently lower the laser induced damage threshold.[8]

Light intensity is the average of the electromagnetic field energy-flux density in a period of the incident wave. The modulation caused by microwaviness on the machined surface will lead to a non-uniform distribution of light intensity. If the light intensity peak reaches a critical value in a certain area, it will provide the electrons with enough energy to transmit to the conduction band, leading to a lot of effects like multi-photon ionization, collision ionization, heating, etc. If there are some deficiencies in such an area, the induced damage probability will be higher, too.[1,3] When analysing the spatial frequency on a KDP element surface by the power spectral density (PSD) method, we find that the spatial period corresponding to the maximum of PSD is very close to the working feed, which means that we can control the spatial frequency distribution by choosing different feeds.[9] Therefore, making an intensive study

*Project supported by the National Natural Science Foundation of China (Grant No. 50875066) and the National High Technology Research and Development Program of China (Grant No. 2009AA044305).

†Corresponding author. E-mail: chenmj@hit.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

of the microwaviness near-field modulation properties can not only provide the theoretical basis for increasing LIDT, but can also guide the choice of effective working parameters. In this paper we discuss the light intensity modulation properties of the microwaviness period and amplitude using the Fourier modal method.

2. Model and theory

The dimensions of the microwaviness and area scale for study in the light transmission direction are both very small. Therefore, scalar diffraction theory and approximate conditions will no longer be tenable, and we should use the vector calculus method based on rigorous electromagnetic field theory.[10,11] The Fourier modal method is based on the rigorous electromagnetic field theory without any approximation and hypothesis, and the basic method is to expand the electromagnetic field of the phase modulation area by Fourier modals, establish the eigenequation, solve the modal coefficient by boundary conditions, and finally solve the whole electromagnetic field.[12–14]

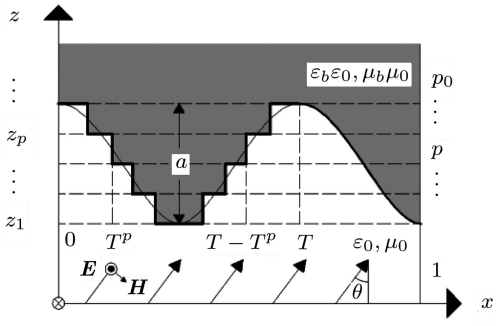


Fig. 1. Physical model of microwaviness (one and a half periods).

The ideal pattern of microwaviness is approximated by a sine wave and the sine wave in turn is approximated by multilayer lamellar steps. Various notations are shown in Fig. 1.[15] The microwaviness cross section is in the x–z plane with the y-axis parallel to the microwaviness line, the z-axis is perpendicular to the substrate layer plane. The $1^{\rm st}$ and $P_0^{\rm th}$ layers are two semi-infinite media, i.e. incident air layer and substrate layer, respectively. The other layers are microwaviness layers.

In each layer, the relative dielectric constant and the relative permeability are periodic with period T, i.e., $\varepsilon(x) = \varepsilon(x+T)$, and $\mu(x) = \mu(x+T)$. In each layer p,

$$\varepsilon(x)^p, \mu(x)^p = \begin{cases} 1 & x \in (0, T^p) \cup (T-T^p, T), \\ \varepsilon_b, \mu_b & x \in (T^p, T-T^p). \end{cases} \tag{1}$$

Writing Eq. (1) in Fourier expansion form as follows:

$$[\varepsilon, \mu]^p = [\varepsilon_n, \mu_n]^p \exp\Big(\mathrm{i}\,\frac{2\pi n}{T}x\Big), \tag{2}$$

where T^p is the interface coordinate between air and crystal in one period, from geometrical relationship, we can obtain

$$T^p = \frac{T}{2\pi}\arccos[(z^p + z^{p-1} - 1)/a], \tag{3}$$

where z^p is the upper interface coordinate of layer p, and a is the amplitude of the microwaviness. In layer p, the dielectric constant and the permeability are $\varepsilon(x)^p\varepsilon_0$ and $\mu(x)^p\mu_0$.

Because of the periodicity of microwaviness, the distribution of the electromagnetic field is also periodic with period T, i.e., $\boldsymbol{E}(x) = \boldsymbol{E}(x+T)$, and $\boldsymbol{H}(x) = \boldsymbol{H}(x+T)$, which, in layer p, are written in Fourier expansion form as follows:

$$[\boldsymbol{E}, \sigma\boldsymbol{H}]^p = \exp(\mathrm{i}k_0\alpha_m x)[e_{xm}e_{ym}e_{zm}, -h_{xm}h_{ym}h_{zm}]^p \times \exp[\mathrm{i}k_0\gamma^p(z-z^p)], \tag{4}$$

where $\alpha_m = \alpha_0 + \lambda m/T$, $m = 0, \pm1, \pm2, \ldots, \pm M, \ldots$; γ^p is for the evaluation related to the z-axis component of the wave number in layer p; $\alpha_0 = \sin\theta$, and θ is the angle between the incident light and axis z.[16] In each layer, the electromagnetic field satisfies the Maxwell equations:

$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \quad \boldsymbol{\nabla}\times\boldsymbol{H} = \frac{\partial\boldsymbol{D}}{\partial t}. \tag{5}$$

Concerning the discontinuation of two different media in microwaviness layers in the axis x, we use the Fourier "inverse regulation" factorization theory.[17] Substituting Eqs. (2) and (4) into Eq. (5), we obtain the component Maxwell equations of matrix style (omitting layer number p):

$$\begin{cases} \bar{\mu}e_y\gamma = h_x, \\ e_x\gamma - \alpha e_z = \mu h_y, \\ \alpha e_y = \mu h_z, \end{cases} \text{and} \quad \begin{cases} -\bar{\varepsilon}h_y\gamma = e_x, \\ h_x\gamma + \alpha h_z = \varepsilon e_y, \\ \alpha h_y = -\varepsilon e_z, \end{cases} \tag{6}$$

where ε, $\bar{\varepsilon}$, μ and $\bar{\mu}$ are $(2M+1)$ square matrixes obtained according to the "inverse regulation" theory; e_x, e_y, e_z, h_x, h_y and h_z are $(2M+1)\times 1$ matrixes (the matrix element of e_x is e_{xm}) for evaluation by the eigenequation; α is a $(2M+1)$ diagonal matrix

with a matrix element of α_m; γ is a value for evaluation by the eigenequation. From Eq. (6) we obtain the eigenequation of the TE wave as follows:

$$\begin{aligned} &(\varepsilon - \alpha\mu^{-1}\alpha)e_y = \bar{\mu}e_y\gamma^2, \\ &h_x = \bar{\mu}e_y\gamma, \\ &h_z = \mu^{-1}\alpha e_y. \end{aligned} \tag{7}$$

Define $A = \bar{\mu}^{-1}(\varepsilon - \alpha\mu^{-1}\alpha)$, then the eigenequation can be rewritten as

$$Ae_y = e_y\gamma^2. \tag{8}$$

Equation (8) is a generalized eigenequation with $(2M+1)$ eigenvalues and eigenvectors.[17] After equation (8) is solved, we obtain the eigenvalue matrixes Γ_+ and Γ_-, and the eigenvector matrix W_{ey}, which are related by

$$AW_{ey} = W_{ey}\Gamma_{+/-}^2, \tag{9}$$

where W_{ey} is a $(2M+1)$-square matrix composed of $(2M+1)$ eigenvectors. Γ_+ and Γ_- are two $(2M+1)$ diagonal matrixes composed of $(2M+1)$ eigenvalues, Γ_+ represents an up-going wave with positive eigenvalues and Γ_- represents a down-going wave with negative eigenvalues.

After the eigenmode field is solved, the general solution of the modal field is the linear superposition of eigenmode fields. In layer p, the y component of electric-field intensity and the x component of magnetic-field intensity can be written as follows:

$$\begin{cases} E_y^p = \exp(\mathrm{i}k_0\alpha x)W_{ey}^p\{\exp[\mathrm{i}k_0\Gamma_+^p(z-z^p)]u^p + \exp[\mathrm{i}k_0\Gamma_-^p(z-z^p)]d^p\}, \\ \sigma H_x^p = \exp(\mathrm{i}k_0\alpha x)(-W_{hx}^p)\{\exp[\mathrm{i}k_0\Gamma_+^p(z-z^p)]u^p - \exp[\mathrm{i}k_0\Gamma_-^p(z-z^p)]d^p, \end{cases} \tag{10}$$

where u^p and d^p are two column vectors composed of eigenmode field amplitude coefficients of the up-going wave and the down-going wave, respectively, which can be solved by an algorithm of a recursion reflection–transmission coefficient matrix (RTCM).[18,19]

Light intensity can be obtained from the following equation (in units of W/m^2):

$$I = \frac{1}{2}|Re(\boldsymbol{E} \times \boldsymbol{H}^*)|. \tag{11}$$

3. Result and discussion

To further study the light intensity modulation properties of microwaviness and simplify the mathematic model, the incident light wave is assumed to be a unit TE wave with vertical incidence. The refractive index of KDP is 1.49; incident light wave length is $\lambda = 1064$ nm; the thickness of the KDP crystal is 10 mm;[20] $z_1 = 0$. Now we come to discuss the light intensity modulation properties of the microwaviness period and amplitude separately.

3.1. Light intensity modulation property of microwaviness amplitude

The incident wave generates a new diffraction field inside the crystal after the phase modulation of the microwaviness. Denote the light intensity (inside the crystal) as I_{perf} under ideal conditions (without microwaviness), then the light intensity modulation degree will be defined as

$$M_{\mathrm{I}} = I_{\max}/I_{\mathrm{perf}}, \tag{12}$$

where $I_{\max}$ is the light intensity maximum after microwaviness modulation. From the viewpoint of failure resistance, the modulation degree is used to identify the system security, which means that the bigger the modulation degree is, the more easily the optical component can be destroyed (the definition of modulation degree here is different from that in Ref. [12]).

Generally, using the single point diamond turning method, choosing a special machine, optimal working parameter combination and appropriate working direction, the microwaviness amplitude is under 200 nm.[21–23] Figure 2 shows the light intensity modulation degree curves of microwaviness amplitude. We can conclude that the modulation degree becomes bigger as amplitude increases, the reason for which is that the incident wave diffracts after the modulation of the microwaviness so that the ideal light field is distorted,

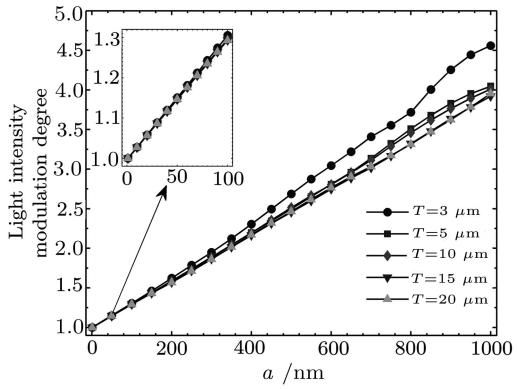


Fig. 2. Modulation property curves of microwaviness amplitude.

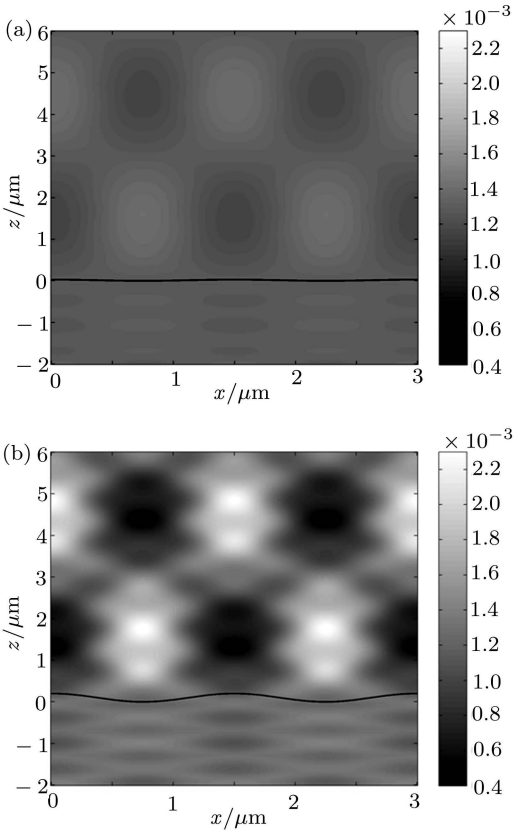


Fig. 3. Light intensity distributions of different microwaviness amplitudes (the black line represents the boundary of microwaviness, the area above it is inside the crystal) at $a = 30$ nm, $T = 1.5$ μm (a), and $a = 200$ nm, $T = 1.5$ μm (b).

and a bigger amplitude will lead to a more serious distortion. In some areas the light waves interfere constructively and the light intensity is strengthened; if it reaches the electron transition critical value or there exist deficiencies in such areas, the induced damage may occur. Moreover, in the common range of the microwaviness amplitude, the light intensity modulation degree increases linearly with increasing microwaviness amplitude, at about 0.03/10 nm.

The light intensity distributions at $a = 30$ nm and 200 nm with the same period of $T = 1.5$ μm are given in Fig. 3. From Figs. 3(a) and 3(b) we can see that the light intensity at $a = 30$ nm distributes more uniformly than that at $a = 200$ nm, and the modulation is 1.105. However, the light intensity at $a = 200$ nm is badly distorted with a modulation of 1.82, which is because the bigger amplitude will involve light intensity concentration and result in high light intensity peak in the local area.

3.2. Light intensity modulation property of microwaviness period

The light intensity modulation degree curve of the microwaviness period is is shown in Fig. 4. It can be found that the modulation degree increases slowly with increasing period when the microwaviness period is below 0.712 μm, increases rapidly above 0.712 μm until 1.064 μm where the modulation degree reaches the maximum, descends between 1.064 μm and 3 μm for except an extreme point of 1.6 μm, remains unchanged between 3 μm and 150 μm, and descends above 150 μm to 920 μm above which the modulation is less than 1.01.

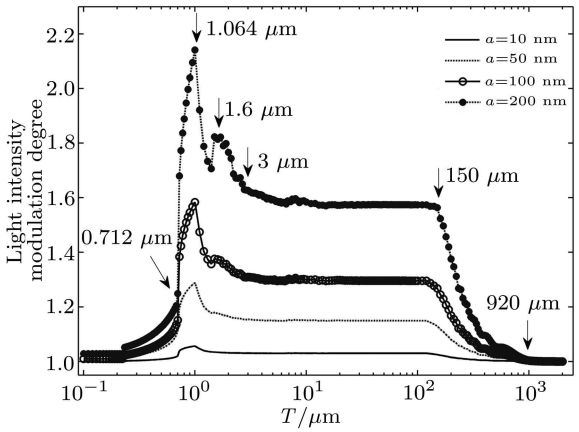


Fig. 4. Modulation property curves of microwaviness period.

The main reason for the modulation property above is that as the microwaviness period varies, the number of diffraction orders and the diffraction efficiency of each order vary. Table 1 shows the diffraction efficiencies of each diffraction order for different microwaviness periods. From the table we can see that with increasing microwaviness period the number of the diffraction orders increases, but the ±1 order diffraction efficiencies are much higher than other orders, so we will focus on them.

Table 1. Diffraction efficiencies of each order.

microwaviness	diffraction efficiency/%					
period	0	±1	±2	±3	±4	±5
/μm	order	order	order $\times 10^{-4}$	order$\times 10^{-4}$	order$\times 10^{-7}$	order$\times 10^{-5}$
1	92.86	1.78	0	0	0	0
2	94.87	0.60	29.8	0	0	0
2.5	94.97	0.56	0.46	1.81	0	0
3	95.02	0.54	2.47	1.21	18	0
4	95.06	0.53	4.57	0.63	2.52	2.71

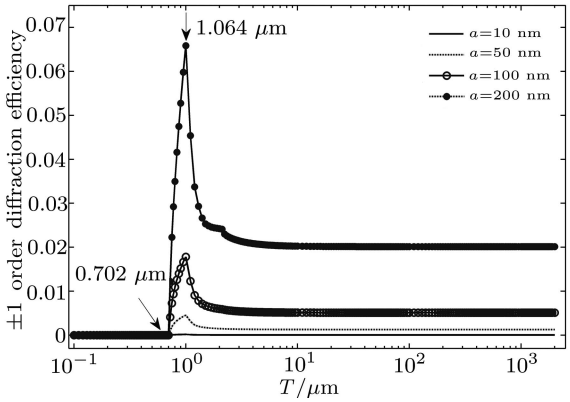


Fig. 5. The ±1 order diffraction efficiencies.

Figure 5 shows the ±1 order diffraction efficiency curves. The ±1 order diffraction efficiencies are both zeros while the microwaviness periods are below 0.712 μm and increase rapidly until 1.064 μm where they reach their maxima, descend between 1.064 μm and 3 μm, and remain unchanged above 3 μm. From Figs. 4 and 5 we can see obviously that the changes of light intensity modulation degree with microwaviness period are quite similar to those of ±1 order diffraction efficiencies. We can conclude that ±1 order diffraction efficiencies are the main reason for inducing light intensity modulation inside KDP crystals, but there are still some differences between Figs. 4 and 5 which we can explain as follows: a) the difference below 0.712 μm is caused by the evanescent wave, which attenuates rapidly along the light propagation direction (Fig. 6(a)); b) the difference at 1.6 μm is caused by the diffractions of other orders, whose diffraction efficiencies are only obvious when the microwaviness amplitude is bigger; c) the difference above 150 μm is caused by the periodic distribution of the light intensity in the z direction (Fig. 6(b)), and the longer the microwaviness period is, the longer the period of the light intensity in the z direction will be because the thickness of the KDP crystal is a constant, and when microwaviness period is longer than 150 μm the peak value of the light intensity is outside the crystal so that the maximum light intensity inside the crystal becomes smaller.

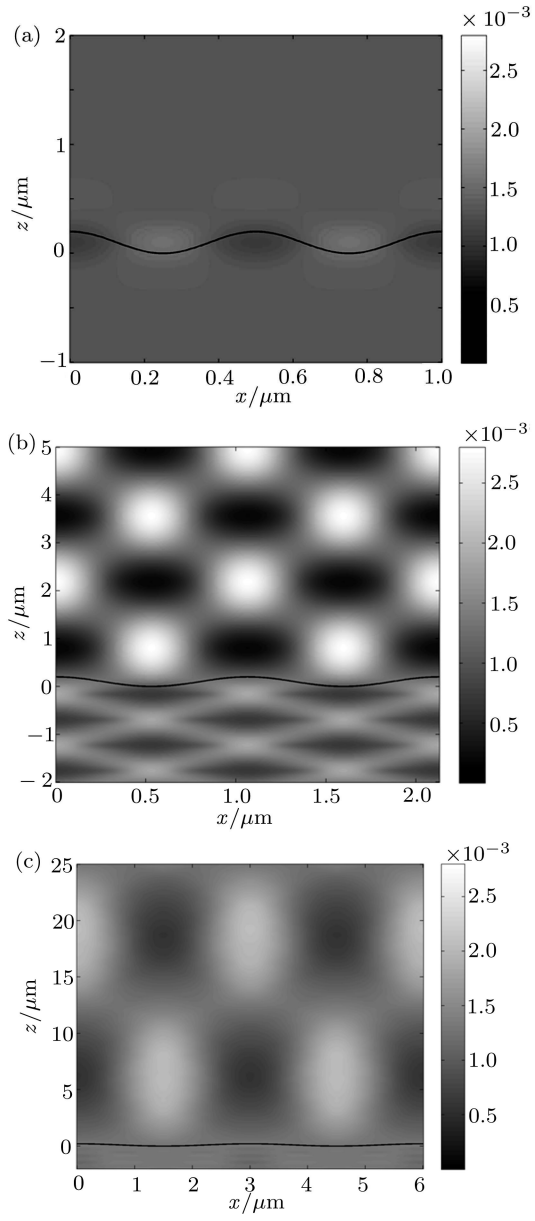


Fig. 6. Light intensity distributions of different microwaviness periods (the black line represents the boundary of microwaviness, the area above it is inside the crystal) in the cases of (a) $a = 100$ nm and $T = 0.5$ μm, (b) $a = 100$ nm and $T = 1.064$ μm, and (c) $a = 100$ nm and $T = 3$ μm.

The light intensity distributions in the cases of $a = 100$ nm, and $T = 0.5$ μm, 1.064 μm and 3 μm are given in Fig. 6, respectively. From Fig. 6(a) we can see that when microwaviness period is 0.5 μm (below 0.712 μm), there is only a 0 order diffraction wave, so the light intensity modulation is just around the microwaviness in the form of an evanescent wave, but not inside the crystal; from Figs. 6(b) and 6(c) we can see that ± 1 and higher order diffraction efficiencies will cause non-uniform distributions of the light intensity and periodic distributions both in the x direction and in the z direction. Moreover, the microwaviness periods around 1.064 μm and 1.6 μm are sensitive periods, others are insensitive periods. From Figs. 6(b) and 6(c) we can also see that the peak value of light intensity in the sensitive period of 1.064 μm is much higher than that in the insensitive period of 3 μm, with modulation degrees of 2.2 and 1.6 respectively. Therefore, we must take an effective measure to make sure the microwaviness periods are not in sensitive periods so as to lower the light intensity modulation degree and enhance the induced damage resistance.

4. Conclusions

Using the Fourier modal method based on rigorous electromagnetic field theory, the modulation properties of microwaviness inside a KDP crystal with microwaviness amplitudes ranging from 10 nm to 1000 nm and periods covering from 0.1 μm to 2 mm are discussed. The study results are as follows.

(i) The ± 1 order diffraction efficiencies are the main reason for the light intensity modulation inside KDP crystals.

(ii) A large microwaviness amplitude will increase the ± 1 order diffraction efficiencies and lead to a serious distortion of the diffractional field. In some areas the light waves interfere constructively and the light intensity is strengthened.

(iii) In the common range of the microwaviness amplitude, the light intensity modulation degree increases about 0.03 whenever the microwaviness amplitude increases 10 nm.

(iv) The light intensity modulation is just around the microwaviness in the form of an evanescent wave when the microwaviness period is below 0.712 μm; there are two peak values of modulation degree in 1.064 μm and 1.6 μm; the modulation degree remains unchanged between 3 μm and 150 μm, and descends above 150 μm to 920 μm above which modulation is less than 1.01.

References

[1] Xia Z L, Shao J D and Fan Z X 2007 *Acta Phys. Sin.* **56** 400 (in Chinese)

[2] Negres R A, Kucheyev S O, DeMange P, Bostedt C, Buuren T V, Nelson A J and Demos S G 2005 *Appl. Phys. Lett.* **86** 171107

[3] Diao L C, Zhang K C and Chang X A 2002 *J. Synth. Cryst.* **31** 99 (in Chinese)

[4] Tang Z Y, Yang J L, Wen S H, Wang G X, Guo Y Z, Yang H Q and Ma C 1999 *Chin. Phys.* **8** 913

[5] Carr C W, Feit M D, Johnson M A and Rubenchik A M 2006 *Appl. Phys. Lett.* **89** 131901

[6] DeMange P, Negres R A, Radousky H B and Demos S G 2006 *Opt. Engin.* **45** 104205

[7] Zhang H Y, Lu Z Z, Fan R W and Chen D Y 2008 *Chin. Phys.* B **17** 4504

[8] Yang L 2001 *Advanced Optical Manufacturing Technology* (Beijing: Science Press) pp. 272–294 (in Chinese)

[9] Chen M J, Pang Q L, Wang J H and Cheng K 2008 *International Journal of Machine Tools & Manufacture* **48** 905

[10] Gao F H, Wang C C, Tang X G, Ma C, Cui Z and Guo Y K 2006 *Microelectronic Engineering* **83** 1062

[11] Kong W J, Yun M J, Sun X, Liu J H, Fan Z X and Shao J D 2008 *Acta Phys. Sin.* **57** 4904 (in Chinese)

[12] Yao X, Gao F H, Li J F, Zhang Y X, Wen S L and Guo Y K 2008 *Acta Phys. Sin.* **57** 4891 (in Chinese)

[13] Liu M M, Zhang G P and Zhou M 2006 *Acta Phys. Sin.* **55** 4608 (in Chinese)

[14] Yao X, Gao F H, Wen S L, Zhang Y X, Li J F and Guo Y K 2007 *Acta Phys. Sin.* **56** 6945 (in Chinese)

[15] Li L F 1998 *J. Opt. Soc. Am.* A **13** 1313

[16] Bayanheshig, Qi X D and Tang Y G 2003 *Acta Phys. Sin.* **52** 1157 (in Chinese)

[17] Thomas Hagen 2005 *Nonlinear Analysis: Real World Applications* **6** 429

[18] Fu K X, Wang Z H, Zhang D Y, Zhang J and Zhang Q Z 1999 *Sci. Chin.* Ser. A **42** 636

[19] Fu K X, Zhang D Y, Wang Z H, Zhang Q Z and Zhang J 1998 *Acta Phys. Sin.* **47** 1278 (in Chinese)

[20] DeMange P, Negres R A, Carr C W, Radousky H B and Demos S G 2006 *Opt. Express* **14** 5313

[21] Chen M J, Wang J H, Liang Y C and Yuan D Y 2007 *Key Engineering Materials* **339** 1

[22] Wang J H, Chen M J, Dong S, Wang H X, Zhang J H and Zong W J 2007 *Key Engineering Materials* **329** 409

[23] Wang J H, Dong S, Wang H X, Chen M J, Zong W J and Zhang L J 2007 *Key Engineering Materials* **339** 78