Multi-window invisible cloaks*

Wang Xin-Hua(王新华)[a)†], Qu Shao-Bo(屈绍波)[a)b)‡], Xia Song(夏 颂)[b)], Wang Bin-Ke(王斌科)[a)], Xu Zhuo(徐 卓)[b)], Ma Hua(马 华)[a)], Wang Jia-Fu(王甲富)[a)], Gu Chao(顾 超)[a)], Wu Xiang(吴 翔)[a)], Lu Lei(鲁 磊)[a)], and Zhou Hang(周 航)[a)]

[a)] *College of Science, Air Force Engineering University, Xi'an* 710051, *China*
[b)] *Electronic Materials Research Laboratory, Key Laboratory of Ministry of Education, Xi'an Jiaotong University, Xi'an* 710049, *China*

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This paper reports that a general method of designing invisible cloaks is using variant constitutive material parameters to realize the space transformation. A hollow region can be hidden after this transformation. It was recently shown (Ma H, Qu S B, Xu Z and Wang J F 2009 *Appl. Phys. Lett.* **94** 103501) that when the original point moves to the boundary of a cloak, the cloak can be designed to be open. Based on this theory, we propose multi-window invisible cloaks which can conceal a group of objects. Full wave simulations for invisible cloaks with regular and irregular shapes verified this method.

Keywords: invisible cloaks, transformation optics, constitutive material parameters

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1. Introduction

Transformation optics theory gives a new way of designing novel electromagnetic devices such as invisible cloaks,[1−6] concentrators,[5] field rotators,[7] super-scatterers,[8] and wave-shape-keeping media.[9] The invisible cloak was firstly proposed by Pendry *et al.*[1] and soon it received much attention.[2−6] By means of coordinate transformation, the transformation of space can be equivalent to that of the material parameter tensors. This method was verified by full wave simulations[3] and experiments.[4] The coordinate transformation theory is based on the form-invariance of Maxwell's equations.[5] Then the analytical material parameters of the elliptical cylindrical cloaks were calculated by Ma *et al.* and Jiang *et al.*[10−12] Acoustic invisible cloaks were designed by making an analogy between the acoustic equations and Maxwell's equations.[13−16] Two-dimensional (2D) electromagnetic cloaks with arbitrary geometries[17,18] were obtained by the coordinate transformation method. An approximation approach and numerical method of designing invisible cloaks with arbitrary shapes were proposed.[19,20] Laplace's equation[21] and Helmholtz's equation[22] were used to solve the material constitutive parameters of 2D invisible cloaks with arbitrary shapes.

The above-mentioned invisible cloaks are closed cloaks which cannot exchange information and materials between the outer and inner region of the cloaks. Ma *et al.* proposed open cloaks[23] by the background parameters substitution method. In this paper, we solved Laplace's equation and designed multi-window invisible cloaks which can conceal a group of objects. Full wave simulations verified the multi-window invisible cloaks.

2. Method of design

According to Ref. [6], if the original space x is distorted to a new space x', the constitutive parameters of the invisible cloak can be described as follows:

$$\begin{aligned} \varepsilon^{i'j'} &= |\det(\Lambda_i^{i'})|^{-1}\Lambda_i^{i'}\Lambda_j^{j'}\varepsilon^{ij}, \\ \mu^{i'j'} &= |\det(\Lambda_i^{i'})|^{-1}\Lambda_i^{i'}\Lambda_j^{j'}\mu^{ij}, \end{aligned} \tag{1}$$

where ε^{ij} and μ^{ij} are the permittivity and permeability in the original space, respectively, $\varepsilon^{i'j'}$ and $\mu^{i'j'}$ are the permittivity and permeability in the transformed space, respectively, $\Lambda_i^{i'}$ is the gradient coefficient of inverse mapping $x_i' \to x_i$.

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†Corresponding author. E-mail: wang12051205@163.com
‡E-mail: qushaobo@mail.xjtu.edu.cn

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The key point of the open cloaks[23] is that the original point O is moved to the boundary of the cloaks. Now we set several points near the boundaries as shown in Fig. 1.

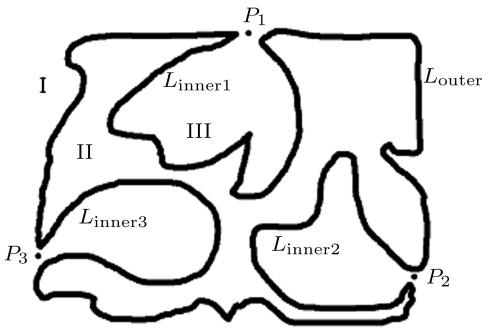


Fig. 1. The space transformation of the multi-window cloaks.

In Fig. 1, regions I and II denote, respectively, the free space and a section of invisible cloaks with its boundaries L_{outer} and L_{inner_k} $(k = 1, 2, 3\ldots)$. Region III is shielded by the cloak, which can be seen as the expansion of the point P_k $(k = 1, 2, 3\ldots)$. In this transformation, the inner boundary $L_{\text{inner}_k}(k = 1, 2, 3\ldots)$ is transformed from the point P_k $(k = 1, 2, 3\ldots)$. So the boundary condition can be described by the equation:

$$\begin{aligned} &x_i'|_{L_{\text{outer}}} = x_i|_{L_{\text{outer}}}, \\ &x_i'|_{L_{\text{inner}k}} = x_i|_{P_k}, \quad k = 1, 2, 3\ldots. \end{aligned} \tag{2}$$

According to Ref. [21], the transformation matrix can be determined by Laplace's equation. So the material parameters of transformation materials can be calculated as follows:

$$\begin{aligned} \varepsilon' &= \varepsilon_0 \,\text{diag}\left[\frac{\lambda_1}{\lambda_2\lambda_3}, \frac{\lambda_2}{\lambda_3\lambda_1}, \frac{\lambda_3}{\lambda_1\lambda_2}\right], \\ \mu' &= \mu_0 \,\text{diag}\left[\frac{\lambda_1}{\lambda_2\lambda_3}, \frac{\lambda_2}{\lambda_3\lambda_1}, \frac{\lambda_3}{\lambda_1\lambda_2}\right], \end{aligned} \tag{3}$$

where λ_i $(i = 1,2,3)$ is the eigenvalue of the tensor V and $V^2 = AA^{\text{T}}$ (A is the Jacobian transformation matrix).

In order to determine λ_i, we should solve Laplace's equation first. The general Laplace's equation has the following form:

$$\left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2} + \frac{\partial^2}{\partial x_3'^2}\right) U_i = 0, \quad i = 1, 2, 3. \tag{4}$$

The λ_i can be obtained by solving Eqs. (4) and (2). The material parameters can be calculated from Eq. (3). By means of this flexible method, we design multi-window invisible cloaks which can conceal a group of objects. In the following section, we will use Laplace's equation to design multi-window cloaks.

3. Multi-window cloaks with regular and irregular shapes

This design includes a two-step process. The particular solution of Eq. (4) with its boundary conditions Eq. (2) can be obtained. Then the material parameters can be acquired according to Eq. (3).

The commercial software Comsol Multiphysics is used to design multi-window invisible cloaks. First, the partial differential equation module is used to set the boundary conditions and obtain the particular solution of Laplace's equation to determine the material parameters of the cloaks. Then, the radio frequency module is used to illuminate the cloaks by a plane wave. This modeling process is similar to that in Ref. [21].

In the computational domain, the inner boundary of the cloak layer is a perfect electric condition and the outer boundary of the cloak layer is a continuous boundary condition. A time-harmonic uniform plane wave with a frequency of 1 GHz is incident onto the cloak, and then propagates along the cloak but does not penetrate through the inner region of the multi-window invisible cloak. The objects in the cloak cannot be detected from exterior electromagnetic waves. A cylindrical cloak with four windows is designed as a case of an invisible cloak with regular shapes, as shown in Fig. 2.

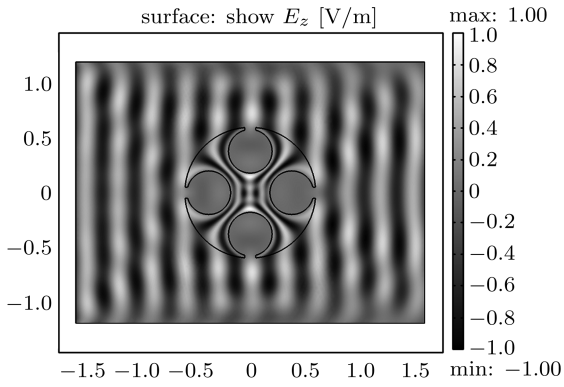


Fig. 2. The z component of the electric field.

From Fig. 2, we can see that in the cloak layer, the z component of the electric field is zero in total and the wave front excluded by the cloak is invariable through the whole propagation process. Despite there being four windows in the cloak, the electromagnetic

wave does not penetrate into the window region. This cloak can make four objects invisible on the inside and can also allow these objects to exchange information and matters with the outer region of the cloak.

In order to improve the practicality of multi-window cloaks, we design multi-window cloaks with irregular shapes, as shown in Fig. 3.

From Fig. 3, the electric field propagates along the multi-window cloak and the wave shape does not change. This makes the objects inside the cloak unable to be detected.

The material parameters can be extracted according to Eq. (1). The values of the permittivity and permeability tensors ε_{zz}, μ_{xx}, μ_{xy} and μ_{yy} of the cloak are shown in Fig. 4.

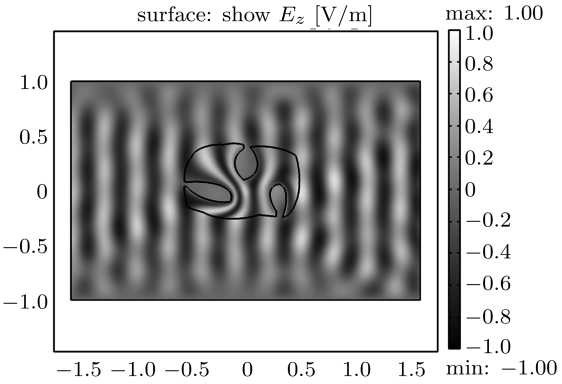


Fig. 3. The z component of the electric field.

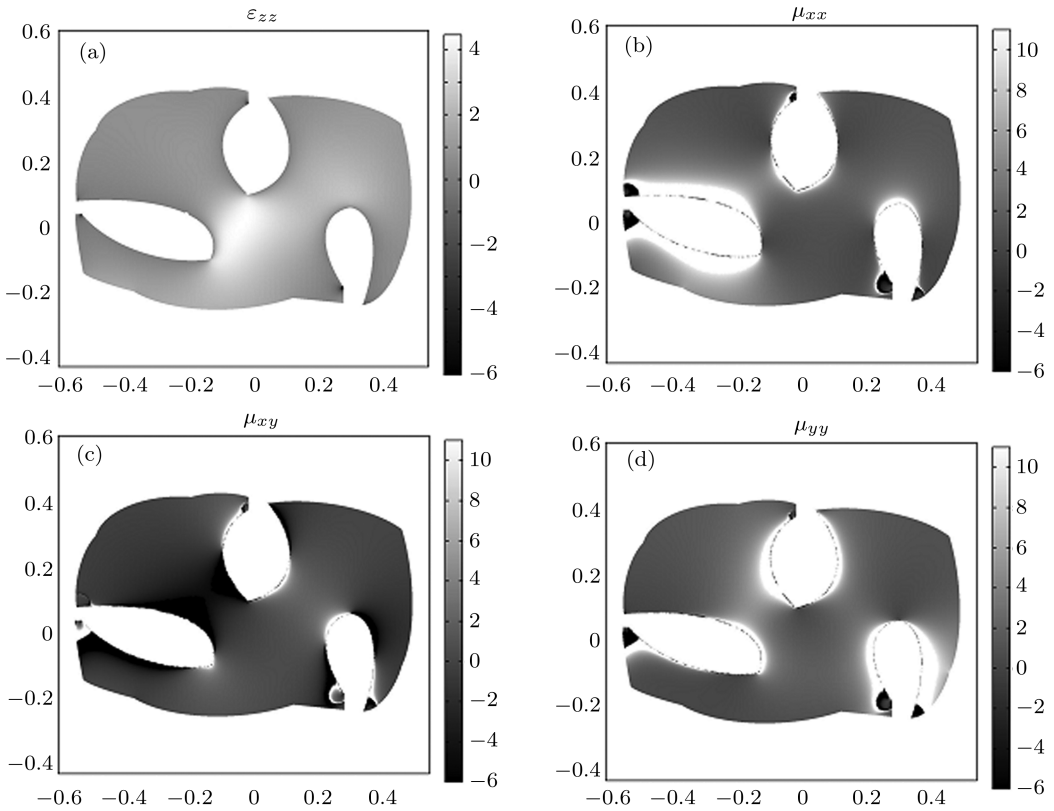


Fig. 4. The material parameters of the multi-window cloak: (a) ε_{zz}; (b) μ_{xx}; (c) μ_{xy}; (d) μ_{yy}.

From Fig. 4, the material parameters of the cloak are discontinuous near the inner cloak region because singular points exist here which cannot be realized in practice. So in the cloak design, singular points should be avoided. In coordinate transformation, the inner boundary is transformed from that of a point and this results in a singular point in mathematics. In order to eliminate these singular points, a point can be replaced by a small region in coordinate transformation.

4. Conclusions

In conclusion, by solving Laplace's equation and the corresponding boundary conditions, the material parameters can be determined. By means of this method, multi-window cloaks can be designed to conceal a

group of objects. Simulations verified the cloaking effects of multi-window cloaks with regular and irregular shapes. These cloaks can exchange information and matters with the outer region of the cloaks and meanwhile shield more than one object.

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