Linear-control-based synchronization of coexisting attractor networks with time delays*

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This paper introduces the concept of linear-control-based synchronization of coexisting attractor networks with time delays. Within the new framework, closed loop control for each dynamic node is realized through linear state feedback around its own arena in a decentralized way, where the feedback matrix is determined through consideration of the coordination of the node dynamics, the inner connected matrix and the outer connected matrix. Unlike previously existing results, the feedback gain matrix here is decoupled from the inner matrix; this not only guarantees the flexible choice of the gain matrix, but also leaves much space for inner matrix configuration. Synchronization of coexisting attractor networks with time delays is made possible in virtue of local interaction, which works in a distributed way between individual neighbours, and the linear feedback control for each node. Provided that the network is connected and balanced, synchronization will come true naturally, where theoretical proof is given via a Lyapunov function. For completeness, several illustrative examples are presented to further elucidate the novelty and efficacy of the proposed scheme.

Keywords: linear feedback control, Lyapunov function, the Newton–Leipnik equation attractor, time delays

PACC: 0545, 0547

1. Introduction

Various large-scale and complicated systems can be modelled by networks, such as the Internet, WWW, and many others. The most striking recent advances are the occurrence of the small world network model[1] and scale free network model,[2] which have been declared to be closer to most real-world networks as compared with their earlier counterpart, the random-graph model.[3] Since that time, many scientists and scholars have contributed to this topic, and answered some questions like network stability as well as stabilization,[4−7] network spreading[8] and network growing.[9] In addition, network synchronization has also been concentrated on, for its widespread applications in various fields of sciences.[10−23] Among the available results, synchronization cost, local synchronization with and without time varying topology for both continuous and discrete networks, phase synchronization and adaptive synchronization have been investigated in detail. It is undoubted that those contributions are important to further research. However, either the linear feedback control action is added through the inner connected matrix, like that in Ref. [20], which reduces the flexibility of the feedback gain selection, or the node dynamics is too easy to be synchronized, like that in Ref. [21]. These may be some of the possible reasons for their limited usage, and moreover, unlike the results in Ref. [22], the linear feedback control action here, which is around its own arena, is free of time delays.

In this paper, linear feedback control action for each node is added in a decoupled way with the inner connected matrix, and the node dynamics has coexisting attractors, which is much more complex. The coexisting attractor network synchronization comes true under the synergy of the node dynamics, the linear feedback control, the inner connected matrix and the outer connected matrix. In the case of theoretical rigidity and completeness, the Lyapunov function is employed to prove the validity of the suggested strategy.

The rest of the paper is organized as follows: In Section 2, complex networks with coexisting attractors

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are considered and the corresponding theorem of synchronization for networks with delays is established. Illustrative examples are given in Section 3, and some remarks will be provided in Section 4.

2. Synchronization stability for coexisting attractor networks with time delays

In this paper we will use a linear control scheme, which works in a more independent way to steer the coexisting attractor networks: the Newton–Leipnik chaotic system network, where all nodes are identical, i.e. each node of the networks has the same dynamics, which is obtained by Newton and Leipnik[24] through the addition of a linear feedback to Euler's body equations in 1981, and its control problem has been investigated by several authors.[25−28] The Newton–Leipnik (N-L) system is described by

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = f(x) = \begin{bmatrix} -\gamma x_1 + x_2 + 10x_2x_3 \\ -x_1 - 0.4x_2 + 5x_1x_3 \\ \sigma x_3 - 5x_1x_2 \end{bmatrix}. \quad (1)$$

Equation (1) has two coexisting chaotic attractors when $\gamma = 0.4$ and $\sigma = 0.175$, the equilibrium points of the system (1) with nominal parameters are

$$\begin{aligned} x_{\rm e}^{(1)} &= (0,0,0)^{\rm T}, \\ x_{\rm e}^{(2)} &= (0.2390, 0.0308, 0.2103)^{\rm T}, \\ x_{\rm e}^{(3)} &= (-0.2390, -0.0308, 0.2103)^{\rm T}, \\ x_{\rm e}^{(4)} &= (0.0315, -0.1224, -0.1103)^{\rm T}, \\ x_{\rm e}^{(5)} &= (-0.0315, 0.1224, -0.1103)^{\rm T}. \end{aligned}$$

All these equilibrium points are unstable. Note that the points $x_{\rm e}^{(2)}$ and $x_{\rm e}^{(3)}$ lie in the eye-centres of UA, while $x_{\rm e}^{(4)}$ and $x_{\rm e}^{(5)}$ are situated in the eye-centres of LA, and its phase locus is shown in Fig. 1.

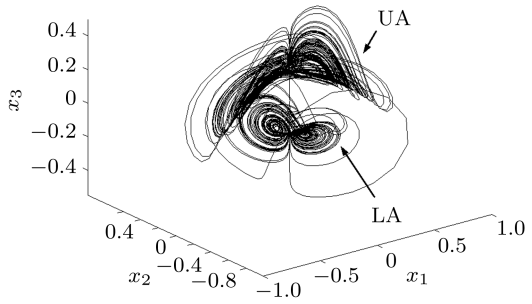


Fig. 1. The coexisting strange attractors of the Newton–Leipnik system.

Considering a dynamic network consisting of linearly and diffusively coupled identical N-L nodes with time delays, where each node is an n-dimensional dynamic system, the state equations of this network can be described by

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} g_{ij} A x_j(t-\tau), \quad i = 1, \ldots, N, \quad (2)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\rm T} \in R^n$ are the n-dimensional state variable of node i, $f(x_i(t))$ is a given nonlinear smooth vector-valued function describing the dynamics of the nodes, τ is the coupling time-delay, $A = (a_{ij}) \in R^{n\times n}$ is the constant fixed inner-coupling matrix between nodes, ε is a positive constant representing the coupling strength, and $G = (g_{ij}) \in R^{N\times N}$ is the coupling configuration matrix representing the topological structure of the network, in which $g_{ij} \in R$ is defined as follows: if there is a coupling from node j to node i ($i \neq j$), then the coupling strength $g_{ij} = g_{ji} = 1$; otherwise $g_{ij} = 0$, and the diagonal elements of G are defined by

$$g_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} g_{ij}, \quad i = 1, 2, \ldots, N. \quad (3)$$

Note that, the coupling configuration matrix G is assumed to be symmetrical and irreducible. Since the row sums of the matrix G are all zero, zero is an eigenvalue of G associated with eigenvector $(1, 1, \ldots, 1)^{\rm T}$. Hereafter, the network (2) with delays is said to achieve asymptotical synchronization if

$$x_i(t) = s(t) \quad (4a)$$

as $t \to +\infty, i = 1, 2, \ldots, N$, in which $s(t) \in R^n$ satisfies

$$\dot{s}(t) = f(s(t)), \quad (4b)$$

where $s(t)$ can either be an equilibrium point, a (quasi-) periodic orbit, or an orbit of a chaotic attractor. In order to achieve the control purpose, our strategy is to utilize linear feedback controllers u_i on the nodes in the network (2). Then the controlled network is described by

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} g_{ij} A x_j(t-\tau) + u_i, \quad i = 1, \ldots, N, \quad (5)$$

where u_i are designed as

$$u_i = -K(x_i(t) - s(t)), \quad i = 1, \ldots, N, \quad (6)$$

where K is a parameter matrix to be designed. Substituting Eq. (6) into Eq. (5), one has

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} g_{ij} A x_j(t-\tau) - K(x_i(t) - s(t)), \quad i = 1, \ldots, N. \tag{7}$$

Lemma 1 Suppose that $G = (g_{ij})_{N\times N}$ is a real symmetry and irreducible matrix, where $g_{ij} = g_{ji} \geq 0$ $(i \neq j)$, $g_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} g_{ij}$, then

(i) 0 is an eigenvalue of G with multiplicity 1, associated with the eigenvector $(1,1, \ldots, 1)^{\mathrm{T}}$;

(ii) All the other eigenvalues of G are real and less than 0;

(iii) There exists a unitary matrix, $\Phi = (\Phi_1, \Phi_2, \ldots, \Phi_N)$ such that $G^{\mathrm{T}}\Phi_k = \lambda_k \Phi_k, k = 1, 2, \ldots, N$, where $0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$ are the eigenvalues of G.

Theorem 1 Consider the delayed dynamical network (7), and let $0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$ be the eigenvalues of the coupling configuration matrix G. If the following $(N-1)$ systems of n-dimensional linear time-varying delayed differential equations are asymptotically stable about their zero solutions

$$\dot{\eta}(t) = (Df(s(t)) - K)\eta(t) + \varepsilon\lambda_i A\eta(t-\tau), \quad 2 \leq i \leq N, \tag{8}$$

where $Df(s(t)) \in R^{n\times n}$ is the Jacobian of $f(s(t))$ at $s(t), \eta(t) \in R^n, \eta(t-\tau) \in R^n$, then the synchronized states of Eq. (4) are asymptotically stable.

Proof To investigate the stability of the synchronized states of Eq. (4), let

$$x_i(t) = s(t) + e_i(t). \tag{9}$$

Substituting it into Eq. (7) gives

$$\dot{e}_i(t) = f(s(t) + e_i(t)) - f(s(t)) - Ke_i(t) + \varepsilon \sum_{j=1}^{N} g_{ij} A e_j(t-\tau), \quad 1 \leq i \leq N. \tag{10}$$

For $e(t) = (e_1(t), e_2(t), \ldots, e_N(t)) \in R^{n\times N}$, one has

$$\dot{e}(t) = (Df(s(t) - K)e(t) + \varepsilon A e(t-\tau)G^{\mathrm{T}}, \tag{11}$$

where $e(t-\tau) = (e_1(t-\tau), e_2(t-\tau), \ldots, e_N(t-\tau)) \in R^{n\times N}$. By Lemma 1, there exists a nonsingular matrix Φ, such that $G^{\mathrm{T}}\Phi = \Phi\Gamma, \Gamma = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$. If one sets $e(t)\Phi = v(t) = (v_1(t), v_2(t), \ldots, v_N(t)) \in R^{n\times N}$, then equation (11) can be transformed into the following matrix equation

$$\dot{v}(t) = (Df(s(t) - K)v(t) + \varepsilon A v(t-\tau)\Gamma, \quad i = 1, \ldots, N. \tag{12}$$

Note that $\lambda_1 = 0$ corresponds to the synchronization of the system (4). If the following $N-1$ pieces of the n-dimensional linear time-varying delayed differential equations

$$\dot{v}_i(t) = (Df(s(t) - K)v_i(t) + \varepsilon\lambda_i A v_i(t-\tau), \quad i = 2, \ldots, N \tag{13}$$

are asymptotically stable, then $e(t)$ will tend to zero asymptotically, which shows that the synchronized states of Eq. (4) are asymptotically stable. This completes the proof.

Theorem 2 Assume that all eigenvalues of the matrix G are listed in order, $0 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_N$. If there exist i, j and K such that $\begin{bmatrix} -\lambda_j(A + A^{\mathrm{T}}) & \lambda_i A \\ \lambda_i A^{\mathrm{T}} & 0 \end{bmatrix} \leq 0, \lambda_{\max}(Df(s(t)) - K + \lambda_j A) < 0$ for all $t \geq t_0$, where $i = 2, \ldots, N, j = 1, \ldots, N$, then the synchronous states (4) of the dynamical network (7) are exponentially stable for any fixed delay $\tau > 0$.

Proof Consider the linear time-varying system $\dot{x}(t) = (Df(s(t)) - K)x(t) + \varepsilon\lambda_i A x(t-\tau), \ 2 \leq i \leq N$. Since

$$\begin{aligned}
&\mathrm{d}\left\|x(t)\right\|^2/\mathrm{d}t \\
&= x(t)^{\mathrm{T}}(Df(s(t)) - K)x(t) + \varepsilon\lambda_i x(t)^{\mathrm{T}} A x(t-\tau) + x(t)^{\mathrm{T}}(Df(s(t)) - K)^{\mathrm{T}} x(t) + \varepsilon\lambda_i x(t-\tau)^{\mathrm{T}} A^{\mathrm{T}} x(t) \\
&= x(t)^{\mathrm{T}} \left\{[(Df(s(t)) - K + \lambda_j A)]^{\mathrm{T}} + [(Df(s(t)) - K + \lambda_j A)]\right\} x(t) \\
&\quad + \varepsilon \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\lambda_j(A + A^{\mathrm{T}}) & \lambda_i A \\ \lambda_i A^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix} \\
&\leq x(t)^{\mathrm{T}} \left\{[(Df(s(t)) - K + \lambda_j A)]^{\mathrm{T}} + [(Df(s(t)) - K + \lambda_j A)]\right\} x(t) \\
&\leq 2\lambda_{\max}[Df(s(t)) - K + \lambda_j A] \left\|x(t)\right\|^2,
\end{aligned}$$

where $\lambda_{\max}(\bullet)$ denotes the maximum eigenvalue of matrix $\bullet$, all the linear time-varying systems of Eq. (8) are exponentially stable around their zero solutions. From Theorem 1, the synchronous states of dynamical network (7) are exponentially stable. The proof is thus complete.

3. Illustrative examples

A five-node network is investigated here, in which each node is a coexisting attractor chaos system, i.e. the N–L system, the Jacobian matrix of which is

$$Df(x(t)) = \begin{bmatrix} -\gamma & 1+10x_3 & 10x_3 \\ -1+5x_3 & -0.4 & 5x_1 \\ -5x_2 & -5x_1 & \sigma \end{bmatrix}.$$

If we choose $\varepsilon = 1$, $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $K = \begin{bmatrix} 0 & 0 & 0 \\ \kappa_1 & 0 & 0 \\ \kappa_2 & \kappa_3 & \kappa_4 \end{bmatrix}$, then

$$Df(s(t))|_{s(t)=x(t)} - K + \lambda_j A = \begin{bmatrix} -\gamma & 1+10x_3 & 10x_3 \\ -1+5x_3 & -0.4 & 5x_1 \\ -5x_2 & -5x_1 & \sigma \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ \kappa_1 & 0 & 0 \\ \kappa_2 & \kappa_3 & \kappa_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_j \end{bmatrix}$$
$$= \begin{bmatrix} -\gamma & 1+10x_3 & 10x_3 \\ -1+5x_3-\kappa_1 & -0.4 & 5x_1 \\ -5x_2-\kappa_2 & -5x_1-\kappa_3 & \sigma-\kappa_4+\lambda_j \end{bmatrix}.$$

As to the origin equilibrium, one has

$$Df(s(t))|_{x_e^{(1)}=(0,0,0)^{\mathrm{T}}} - K + \lambda_j A = \begin{bmatrix} -\gamma & 1+10x_3 & 10x_3 \\ -1+5x_3-\kappa_1 & -0.4 & 5x_1 \\ -5x_2-\kappa_2 & -5x_1-\kappa_3 & \sigma-\kappa_4+\lambda_j \end{bmatrix}_{x_e^{(1)}=(0,0,0)^{\mathrm{T}}}$$
$$= \begin{bmatrix} -\gamma & 1 & 0 \\ -1-\kappa_1 & -0.4 & 0 \\ -\kappa_2 & -\kappa_3 & \sigma-\kappa_4+\lambda_j \end{bmatrix}.$$

It is obvious that, if $\kappa_1 = 1, \kappa_2 = \kappa_3 = 0$ and $\kappa_4 > \sigma$, then $\lambda_{\max}(Df(s(t)) - K + \lambda_j A) < 0$ will hold naturally. During simulation, $\kappa_1 = 1, \kappa_2 = \kappa_3 = 0, \kappa_4 = 5$ is employed. As to the second equilibrium, one has

$$Df(s(t))|_{x_e^{(2)}=(0.2390,0.0308,0.2103)^{\mathrm{T}}} - K + \lambda_j A$$
$$= \begin{bmatrix} -\gamma & 1+10x_3 & 10x_3 \\ -1+5x_3-\kappa_1 & -0.4 & 5x_1 \\ -5x_2-\kappa_2 & -5x_1-\kappa_3 & \sigma-\kappa_4+\lambda_j \end{bmatrix}_{x_e^{(2)}=(0.2390,0.0308,0.2103)^{\mathrm{T}}}$$
$$= \begin{bmatrix} -\gamma & 3.103 & 2.103 \\ 0.05105-\kappa_1 & -0.4 & 1.195 \\ -0.154-\kappa_2 & -1.195-\kappa_3 & \sigma-\kappa_4+\lambda_j \end{bmatrix}.$$

It is obvious that if $\kappa_1 = 0.05105, \kappa_2 = -0.154, \kappa_3 = -1.195$ and $\kappa_4 > \sigma$, then $\lambda_{\max}(Df(s(t)) - K + \lambda_j A) < 0$ will hold naturally. During simulation, $\kappa_1 = 0.05105, \kappa_2 = -0.154, \kappa_3 = -1.195, \kappa_4 = 50$ is employed and the time delay is uniformly selected as $\tau = 1$. The initial condition in this work is chosen as

$$x_{10} = [-0.2, -0.1, 0, 0.1, 0.2]^{\mathrm{T}}, \quad x_{20} = [-0.4, -0.3, 0, 0.3, 0.4], \quad x_{30} = [-0.6, -0.5, 0, 0.5, 0.6].$$

Remark 1 The inner connected matrix has merely one non-zero cell, which implies that there is only one element of the state variables of the node being taken to be coupled with its neighbour nodes. When the scale of the network is large, this can cut the expense of the connection greatly.

Remark 2 The gain matrix K here can have much more freedom compared with the gain inner matrix coupled strategy, which enables the flexibility of the linear state feedback to be controlled within node level.

In order to explain the linear control mechanism further, three examples of co-existing attractor networks corresponding to star-, ring- and chain-connected topologies are studied.

3.1. Star-connected network

The star-connected coexisting attractor network is shown in Fig. 2, the nodes are labelled as 1 to 5. Here $N = 5$.

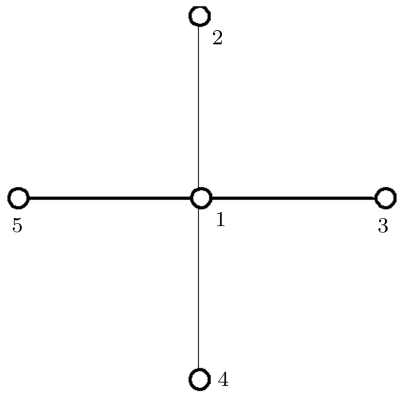


Fig. 2. Star-connected coexisting attractor network.

The inter-coupling matrix $\boldsymbol{G}$ is

$$\boldsymbol{G} = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{14}$$

Synchronization at the manifold of origin is illustrated in Fig. 3.

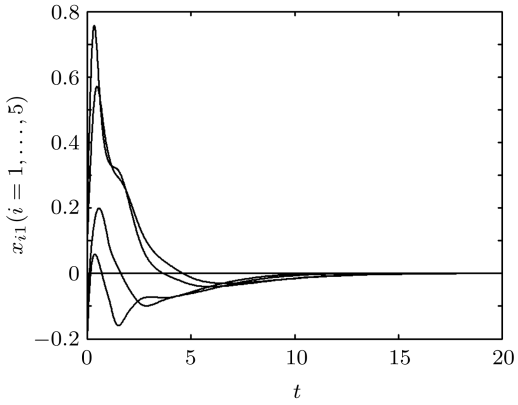


Fig. 3. Synchronization at the manifold of origin of the star-connected network.

Note Without loss of generality, and in the case of saving length, only synchronization of the first variables of the nodes is given throughout this paper.

Remark 3 The control action among the nodes includes two different parts: one comes from its own arena, we call this action "in control"; and the other comes from its neighbours, we call this control action "net control". For example, the control action added on the second node has two distinct parts, one is linear feedback control from its inner part, i.e.,

$$\begin{aligned} u_2^{\text{in}} &= \begin{bmatrix} 0 & 0 & 0 \\ \kappa_1 & 0 & 0 \\ \kappa_2 & \kappa_3 & \kappa_4 \end{bmatrix} \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} \Bigg|_{i=2} \\ &= \begin{bmatrix} 0 \\ \kappa_1 x_{21} \\ \kappa_2 x_{21} + \kappa_3 x_{22} + \kappa_4 x_{23} \end{bmatrix} \Bigg|_{\kappa_1=1, \kappa_2=\kappa_3=0, \kappa_4=5} \\ &= \begin{bmatrix} 0 \\ x_{21} \\ 5x_{23} \end{bmatrix}, \end{aligned}$$

and the other is from its neighbours. Because it has only one neighbour, i.e., the first node, we have

$$\begin{aligned} u_2^{\text{net}} &= \sum_{j \in N_i} g_{ij} A(x_j(t-\tau) - x_i) \Big|_{i=2, j=1} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} (x_1(t-\tau) - x_2) \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11}(t-\tau) - x_{21} \\ x_{12}(t-\tau) - x_{22} \\ x_{13}(t-\tau) - x_{23} \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ x_{13}(t-\tau) - x_{23} \end{bmatrix} \Bigg|_{\tau=1}$$

$$= \begin{bmatrix} 0 \\ 0 \\ x_{13}(t-1) - x_{23} \end{bmatrix},$$

where N_i denotes the number of node i's neighbours.

If we want to drive the coexisting network to synchronize at the other non-zero manifold, for example, the second equilibrium, we just need to turn the reference signal into the second equilibrium. Synchronization at the manifold of the second equilibrium is illustrated in Fig. 4.

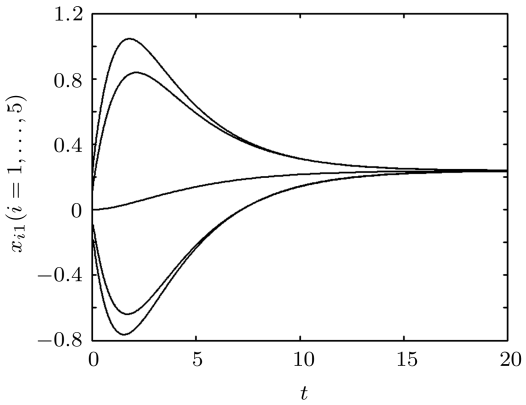


Fig. 4. Synchronization at the manifold of the second equilibrium of the star-connected network.

Remark 4 After the node dynamics has been stabilized to the second equilibrium, when the extra control action of linear state feedback is cut off, all the nodes will be synchronized at the upper chaotic attractor.

3.2. Ring-connected network

The ring-connected coexisting attractor network is depicted in Fig. 5. For simplicity, we only display the outer connected matrix and the results obtained here

$$\boldsymbol{G} = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}. \tag{15}$$

The corresponding synchronization results are shown in Figs. 6 and 7.

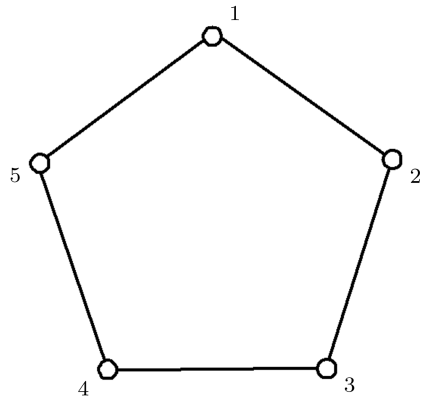


Fig. 5. Ring-connected coexisting attractor network.

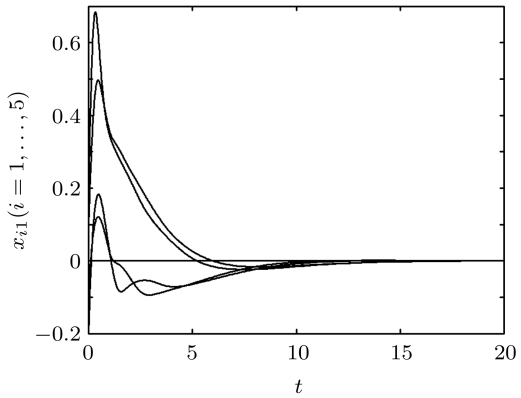


Fig. 6. Synchronization at the manifold of origin of the ring-connected network.

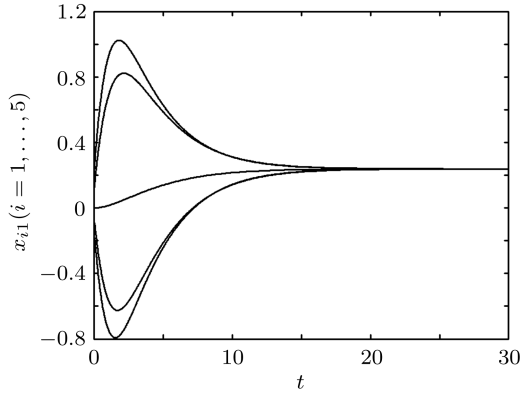


Fig. 7. Synchronization at the manifold of the second equilibrium of the ring-connected network.

3.3. Chain-connected network

The chain-connected coexisting attractor network is given in Fig. 8. For the same reason as for the ring-connected case, we here only give the outer connected matrix and the results obtained.

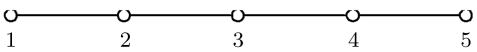


Fig. 8. Chain-connected coexisting attractor network.

$$\boldsymbol{G} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \quad (16)$$

The corresponding synchronization results are illustrated in Figs. 9 and 10.

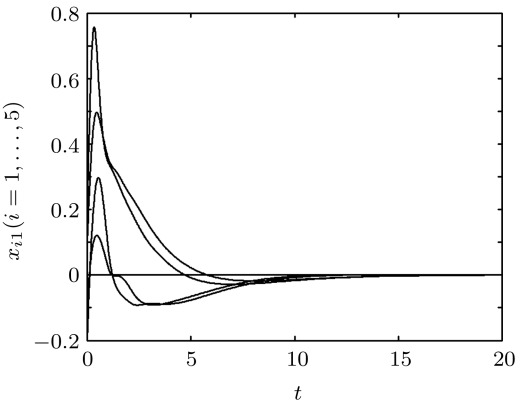


Fig. 9. Synchronization at the manifold of origin of the chain-connected network.

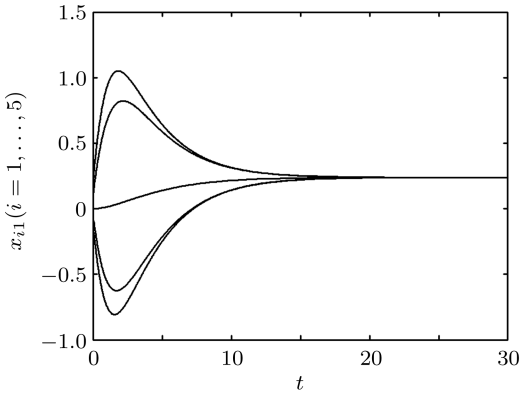


Fig. 10. Synchronization at the manifold of the second equilibrium of the chain-connected network.

4. Conclusions

Synchronization of coexisting attractor networks with time delays has been studied and discussed in this paper. Unlike the existing methods, a linear control strategy is used here, which works in a decentralized way. Due to its freedom from limits of couplings with the inner connected matrix, the feedback gain matrix selection can be made more flexible.

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