Nonsingularity in two-dimensional cylindrical invisible cloaks*

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The method of designing electromagnetic invisible cloaks is usually based on the form-invariance of Maxwell's equations in coordinate transformation. The exterior boundary of a cylindrical invisible cloak is unchanged and the interior boundary is extended from that of a point to that of a cylindrical region in coordination transformation. This transformation process makes perfect cloaks, but it causes singularity in the constitutive material parameters of cloaks. This singularity makes the cloaks impossible to realize in practice. In order to remove this singularity, this paper sets a small cylindrical region replacing a point in the space transformation. The cylindrical region is so small that it does not affect the invisibility effects, but it can remove the singularity for material parameters. Full wave simulations based on the finite element method were used to verify the designed cloaks.

Keywords: invisible cloak, arbitrary shapes, singularity, constitutive material parameters

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1. Introduction

The coordinate transformation method for invisible cloak design is mainly based on the form-invariance of Maxwell's equations in coordinate transformation.[1−7] This theory was first proposed by Pendry *et al.*[1] who obtained the spherical analytical solution. Then analytical cylindrical and elliptical cylindrical cloaks were obtained and verified by full wave simulations with the finite element method.[3−14] In order to solve the problem of cloaks with arbitrary boundary shapes, numerical methods and electromagnetic equations have been presented.[15−21] The exterior boundary of these cloaks is unchanged and the interior boundary is extended from that of a point to that of a region in coordinate transformation. A point transformation in topology makes the effects of the cloak perfect, but at the same time, it causes singularity in the constitutive material parameters. It is impossible to realize these cloaks because of the infinite material parameters. In order to solve this problem, a line transformation was proposed to solve elliptical cylindrical cloaks.[22] A line transformation can solve the problem of singularity in the line direction, but it brings a lot of electromagnetic wave inside the cloak layer in the other directions, and the effects of invisibility disappear. By means of projection from a three-dimensional (3D) cloak,[23] a nonsingular (2-dimentional) cloak of complex shapes was obtained.

In this paper, we set a small cylindrical region which replaces a point in space transformation. The cylindrical region is so small that it does not affect the invisibility effects, but it can remove the singularity for the material parameters of cloaks. This method is easier than the one described in Ref. [23]. Furthermore, this method can also be used for cloaks with arbitrary boundary shapes. In the following section, we will design a cylindrical cloak as an example.

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2. Method of design for non-singularity

Variant constitutive material parameters are used to carry out space transformation in coordinate transformation. In space transformation, the boundary condition is transformed as illustrated by the following figures. The exterior boundary of these cloaks is unchanged and the interior boundary is extended from that of a point to that of a region in space transformation. A point transformation makes cloaks perfect, but it causes singularity in material parameter tensors. The singularity will result in infinite parameters. It is impossible to realize physically infinite parameters.

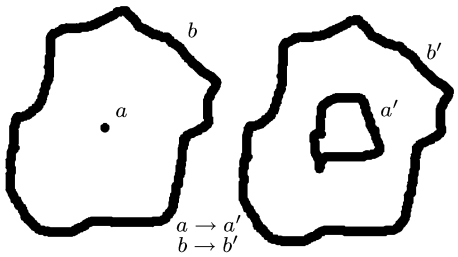


Fig. 1. The boundary condition in space transformation.

In order to remove the singularity, we replace a point by a small cylindrical region in space transformation as shown in Fig. 2. This small cylindrical region can remove singularity, but it will reduce the efficiency of invisibility. If the region is small enough, the invisibility effects can be quite desirable. By this transformation, we can design 2D invisible cloaks without singularity.

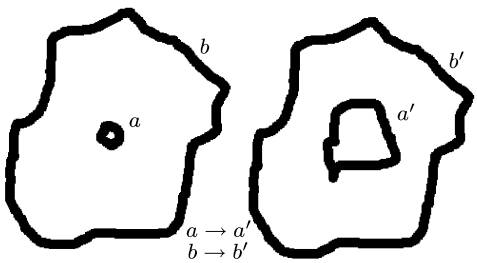


Fig. 2. The boundary condition in space transformation.

This space transformation can be described by the Jacobian transformation matrix A with the components defined as

$$A_{\xi\eta} = \partial\xi'/\partial\eta', \quad \xi, \eta = x, y, z, \tag{1}$$

where ξ, η represent the components of space and ξ', η' represent the components after coordinate transformation.

According to the coordinate transformation theory,[1−6] the constitutive material parameters of the cloak region can be described via

$$\alpha'(x', y', z') = A\alpha(x, y, z)A^{\mathrm{T}}/\det(A), \tag{2}$$

where $\alpha'(x', y', z')$ and $\alpha(x, y, z)$ denote the constitutive material parameters after and before the coordinate transformation, respectively.

By solving Eq. (2), we can design invisible cloaks without singularity. By means of a small region transformation, all the material parameters can be designed with finite values.

3. Designing 2D cylindrical cloaks with non-singularity

According to Ref. [20], by solving Laplace's equation and its boundary condition, we can determine the constitutive material parameters of the cloak layer. The design process includes two steps. The first step is to solve Laplace's equation and its boundary condition to get the material parameters of the cloak layer. Then the second step is to verify the cloak effects by a plane wave.

The commercial software Comsol Multiphysics is used to design 2D cylindrical cloaks. The partial differential equation module is used to solve the material parameters of the cloak layer, and then the radio frequency module is used to verify the invisibility effects.

In order to compare the material parameters of a region transformation with those of a point transformation, we design a point transformation first and then a region transformation is designed for comparison.

In the computational domain, the boundary of the cloak layer is a continuous boundary condition. A time-harmonic plane wave with a frequency of 1.5 GHz is incident onto the cloak, and then propagates along the cloak. The boundary of a point transformation is shown in Fig. 1. In this case, the cloak effect is perfect, as shown in Fig. 3.

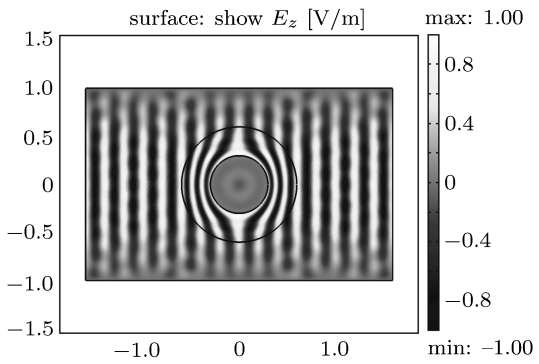


Fig. 3. The z component of the electric field.

From Fig. 3, the z component of the electric field in the cloak layer is small enough and the wave front does not change after the cloak. This property can make the objects inside invisible. A point transformation can make the invisible cloak perfect, but at the same time, the point transformation can cause singularity in the material parameters of the cloak, as shown in Fig. 4.

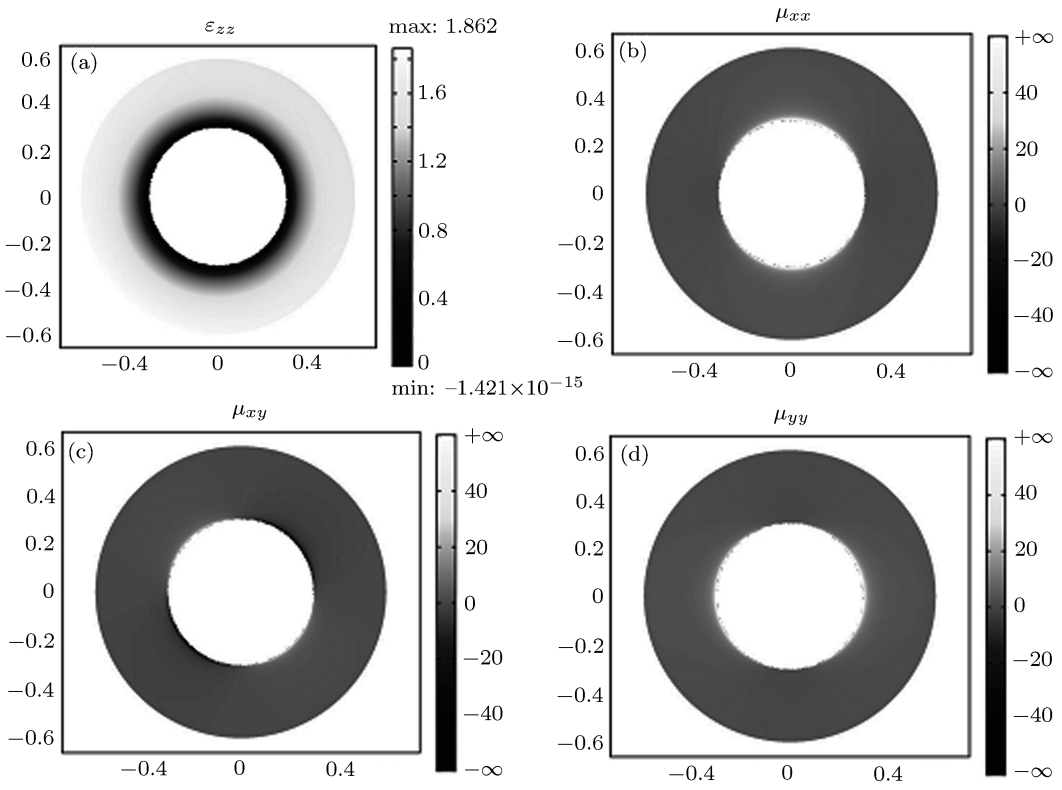


Fig. 4. The material parameters of the cloak layer: (a) ε_{zz}; (b) μ_{xx}; (c) μ_{xy}; (d) μ_{yy}.

As shown in Fig. 4, μ_{xx}, μ_{xy} and μ_{yy} are infinite. These parameters cannot be fulfilled in reality. In order to solve this problem, we use a region transformation to carry out space transformation. The boundary condition of the cloak layer is a continuous boundary condition. A plane TE wave with a frequency of 1.5 GHz is incident on the cloak layer after a region transformation. This design process is similar to a point transformation.

From Fig. 5, the z component of the electric field in the cloak layer is unequal to zero and the cloak effect is imperfect. However, this component is so small that it does not affect the cloak effects. The wave front does not change after the cloak layer. Furthermore, a region transformation can also make nonsingular 2D invisible cloaks. The material parameters of a region transformation are shown in Fig. 6.

From Fig. 6, we can see that the material parameters of the cloak layer are nonsingular. The parameters are all finite. These nonsingular material parameters can be realized in practice, and a region transformation is very easy for calculation.

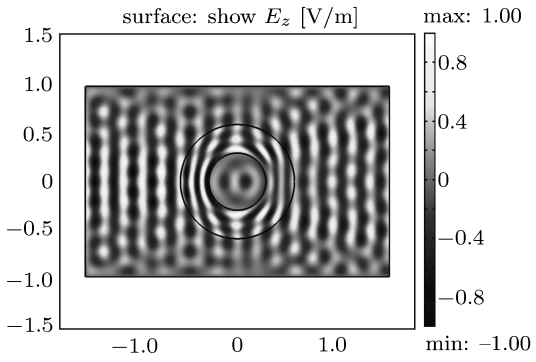


Fig. 5. The z component of the electric field.

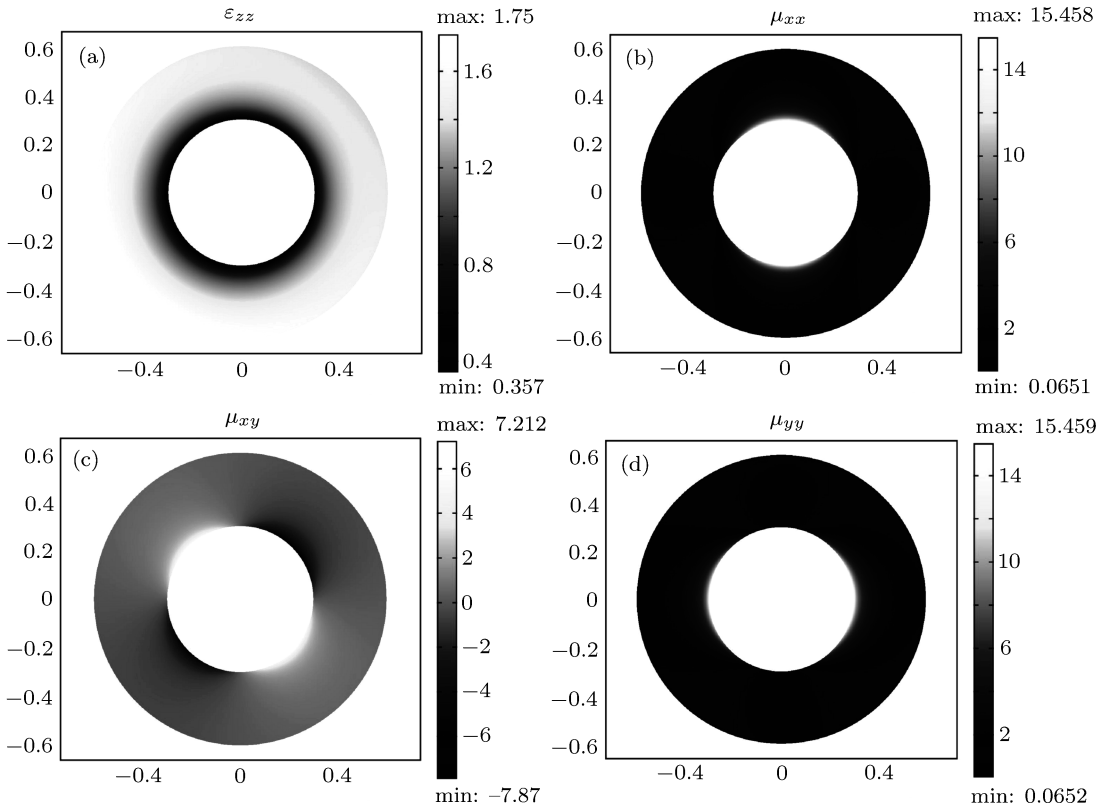


Fig. 6. The material parameters of the cloak layer: (a) ε_{zz}; (b) μ_{xx}; (c) μ_{xy}; (d) μ_{yy}.

4. Conclusions

In conclusion, a region transformation is used to design a 2D invisible cloak. This design can remove the material parameter singularity of the cloak in a point transformation. A region transformation does not affect the invisibility effects and can also remove the singularity. Full wave simulations were used to verify this method.

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