Size transition of spiral waves using the pulse array method

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The domain size of spiral waves is an important issue in studies of two-dimensional (2D) spatiotemporal patterns. In this work, we use the 2D complex Ginzburg–Landau equation (CGLE) as our model and find that an initially big spiral can successfully transfer to several small spirals by applying a pulse array method. The impacts of several important factors, such as array density, controlling intensity and pulsing time, are investigated. This control approach may be useful for the control of 2D spatiotemporal patterns and has potential applications in the control of some realistic systems, such as meteorological and cardiac systems.

Keywords: spiral wave, size transition, pulse array method
PACC: 0565, 8220M, 5235M

1. Introduction

Spiral waves are coherent structures spontaneously appearing in two-dimensional (2D) spatiotemporal systems far from equilibrium. Spiral waves can be frequently observed in numerous natural and experimental systems, such as meteorological models,[1] cardiac tissue,[2,3] chemical reactions,[4] and hydrodynamic systems.[5] It has been reported that spiral waves present complex behaviours and quasi-periodic oscillation of anti-spiral oscillations,[6,7] multi-armed spiral rotations,[8] synchronization defect lines,[9] and segmented spiral waves.[10] In some cases the spirals are harmful and should be suppressed or removed, such as in ventricular fibrillation of cardiac diseases and tornados in extreme atmosphere conditions. Therefore the mechanism of breakup and suppression of spiral waves is an important issue in the research of 2D patterns, and the control of spiral waves has become an important topic, because a deeper understanding of the various controlling techniques may shed light on the potential applications of the spiral patterns.[11−21] The spiral waves can be destroyed by core meandering by the Doppler effect,[15,16] or far field breakup by the Eckhaus instability[17,18] or the injection of external signals.[19,20] It has also been reported that a gradient force can be imposed to produce spiral drift and deformation.[21] Thus it is natural that most existing works have focused on frequency and wavelength analysis to find stable wave conditions for the spiral patterns. On the other hand, the domain size of a spiral is an important issue for the characterization of frozen spiral patterns, in which the borders of the spirals are steady and unchanged during the evolution.[22−24] To be specific, a single spiral (SS) wave of big size can be produced in the whole system space when a perpendicular-gradient initial condition is adopted,[25] and a pattern of multiple spiral (MS) waves of small size areas, which are divided by hyperbolic lines, is developed when the random initial condition is used instead.[22,23] The above SS and MS waves have the same oscillatory frequencies and wavelengths, while the wave sizes of the frozen spiral patterns are uniquely determined by the initial conditions. Because the frozen state could qualitatively describe the steadiness of a spiral-shaped hurricane in a comparatively large time scale, and spiral size is an important concern of many researchers in the study of the production and development of meteorological vortices, it is of crucial importance to study the transition between SS and MS, and the stabilities and the competitions between different size spirals. In this study, we will propose a pulse array method by taking the 2D complex Ginzburg–Landau equation (CGLE) as the model, and study the feasibility and stability of transition from SS to MS waves.

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2. Model and controlling scheme

We consider the following CGLE system

$$\frac{\partial A}{\partial t} = A + (1 + \mathrm{i}c_1)\nabla^2 A - (1 + \mathrm{i}c_2)|A|^2 A, \quad (1)$$

which is a general model describing the onset of Hopf bifurcations for the extended systems.[26−28] A is the complex variable, and c_1 and c_2 are real system parameters. In the 2D case, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. When the polar coordinates system is considered, $(x, y) \to (r, \theta)$, system (1) admits the following spiral wave solution[29]

$$A(r, \theta, t) = F(r) \exp\{\mathrm{i}[m\theta - \omega t + \psi(r)]\}, \quad (2)$$

where the real functions $F(r)$ and $\psi(r)$ denote the amplitude and phase of the spiral, respectively. The integer m is called the topological charge, and ω is the rotation frequency. In the CGLE system, spiral solutions with $m \neq \pm 1$ in Eq. (2) are unstable. In particular, $m = 0$ corresponds to the centrically radiate wave (target wave), which can only be stabilized with the local inhomogeneity or stimulus or special boundary configurations.[30,31] Moreover, the amplitude and the phase functions have the following behaviours: $F(r) \sim \psi' \sim r$ as $r \to 0$, and $F(r) \to \sqrt{1 - k^2}$, $\psi' \to k$ as $r \to +\infty$, where k is the asymptotic wave number of the spiral, and has the dispersion relation $\omega = c_2 + (c_1 - c_2)k^2$.

Considering that system (1) is confined in a square area $L \times L$ with the no-flux boundary condition and the system variables are imposed by a special initial condition, in which the real and imaginary parts are of perpendicular-gradient distribution,[25] we integrate the CGLE and observe the evolution of the spatiotemporal pattern. A spiral wave is produced from the centre of the system and its wave front spreads outwards and occupies all the space to form an SS wave in the end. Assuming that the whole system has been well dominated by the SS wave, as shown in Fig. 1(a), now our task is to destroy the big-size spiral pattern and induce small-size MS in the system. This idea can be potentially realized in the atmospheric field to suppress or remove large area hurricanes or tornadoes because of their similar spiral structures. External electromagnetic pulse injections may influence the atmospheric field, change the path of a hurricane or even demolish the big spiral structure of a tornado.[32] To test the above ideas in numerical simulations, we suggest a scheme of applying spatially distributed pulses to model an external perturbation and study the feasibility of driving SS to MS waves. We modify system (1) as

$$\begin{aligned}\frac{\partial A}{\partial t} = {} & A + (1 + \mathrm{i}c_1)\nabla^2 A - (1 + \mathrm{i}c_2)|A|^2 A \\ & + \varepsilon[H(t) - H(t - T)]\sum_{p=1}^{M}\delta(\boldsymbol{r} - \boldsymbol{r}_p), \quad (3)\end{aligned}$$

where ε is the controlling intensity (pulse strength). $H(t)$ is the Heaviside function with $H(t) = 0$ as $t < 0$ and $H(t) = 1$ as $t \geq 0$. $\delta(r)$ is the Dirac function. $\boldsymbol{r}_p = (x_p, y_p)$ $(p = 1, 2, \ldots, M)$ are the locations of the injected pulses and M is the pulse quantity, which is an integer. At $t = 0$, we introduce M injected pulses, which are located at the controlling sites $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, ..., and $\boldsymbol{r}_M$ combined as an array and distributed in the system space, and study the evolution of the controlled Eq. (3). At $t = T$, the pulsing operation ends and the system runs freely as the uncontrolled form Eq. (1). This scheme adopts an array of pulse injections to destabilize the original SS pattern, which can therefore be called the pulse array method. These local pulses are modelled as the Dirac function array, as shown in Eq. (3). To be specific, in the following study the pulses are distributed evenly and the controller number is $M = N \times N$. Thus the distance of the neighbouring controlling sites of the pulse array is the same $\Delta = L/N$. Extension of the following investigations to other choices of controlling sites (x_p, y_p) is straightforward.

3. Simulation results

Now we investigate the controllability of the pulse array operation. In the following numerical studies, the Euler algorithm with the time iteration 0.02 and the space step 0.5 is adopted. We integrate the controlled system Eq. (3) with the initiation shown in Fig. 1(a), and find that the big size SS is perturbed by the injected pulse array in Fig. 1(b). After the pulse operation is switched off, the original big and single spiral cannot be fully recovered [Fig. 1(c)], and the system evolves into small and multiple spirals [Fig. 1(d)]. The big SS pattern can be successfully changed to small MS waves.

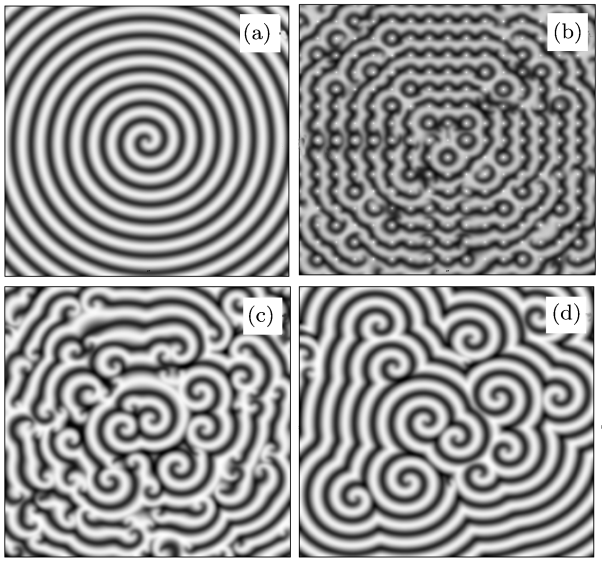


Fig. 1. Snapshots of the system under the control of the pulse array method. The system is a 256×256 square with a no-flux boundary condition. $c_1 = -1.34$, $c_2 = 0.35$. The number of pulses is $M = 16 \times 16$. The pulsing duration is $T = 50$ t.u. (time units) and the controlling intensity is $\varepsilon = 15$ a.u. (arb. units). (a) The big size SS at $t = 0$ t.u. The pulse array is switched on when $t = 50$ t.u. (b) $t = 100$ t.u., (c) $t = 200$ t.u., and (d) $t = 1000$ t.u.; finally, small size MS waves are produced.

The stability and feasibility of the above transition are influenced by several relevant factors, such as pulsing time, controlling intensity and array density. The defect of a spiral wave in the CGLE system is a special point with zero amplitude, and the big (small) defect number roughly indicates the small size (big size) spirals. To observe this spiral size conversion in detail, we study the impact of the pulsing time by counting the change of the defect numbers of the patterns. In Figs. 2(a), 2(b) and 2(c), we present this change with different pulsing time durations. In all the above cases, the SS waves are disturbed by the pulse array. The defect numbers increase rapidly when the array is added. After the control is removed ($t > T$), they monotonically decrease. This phenomenon can be intuitively explained by saying that the existence of pulsing signals introduces extra defect points, as shown in Fig. 2(b), and these defects are not spiral cores and will fade away when the control is removed; therefore, the defect numbers decrease after the pulsing operation finishes. Finally, the pattern of the spirals is frozen and the defect numbers are unchanged with time. Comparing the final defect numbers versus controlling times, as illustrated in Fig. 2(d), we find that a longer pulse array persistence will lead to a bigger defect number statistically. By applying the same technique, we investigate the influence of pulse strength, and find that a bigger pulse intensity and a longer pulsing time have similar effects, as shown in Fig. 3.

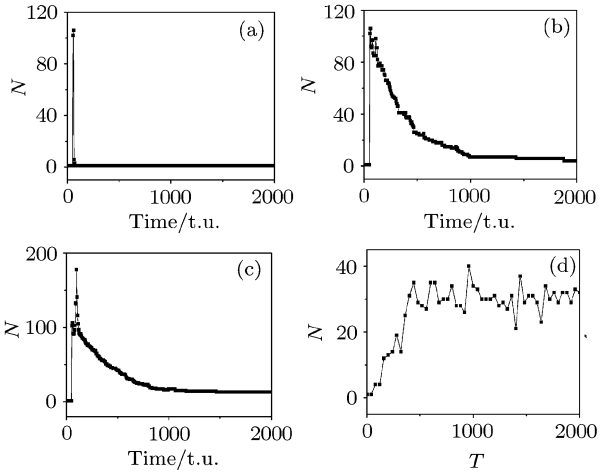


Fig. 2. Defect number N versus pulsing time T. $\varepsilon = 15$ a.u., $M = 16 \times 16$. (a) $T = 10$ t.u., (b) $T = 30$ t.u., (c) $T = 50$ t.u. and (d) N versus T.

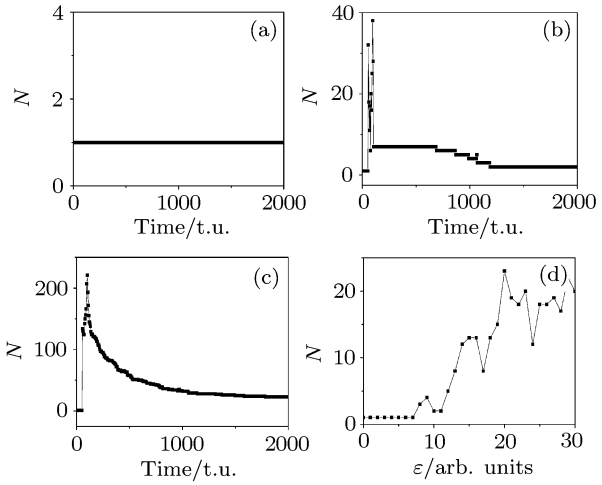


Fig. 3. Defect number versus pulse strength ε. $T = 50$ t.u., $M = 16 \times 16$. (a) $\varepsilon = 3$ a.u., (b) $\varepsilon = 10$ a.u., (c) $\varepsilon = 20$ a.u. and (d) N versus ε.

Finally, we try to find the effects of the pulse densities, which can be quantified by the pulse number M in Eq. (3). We find that a bigger M leads to a bigger defect number, plotted in Fig. 4. Now the impacts of the relevant factors become clear. These findings show that the approach in Eq. (3) is indeed effective in transferring a big-size SS to a pattern of several small-size MS, and the adjustment of the pulse strength, pulsing time and array density change the final results dramatically. As an example, the phase diagram in the T–ε plane for the region of SS or MS is given in Fig. 5.

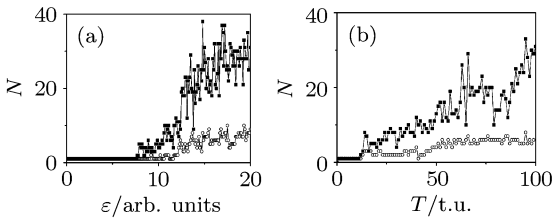


Fig. 4. The defect numbers for different pulse array densities. (a) N versus ε, $T = 100$ t.u., $M = 8\times8$ (white circle), $M = 16 \times 16$ (black square); (b) N versus T, $\varepsilon = 15$ a.u., $M = 8 \times 8$ (white circle), $M = 16 \times 16$ (black square).

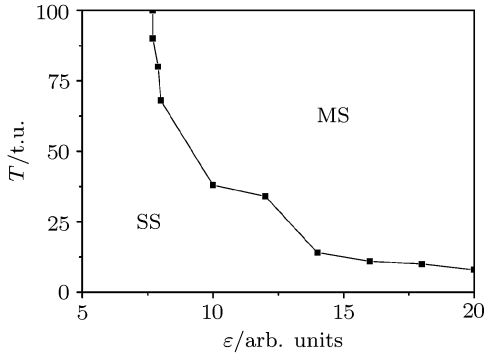


Fig. 5. The pattern distribution in the $\varepsilon - T$ plane. $M = 16 \times 16$. The boundary between the SS and MS waves is plotted as a solid line.

4. Conclusion

In conclusion, we have presented a pulse array approach and found a size transition of the spatiotemporal system from a big spiral to small spirals. Compared to most published work, which focuses on the impact of oscillatory frequencies, the boundary effect or periodic/noise signal injection to investigate spiral instability, our work mainly focuses on the stabilities of spirals with different sizes, which show the other important properties of the spirals and cannot be ignored in many cases. We have also studied the transition by considering the different influences of the pulse strength, operation duration and array density. Based on these observations, the proposed control approach has been proven to be highly effective. The study of realizing this theoretical means in the application of controlling or annihilating spiral waves in real meteorological and cardiac systems is important and is under investigation.

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