Improved delay-dependent globally asymptotic stability of delayed uncertain recurrent neural networks with Markovian jumping parameters*

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In this paper, we have improved delay-dependent stability criteria for recurrent neural networks with a delay varying over a range and Markovian jumping parameters. The criteria improve over some previous ones in that they have fewer matrix variables yet less conservatism. In addition, a numerical example is provided to illustrate the applicability of the result using the linear matrix inequality toolbox in MATLAB.

Keywords: recurrent neural networks, time-varying delays, linear matrix inequality, Lyapunov–Krasovskii functional, Markovian jumping parameters

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1. Introduction

In the past few decades, the mathematical properties of recurrent neural networks (RNNs), such as the stability, the attractivity and the oscillation, have been intensively investigated. RNNs have been successfully applied in many areas, including image processing, pattern recognition, associative memory, and optimization problems. In fact, the stability analysis issue for RNNs with time delays has been an attractive subject of research in the past few years, where the time delays under consideration can be classified as constant delays, time-varying delays, and distributed delays, with various sufficient conditions, either delay-dependent or delay-independent. Since delay-independent criteria tend to be more conservative especially for small size delays, considerable attention has been devoted to delay-dependent ones, see for example Refs. [1]–[6].

Recently, stability analysis issues for neural networks in the presence of parameter uncertainties have been proposed in some papers. Chen *et al.*[7] used free-weighting matrices to study the exponential stability of uncertain stochastic delayed neural networks in mean square. Rakkiyappan *et al.*[8] obtained robust stability results for uncertain stochastic neural networks with discrete interval and distributed time-varying delays by using LMI and free-weighting matrices. The free weighting matrix method, by contrast, can keep a balance between the conservatism and the computational effort; there is still some conservatism (see Refs. [9]–[11]) and the criteria can be simplified. But this method needs many matrix variables; Shao[12] and Kwon[13] got delay-dependent stability criteria for systems with interval time-varying delay with fewer matrix variables than some recently reported ones.

It is worth noting that the RNNs may have finite modes, and the modes may switch (or jump) from one to another at different times. Recently, it has been shown in Ref. [14] that the switching (or jumping) between different RNN modes can be governed by a Markovian chain. Hence, an RNN with such a "jumping" character may be modelled as a hybrid one; that is, the state space of the RNN contains both discrete and continuous states. For a specific mode, the dynamics of the RNN is continuous, but the parameter jumps among different modes may be seen as discrete events. When time delay control systems are subject to the influence of Markovian jumping parameters, the behaviour of the system becomes stochastic. Standard techniques for the design of robust controllers of deterministic systems mentioned above are no longer applicable. Therefore, RNNs with Markovian jumping parameters are of great significance in modelling

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a class of neural networks with finite network modes. Note that the concept of Markovian neural networks or stochastic system has already been used in some papers, see e.g. Refs. [15]–[19].

It should be pointed out that, up to now, the stability analysis problem for uncertain RNNs with Markovian switching has received little research attention, despite its practical importance. In this paper, we are interested in developing criteria for delay-dependent stability of delayed recurrent neural networks with Markovian jumping parameters. By utilizing a Lyapunov–Krasovskii functional and conducting the stochastic analysis, we recast the addressed stability analysis problem into a numerically solvable problem. A unified LMI approach is developed to establish sufficient conditions for the neural networks to be globally asymptotically stable. Note that LMIs can be easily solved by using the Matlab LMI toolbox, and no tuning of parameters is required.[20] The numerical example is provided to show the usefulness of the proposed global stability condition.

Notations: The notations are quite standard. Throughout this paper, R^n denotes the n-dimensional Euclidean space. The superscript "T" denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices means that $X - Y$ is positive-semidefinite (respectively, positive-definite). I_n is the $n \times n$ identity matrix. $|\cdot|$ is the Euclidean norm in R^n. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$ where $\lambda_{\max}(\cdot)$ (respectively, $\lambda_{\min}(\cdot)$) means the largest (respectively, smallest) eigenvalue of A. The symmetric terms in a symmetric matrix are denoted by $\star$.

2. Model descriptions and preliminaries

Consider the following delayed recurrent neural networks with Markovian jumping described by

$$\begin{aligned} \mathrm{d}x(t) = \Big[& - d_m(\eta_t)x_m(t) + \sum_{n=1}^{p} a_{mn}(\eta_t)f_n(x_n(t)) \\ & + \sum_{n=1}^{q} b_{mn}(\eta_t)f_n(x_n(t-h(t))) \\ & + \sum_{n=1}^{p} c_{mn}\int_{t-\tau(t)}^{t} f_n(x_n(s))\mathrm{d}s + I_m\Big]\mathrm{d}t, \\ & m,n = 1,2,\ldots,p, \end{aligned} \tag{1}$$

where $x_m(t)$ is the activation of the m-th neurons, $f_n(.)$ denotes the signal function of the n-th neurons; I_m stands for the external inputs at time t; $d_m(\eta_t)$ is a positive number, and it denotes the charging time constant or passive decay rate of the m-th neurons; $a_{mn}(\eta_t)$, $b_{mn}(\eta_t)$, $c_{mn}(\eta_t)$ stand for the synaptic connection weights. $\eta_t, t \geq 0$ is a homogeneous, finite-state Markovian process with right continuous trajectories and takes values in a finite set $S = \{1,2,\ldots,N\}$ with a given probability space $(\Omega, \mathscr{F}, \mathscr{P})$ and the initial model η_0, and a finite-state Markovian process with right continuous trajectories with generator $\aleph = [\pi_{ij}]$ and transition probability from mode i at time t to mode j at time $t+\delta, i, j \in S$,

$$\begin{aligned} p_{ij} &= Pr(\eta_{t+\delta} = j | \eta_t = i) \\ &= \begin{cases} \pi_{ij}\delta + o(\delta), & i \neq j, \\ 1 + \pi_{ij}\delta + o(\delta), & i = j \end{cases} \end{aligned} \tag{2}$$

with transition probability rates $\pi_{ij} \geq 0$, for $i, j \in S$, $i \neq j$ and $\pi_{ii} = -\sum_{j=1}^{N} \pi_{ij}$, where $\delta > 0$ and $\lim_{\delta \to 0} o(\delta)/\delta = 0$.

To obtain our results, we make the following assumption.

(i) The neuron activation functions in Eq. (1) satisfy

$$|f_n(s_1) - f_n(s_2)| \leq L_n|s_1 - s_2|, s_1, s_2 \in R, \quad s_1 \neq s_2,$$

where L_n $(n = 1,2,\ldots,p)$ are positive constants.

(ii) The neural activation function in Eq. (1) is bounded.

(iii) The time delays $h(t)$ satisfy

$$0 \leq h_1 \leq h(t) \leq h_2, \quad \dot{h}(t) \leq h, \quad 0 \leq \tau(t) \leq \tau,$$

where h_1, h_2, h are constants.

It is clear that under the assumptions (i) and (ii), system (1) has at least one equilibrium. In order to simplify our proof, we shift the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_p^*)^{\mathrm{T}}$ of system (1) to the origin. This transformation

$$g_n(x_n(t)) = f_n(u_n(t)+x^*) - f(x^*), u_m(t) = x_m(t) - x^*$$

puts system (1) into the following system (3)

$$\begin{aligned}\mathrm{d}u_m(t) = \Big[& - d_m(\eta_t)u_m(t) + \sum_{n=1}^{p} a_{mn}(\eta_t)g_n(u_n(t)) \\ & + \sum_{n=1}^{q} b_{mn}(\eta_t)g_n(u_n(t-h(t))) \\ & + \sum_{n=1}^{p} c_{mn}(\eta_t)\int_{t-\tau(t)}^{t} g_n(u_n(s))\mathrm{d}s\Big]\mathrm{d}t, \\ & m,n = 1,2,\ldots,p \end{aligned} \tag{3}$$

with the initial values given by

$$u_m(t) = \varphi_m(t), t \in [-2\mu, 0], \mu = \max\{h, \tau\},$$

where $\varphi_m(t)$ is a continuous function $g_n(.)$ which satisfies the following properties.

(H1) The neuron activation functions $g_n(.)$ ($n = 1,2,\ldots,p$) are bounded on R and Lipchitz continuous, that is, there exist constants L_n such that

$$|g_n(s_1) - g_n(s_2)| \le L_n|s_1 - s_2|,$$

for all $s_1, s_2 \in R$.

Rewriting Eq. (3) into the vector form, we obtain

$$\begin{aligned}\mathrm{d}u(t) = \Big[& - D(\eta_t)u(t) + A(\eta_t)g(u(t)) \\ & + B(\eta_t)g(u(t-h(t))) \\ & + C(\eta_t)\int_{t-\tau(t)}^{t} g(u(s))\mathrm{d}s\Big]\mathrm{d}t, \end{aligned} \tag{4}$$

where

$$\begin{aligned} & u(t) = (u_1(t), u_2(t), \ldots, u_p(t))^{\mathrm{T}}, \\ & D(\eta_t) = \mathrm{diag}(d_1(\eta_t), d_2(\eta_t), \ldots, d_p(\eta_t)), \\ & A(\eta_t) = (a_{mn}(\eta_t))_{p\times p}, \\ & B(\eta_t) = (b_{mn}(\eta_t))_{p\times p}, \quad C(\eta_t) = (c_{mn}(\eta_t))_{p\times p}. \end{aligned}$$

Some lemmas used in this correspondence are listed as follows.

Lemma 1 (Schur complement) Given constant Ω_1, Ω_2, and Ω_3 with appropriate dimensions, where $\Omega_1^{\mathrm{T}} = \Omega_1$ and $\Omega_2^{\mathrm{T}} = \Omega_2 > 0$, then

$$\Omega_1 + \Omega_3^{\mathrm{T}}\Omega_2^{-1}\Omega_3 < 0,$$

if and only if $\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ \star & -\Omega_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -\Omega_2 & \Omega_3 \\ \star & \Omega_1 \end{bmatrix} < 0$.

Lemma 2[16] For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \to R^n$ such that the integrals concerned are well defined, then the following inequality holds

$$\begin{aligned} & \Big[\int_a^b x(s)\mathrm{d}s\Big]^{\mathrm{T}} M \Big[\int_a^b x(s)\mathrm{d}s\Big] \\ & \le (b-a)\int_a^b x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s. \end{aligned}$$

Lemma 3[5] Let A, D and N be real constant matrices with appropriate dimensions; matrix $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \le I$. Then we have:

For any $\epsilon_1 > 0$, $DF(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}} \le \epsilon^{-1}DD^{\mathrm{T}} + \epsilon N^{\mathrm{T}}N$.

Lemma 4[13] For any scalar $h(t) > 0$, and any constant matrix $Q \in R^{n\times n}$, $Q = Q^{\mathrm{T}} > 0$, the following inequality holds:

$$\begin{aligned} -\int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)Q\dot{x}(s)\mathrm{d}s \le & \tau(t)\xi^{\mathrm{T}}(t)\mathscr{X}Q^{-1}\xi(t) \\ & + 2\xi^{\mathrm{T}}(t)[x(t) - x(t-\tau(t))], \end{aligned}$$

where $\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-h_1), x^{\mathrm{T}}(t-h_2), \dot{x}^{\mathrm{T}}(t), z^{\mathrm{T}}(t), G^{\mathrm{T}}(x(t)), G^{\mathrm{T}}(x(t-\tau(t)))]$ and $\mathscr{X}$ is a free-weighting matrix with appropriate dimensions.

In what follows, for simplicity, while $\eta_t = i$, the matrices $D(\eta_t), A(\eta_t), B(\eta_t), C(\eta_t)$ are represented by D_i, A_i, B_i, C_i.

3. Main results

In this section, some sufficient conditions of global stability for systems (3) or (4) are obtained.

Theorem 1 For system (4) satisfying assumptions (i)–(iii), the equilibrium solution of neural network (4) is globally asymptotically stable in the mean square if there exist matrices $Q_i > 0$, $P_k > 0$ ($k = 1,2,3$), $S_1 > 0$, $S_2 > 0$, $P > 0$, and any matrices X_1, X_2, Y_1, Y_2, Z_1, Z_2 as well as positive scalars b_1, b_2 satisfying the following linear matrix inequalities

$$\Omega_i = \begin{bmatrix} \Sigma_{1i} + \mathscr{R} + \mathscr{R}^{\mathrm{T}} & h_1\mathscr{X} & h_2\mathscr{Y} & h_{12}\mathscr{Z} \\ \star & -S_1 & 0 & 0 \\ \star & \star & -S_2 & 0 \\ \star & \star & \star & -S_2 \end{bmatrix} < 0, \tag{5}$$

where

$$\begin{aligned} & \Sigma_{1i} = (\Sigma_{mni})_{8\times 8\times i}, \\ & \Sigma_{11i} = \sum_{j=1}^{s}\pi_{ij}Q_i + P_1 + P_2 + P_3 + b_1L^{\mathrm{T}}L - W_iD_i \\ & \qquad - D_iW_i^{\mathrm{T}}, \\ & \Sigma_{1ki} = 0\ (k = 2,\ldots,4),\ \Sigma_{15i} = -W_i - D_i^{\mathrm{T}}W_i + Q_i, \end{aligned}$$

$\Sigma_{16i} = W_iA_i, \quad \Sigma_{17i} = W_iB_i,$
$\Sigma_{18i} = W_iC_i, \quad \Sigma_{22i} = -(1-h)P_3 + b_2L^{\mathrm{T}}L,$
$\Sigma_{2ki} = 0 \ \ (k = 3, \ldots, 8),$
$\Sigma_{33i} = -P_1, \quad \Sigma_{3ki} = 0 \ \ (k = 4, \ldots, 8)$
$\Sigma_{44i} = -P_2, \quad \Sigma_{4ki} = 0 \ \ (k = 5, 6, 7, 8),$
$\Sigma_{55} = h_1^2S_1 + h_{12}^2S_2 - W_i - W_i^{\mathrm{T}}, \quad \Sigma_{56i} = W_iA_i,$
$\Sigma_{57i} = W_iB_i, \quad \Sigma_{58i} = W_iC_i,$
$\Sigma_{66i} = \tau^2P - b_1I, \quad \Sigma_{6ki}(k = 7, 8) = 0,$
$\Sigma_{77i} = -b_2I, \quad \Sigma_{78i} = 0, \quad \Sigma_{88i} = -P,$
$\mathscr{X} = h_1[X_1^{\mathrm{T}},\ 0,\ X_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},$
$\mathscr{Y} = h_2[0, Y_1^{\mathrm{T}},\ 0,\ Y_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},$
$\mathscr{Z} = h_{12}[0,\ Z_1^{\mathrm{T}},\ Z_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},$
$\mathscr{R} = [\mathscr{X},\ -\mathscr{X} + \mathscr{Z},\ -\mathscr{Z} + \mathscr{Y},\ -\mathscr{Y},\ 0,\ 0,\ 0,\ 0],$
$h_{12} = h_2 - h_1.$

Proof

Construct the following Lyapunov–Krasovskii functional:

$$V(u(t), t, i) = \sum_{j=1}^{3} V_j(u(t), t, i), \tag{6}$$

$$V_1(u(t), t, i) = u^{\mathrm{T}}(t)Q_iu(t) + \int_{t-h(t)}^{t} u^{\mathrm{T}}(s)P_3u(s)\mathrm{d}s + \sum_{i=1}^{2}\int_{t-h_i}^{t} u^{\mathrm{T}}(s)P_iu(s)\mathrm{d}s,$$

$$V_2(u(t), t, i) = \int_{-h_1}^{0}\int_{t+\theta}^{t} h_2\dot{u}^{\mathrm{T}}(s)S_1\dot{u}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} h_{12}\dot{u}^{\mathrm{T}}(s)S_2\dot{u}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_3(u(t), t, i) = \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} g^{\mathrm{T}}(u(s))Pg(u(s))\mathrm{d}s\mathrm{d}\theta.$$

Then it can be obtained by Ito's formula that

$$\begin{aligned}\mathscr{L}(V_1(u(t), t, i)) =\ & 2u^{\mathrm{T}}(t)Q_i\dot{u}(t) + u^{\mathrm{T}}(t)\Big(\sum_{j=1}^{s}\pi_{ij}Q_i\Big)u(t) + \sum_{k=1}^{3}u^{\mathrm{T}}(t)P_ku(t) \\ & -(1-h)u^{\mathrm{T}}(t-h(t)))P_3u(t-h(t)) - \sum_{k=1}^{2}u(t-h_k)^{\mathrm{T}}P_ku(t-h_k), \end{aligned}\tag{7}$$

$$\begin{aligned}\mathscr{L}(V_2(u(t), t, i)) =\ & \dot{u}^{\mathrm{T}}(t)(h_1^2S_1 + h_{12}^2S_2)\dot{u}(t) - \int_{t-h_1}^{t} h_1\dot{u}^{\mathrm{T}}(s)S_1\dot{u}(s)\mathrm{d}s \\ & - \int_{t-h_2}^{t-h_1} h_{12}\dot{u}^{\mathrm{T}}(s)S_2\dot{u}(s)\mathrm{d}s, \end{aligned}\tag{8}$$

$$\begin{aligned}\mathscr{L}(V_3(u(t), t, i)) =\ & \tau^2f^{\mathrm{T}}(u(t))Pf(u(t)) - \tau\int_{t-\tau}^{t} f^{\mathrm{T}}(u(s))Pf(u(s))\mathrm{d}s. \end{aligned}\tag{9}$$

By Lemma 4, we have

$$\begin{aligned} & -\int_{t-h_1}^{t} h_1\dot{u}^{\mathrm{T}}(s)S_1\dot{u}(s)\mathrm{d}s \le h_1^2\xi^{\mathrm{T}}(t)\mathscr{X}S_1^{-1}\mathscr{X}^{\mathrm{T}}\xi(t) + 2h_1\xi^{\mathrm{T}}(t)\mathscr{X}(u(t) - u(t-h_1)), \end{aligned}\tag{10}$$

$$\begin{aligned} & -\int_{t-h_2}^{t-h_1} h_{12}\dot{u}^{\mathrm{T}}(s)S_2\dot{u}(s)\mathrm{d}s \\ =\ & -\int_{t-h_2}^{t-h(t)} (h_2 - h(t))\dot{u}^{\mathrm{T}}(\theta)S_2\dot{u}(\theta)\mathrm{d}\theta - \int_{t-h(t)}^{t-h_1} (h(t) - h_1)\dot{u}^{\mathrm{T}}(\theta)S_2\dot{u}(\theta)\mathrm{d}\theta \\ \le\ & (h_2 - h(t))^2\mathscr{Y}\xi(t)^{\mathrm{T}}\mathscr{S}_2^{-1}\mathscr{Y}^{\mathrm{T}}\xi(t) + 2(h_2 - h(t))\mathscr{Y}(u(t-h(t)) - u(t-h_2)) \\ & + (h(t) - h_1)^2\mathscr{Z}\xi^{\mathrm{T}}(t)S_2^{-1}\mathscr{Z}^{\mathrm{T}}\xi(t) + 2(h(t) - h_1)\mathscr{Z}(u(t-h_1) - u(t-h(t)) \end{aligned}\tag{11}$$

with $\xi^{\mathrm{T}}(t) = [u^{\mathrm{T}}(t), u^{\mathrm{T}}(t-h(t)), u^{\mathrm{T}}(t-h_1), u^{\mathrm{T}}(t-h_2), \dot{u}^{\mathrm{T}}(t), g^{\mathrm{T}}(u(t)), g^{\mathrm{T}}(u(t-h(t))), (\int_{t-\tau(t)}^{t} g(u(s)))^{\mathrm{T}}\mathrm{d}s]$.

From assumption (i), it is obvious that

$$\begin{aligned} & -b_1f^{\mathrm{T}}(x(t))f(x(t)) + b_1x^{\mathrm{T}}(t)L^{\mathrm{T}}Lx(t) \ge 0, \\ & -b_2f^{\mathrm{T}}(x(t-h(t)))f(x(t-h(t))) + b_2x^{\mathrm{T}}(t-h(t))L^{\mathrm{T}}Lx(t-h(t)) \ge 0. \end{aligned}\tag{12}$$

The first equation in Eq. (4) ensures

$$2\Big(u^{\mathrm{T}}(t) + \dot{u}^{\mathrm{T}}\Big)W_i\Big(-\dot{u}(t) - D_iu(t) + A_ig(u(t)) + B_ig(u(t-h(t))) + C_i\int_{t-\tau(t)}^{t} g(u(s))\mathrm{d}s\Big) = 0. \tag{13}$$

Then, combining expressions (7)–(13), it follows that

$$\mathscr{L}V(u(t), t, i) \le \xi^{\mathrm{T}}(t)\Omega_1\xi(t) \tag{14}$$

with

$$\begin{aligned}\Omega_1 =\ & \Sigma_{1i} + \mathscr{R} + \mathscr{R}^{\mathrm{T}} + h_1^2\xi^{\mathrm{T}}(t)\mathscr{X}S_1^{-1}\mathscr{X}^{\mathrm{T}}\xi(t) + (h_2 - \tau(t))^2\mathscr{Y}\xi(t)^{\mathrm{T}}S_2^{-1}\mathscr{Y}^{\mathrm{T}}\xi(t) \\ & + (\tau(t) - h_1)^2\mathscr{Z}\xi^{\mathrm{T}}(t)S_2^{-1}\mathscr{Z}^{\mathrm{T}}\xi(t) \\ \le\ & \Sigma_1 + \mathscr{R} + \mathscr{R}^{\mathrm{T}} + h_1^2\xi^{\mathrm{T}}(t)\mathscr{X}S_1^{-1}\mathscr{X}^{\mathrm{T}}\xi(t) + h_2^2\mathscr{Y}\xi(t)^{\mathrm{T}}S_2^{-1}\mathscr{Y}^{\mathrm{T}}\xi(t) \\ & + h_{12}^2\mathscr{Z}\xi^{\mathrm{T}}(t)S_2^{-1}\mathscr{Z}^{\mathrm{T}}\xi(t). \end{aligned}\tag{15}$$

By using the Schur complement (Lemma 1), we get $\Omega_1 < 0$. Taking the mathematical expectation, we have

$$\frac{\mathrm{d}\mathbb{E}V(u(t), t, i)}{\mathrm{d}t} \le \mathbb{E}[\xi^{\mathrm{T}}(t)\Omega_1\xi(t)] \le -\lambda\mathbb{E}\|u(t)\|^2,$$

where λ is the eigenvalue of Ω_1; this implies that the equilibrium point x^* of system (4) is globally asymptotically stable. This completes the proof.

Remark 1 Theorem 1 solves the stability problem for time-varying delay over a range and presents a delay-range-dependent criterion. Further, for $h_1 = 0$ it implies that $0 \leq h(t) \leq h_2$, which was investigated in Refs. [7] and [16].

Now we consider the robust stability of the uncertain system

$$\begin{aligned} \mathrm{d}u(t) = \Big[& -(D_i + \Delta D_i(t))(u(t) \\ & + (A_i + \Delta A_i(t))g(u(t)) \\ & + (B_i + \Delta B_i(t))g(u(t-h(t))) \\ & + (C_i + \Delta C_i)\int_{t-\tau(t)}^{t} g(u(s))\mathrm{d}s\Big]\mathrm{d}t, \end{aligned} \tag{16}$$

where A_i, B_i, C_i, D_i, for any $i \in S$, are known constant matrices of appropriate dimensions, ΔA_i, ΔB_i, ΔC_i and ΔD_i for $i \in S$, are unknown matrices that represent time-varying parameter uncertainties and are assumed to be of the form:

$$\begin{aligned} & [\Delta D_i, \Delta A_i, \Delta B_i, \Delta C_i] \\ = \; & H_i F_i(t)[E_{1i}, E_{2i}, E_{3i}, E_{4i}], \end{aligned} \tag{17}$$

where H_i, E_{1i}, E_{2i}, E_{3i}, E_{4i} are known real constant matrices and $F_i(t)$, for any $i \in S$ are unknown time-varying matrix functions satisfying

$$F_i^{\mathrm{T}}(t)F_i(t) \leq I, i \in S.$$

Definition 1 The equilibrium solution of the neural networks (16) is said to be globally robustly asymptotically stable for all admissible uncertainties satisfying Eq. (17) in the mean square if the following condition holds:

$$\mathbb{E}\|x(t)\|^2 = 0.$$

The system (16) can be written as

$$\begin{aligned} \mathrm{d}u(t) = \Big[& -D_i(t)(u(t) + A_i(t)g(u(t)) \\ & + B_i(t)g(u(t-h(t)) \\ & + C_i(t)\int_{t-\tau(t)}^{t} g(u(s))\mathrm{d}s\Big]\mathrm{d}t, \end{aligned} \tag{18}$$

Theorem 2 For system (18) satisfying assumptions (i)–(iii), the equilibrium solution of neural network (18) is globally asymptotically stable in the mean square if there exist matrices $Q_i > 0$, $P_k > 0$ $(k = 1, 2, 3)$, $S_1 > 0$, $S_2 > 0$, $P > 0$, and any matrices X_1, X_2, Y_1, Y_2, Z_1, Z_2 as well as positive scalars ε_i, b_1, b_2 that satisfy the following linear matrix inequalities

$$\Omega_1 = \begin{bmatrix} \Gamma_{1i} + \mathscr{R}_1 + \mathscr{R}_1^{\mathrm{T}} & h_1\mathscr{X} & h_2\mathscr{Y} & h_{12}\mathscr{Z} \\ \star & -S_1 & 0 & 0 \\ \star & \star & -S_2 & 0 \\ \star & \star & \star & -S_2 \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} & \Gamma_{1i} = (\Gamma_{mni})_{9\times 9\times i}, \quad \Gamma_{11i} = \Sigma_{11i} + \varepsilon_i E_{1i}^{\mathrm{T}} E_{1i}, \\ & \Gamma_{1ki} = 0 \;\; (k = 2, \ldots, 4), \quad \Gamma_{15i} = \Sigma_{15i}, \\ & \Gamma_{16i} = \Sigma_{16i} + \varepsilon_i E_{1i}^{\mathrm{T}} E_{2i}, \\ & \Gamma_{17i} = \Sigma_{17i} + \varepsilon_i E_{1i}^{\mathrm{T}} E_{3i}, \\ & \Gamma_{18i} = \Sigma_{18i} + \varepsilon_i E_{1i}^{\mathrm{T}} E_{4i}, \quad \Gamma_{19i} = W_i H_i, \\ & \Gamma_{22i} = \Sigma_{22i}, \quad \Gamma_{2ki} = 0 \;\; (k = 3, \ldots, 9), \\ & \Gamma_{33i} = -P_1, \quad \Gamma_{3ki} = 0 \;\; (k = 4, \ldots, 9), \\ & \Gamma_{44i} = -P_2, \quad \Gamma_{4ki} = 0 \;\; (k = 5, 6, 7, 9), \\ & \Gamma_{55i} = \Sigma_{55i}, \quad \Gamma_{56i} = \Sigma_{56i}, \\ & \Gamma_{57i}\Sigma_{57i}, \quad \Gamma_{58i} = \Sigma_{58i}, \\ & \Gamma_{59i} = W_i H, \quad \Gamma_{66i} = \Sigma_{66i} + \varepsilon E_{2i}^{\mathrm{T}} E_{2i}, \\ & \Gamma_{67i} = \varepsilon_i E_{2i}^{\mathrm{T}} E_{3i}, \quad \Gamma_{68i} = \varepsilon_i E_{2i}^{\mathrm{T}} E_{4i}, \\ & \Gamma_{69i} = 0, \quad \Gamma_{77i} = -b_2 I + \varepsilon_i E_{3i}^{\mathrm{T}} E_{3i}, \\ & \Gamma_{78i} = \varepsilon_i E_{3i}^{\mathrm{T}} E_{4i}, \quad \Gamma_{79i} = 0, \\ & \Gamma_{88i} = -P + \varepsilon_i E_{4i}^{\mathrm{T}} E_{4i}, \quad \Gamma_{89i} = 0, \quad \Gamma_{99i} = \varepsilon_i I, \\ & \mathscr{X} = h_1[X_1^{\mathrm{T}},\ 0,\ X_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}}, \\ & \mathscr{Y} = h_2[0,\ Y_1^{\mathrm{T}},\ 0,\ Y_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}}, \\ & \mathscr{Z} = h_{12}[0,\ Z_1^{\mathrm{T}},\ Z_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}}, \\ & \mathscr{R}_1 = [\mathscr{X},\ -\mathscr{X} + \mathscr{Z}, -\mathscr{Z} + \mathscr{Y}, -\mathscr{Y}, 0, 0, 0, 0, 0], \\ & h_{12} = h_2 - h_1. \end{aligned}$$

Proof Replacing D_i, A_i, B_i, C_i in Eq. (4) with $D_i + \Delta D_i(t)$, $A_i + \Delta A_i(t)$, $B_i + \Delta B_i(t)$ and $C_i + \Delta C_i(t)$, we get

$$\begin{aligned} \Gamma_{1i} = \; & \Sigma_{1i} + [E_{1i}, 0, 0, 0, 0, E_{2i}, E_{3i}, E_{4i}]^{\mathrm{T}} F_i^{\mathrm{T}}(t) \\ & [H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0, 0, 0, H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0] \\ & + [H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0, 0, 0, H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0]^{\mathrm{T}} F_i(t) \\ & [E_{1i}, 0, 0, 0, 0, E_{2i}, E_{3i}, E_{4i}]. \end{aligned} \tag{20}$$

From Lemma 1, we know that Γ_{1i} is equivalent to

$$\begin{aligned} \Gamma_{2i} = \; & \Sigma_{1i} + \varepsilon_i^{-1}[H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0, 0, 0, H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0]^{\mathrm{T}} \\ & [H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0, 0, 0, H_i^{\mathrm{T}} W_i^{\mathrm{T}}, 0, 0] \\ & + \varepsilon_i [E_{1i}, 0, 0, 0, 0, E_{2i}, E_{3i}, E_{4i}]^{\mathrm{T}} \\ & [E_{1i}, 0, 0, 0, 0, E_{2i}, E_{3i}, E_{4i}]. \end{aligned} \tag{21}$$

Using the Schur complement, we have that Γ_{2i} is equivalent to Ω_{1i}. This completes the proof.

When $h_1 = 0$, we can get the following result.

Corollary 1 For system (18) satisfying assumptions (i)–(iii), the equilibrium solution of neural network (18) is globally asymptotically stable in the mean square if there exist matrices $Q_i > 0$, $P_k > 0$ $(k = 2, 3)$, $S_1 > 0$, $S_2 > 0$, $P > 0$, and any matrices Y_1, Y_2, Z_1, as well as positive scalars ε_i, b_1, b_2 that satisfy the following linear matrix inequalities

$$\Omega = \begin{bmatrix} \Sigma + \mathscr{R} + \mathscr{R}^{\mathrm{T}} & h_2\mathscr{Y} & h_2\mathscr{Z} \\ \star & -S_2 & 0 \\ \star & \star & -S_2 \end{bmatrix} < 0, \quad (22)$$

where

$$\begin{aligned}
&\Sigma_i = (\Sigma_{mni})_{8\times 8\times i},\\
&\Sigma_{11i} = \sum_{j=1}^{s}\pi_{ij}Q_i + P_2 + P_3 + b_1L^{\mathrm{T}}L\\
&\qquad - W_iD_i - D_iW_i^{\mathrm{T}} + \varepsilon_iE_{1i}^{\mathrm{T}}E_{1i},\\
&\Sigma_{1ki}=0\ (k=2,\ldots,4),\ \Sigma_{15i} = -W_i - D_i^{\mathrm{T}}W_i + Q_i,\\
&\Sigma_{16i} = W_iA_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{2i},\quad \Sigma_{17i} = W_iB_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{3i},\\
&\Sigma_{18i} = W_iC_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{4i},\\
&\Sigma_{19i} = W_iH_i,\quad \Sigma_{22i} = -(1-h)P_3 + b_2L^{\mathrm{T}}L,\\
&\Sigma_{2ki} = 0\ \ (k=3,\ldots,8),\\
&\Sigma_{33i} = -P_2,\quad \Sigma_{3ki} = 0\ \ (k=5,6,7,8),\\
&\Sigma_{44} = h_2^2S_2 - W_i - W_i^{\mathrm{T}},\quad \Sigma_{45i} = W_iA_i,\\
&\Sigma_{46i} = W_iB_i,\quad \Sigma_{47i} = W_iC_i,\\
&\Sigma_{48i} = W_iH,\quad \Sigma_{55i} = \tau^2P - b_1I + \varepsilon E_{2i}^{\mathrm{T}}E_{2i},\\
&\Sigma_{56i} = \varepsilon_iE_{2i}^{\mathrm{T}}E_{3i},\quad \Sigma_{57i} = \varepsilon_iE_{2i}^{\mathrm{T}}E_{4i},\\
&\Sigma_{58i} = 0,\quad \Sigma_{66i} = -b_2I + \varepsilon_iE_{3i}^{\mathrm{T}}E_{3i},\\
&\Sigma_{67i} = \varepsilon_iE_{3i}^{\mathrm{T}}E_{4i},\quad \Sigma_{68i} = 0,\\
&\Sigma_{77i} = -P + \varepsilon_iE_{4i}^{\mathrm{T}}E_{4i},\quad \Sigma_{78i} = 0,\quad \Sigma_{88i} = -\varepsilon_iI,\\
&\mathscr{Y} = h_2[0,\ Y_1^{\mathrm{T}},\ Y_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\mathscr{Z} = h_2[0,\ Z_1^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\mathscr{R} = [0,\ \mathscr{Z},\ -\mathscr{Y},\ 0,\ 0,\ 0,\ 0,\ 0].
\end{aligned}$$

When the information of the time derivative of delay is unknown, by setting $P_3 = 0$ and $P = 0$, we have the following results from Theorem 2.

Corollary 2 For system (18) satisfying assumptions (i)–(iii) the equilibrium solution of neural network (18) is exponentially stable in the mean square if there exist matrices $Q_i > 0$, $P_k > 0$ $(k = 1, 2)$, $S_1 > 0$, $S_2 > 0$ and any matrices X_1, X_2, Y_1, Y_2, Z_1, Z_2 as well as positive scalars ε_i, b_1, b_2 that satisfy the following linear matrix inequalities

$$\Omega = \begin{bmatrix} \Sigma_{1i} + \mathscr{R} + \mathscr{R}^{\mathrm{T}} & \tau_1\mathscr{X} & \tau_2\mathscr{Y} & \tau_{12}\mathscr{Z} \\ \star & -S_1 & 0 & 0 \\ \star & \star & -S_2 & 0 \\ \star & \star & \star & -S_2 \end{bmatrix} < 0, \quad (23)$$

where

$$\begin{aligned}
&\Sigma_{1i} = (\Sigma_{mni})_{9\times 9\times i},\\
&\Sigma_{11i} = \sum_{j=1}^{s}\pi_{ij}Q_i + P_1 + P_2 + b_1L^{\mathrm{T}}L - W_iD_i\\
&\qquad - D_iW_i^{\mathrm{T}} + \varepsilon_iE_{1i}^{\mathrm{T}}E_{1i},\\
&\Sigma_{1ki}=0\ (k=2,\ldots,4),\ \Sigma_{15i} = -W_i - D_i^{\mathrm{T}}W_i + Q_i,\\
&\Sigma_{16i} = W_iA_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{2i},\quad \Sigma_{17i} = W_iB_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{3i},\\
&\Sigma_{18i} = W_iC_i + \varepsilon_iE_{1i}^{\mathrm{T}}E_{4i},\quad \Sigma_{19i} = W_iH_i,\\
&\Sigma_{22i} = b_2L^{\mathrm{T}}L,\quad \Sigma_{2ki} = 0\ \ (k=3,\ldots,9),\\
&\Sigma_{33i} = -P_1,\quad \Sigma_{3ki} = 0\ \ (k=4,\ldots,9),\\
&\Sigma_{44i} = -P_2,\quad \Sigma_{4ki} = 0\ \ (k=5,6,7,9),\\
&\Sigma_{55} = h_1^2S_1 + h_{12}^2S_2 - W_i - W_i^{\mathrm{T}},\quad \Sigma_{56i} = W_iA_i,\\
&\Sigma_{57i} = W_iB_i,\quad \Sigma_{58i} = W_iC_i,\\
&\Sigma_{59i} = W_iH,\quad \Sigma_{66i} = \tau^2P - b_1I + \varepsilon_iE_2^{\mathrm{T}}E_2,\\
&\Sigma_{67i} = \varepsilon_iE_2^{\mathrm{T}}E_3,\quad \Sigma_{68i} = \varepsilon_iE_2^{\mathrm{T}}E_4,\\
&\Sigma_{69i} = 0,\quad \Sigma_{77i} = -b_2I + \varepsilon_iE_3^{\mathrm{T}}E_3,\\
&\Sigma_{78i} = \varepsilon_iE_3^{\mathrm{T}}E_4,\quad \Sigma_{79i} = 0,\\
&\Sigma_{88i} = -P + \varepsilon_iE_4^{\mathrm{T}}E_4,\quad \Sigma_{89i} = 0,\quad \Sigma_{99i} = -\varepsilon_iI,\\
&\mathscr{X} = h_1[X_1^{\mathrm{T}},\ 0,\ X_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\mathscr{Y} = h_2[0, Y_1^{\mathrm{T}},\ 0,\ Y_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\mathscr{Z} = h_{12}[0,\ Z_1^{\mathrm{T}},\ Z_2^{\mathrm{T}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\mathscr{R} = [\mathscr{X}, -\mathscr{X} + \mathscr{Z}, -\mathscr{Z} + \mathscr{Y}, -\mathscr{Y}, 0, 0, 0, 0, 0],\\
&h_{12} = h_2 - h_1.
\end{aligned}$$

Remark 2 Theorem 2 can be applied to both discrete and distributed time-varying delays only if h is known. But when $h(t)$ is not differentiable or h is unknown, Theorem 2 of this paper and Theorem 1 of Ref. [16] fail to work, while Corollary 2 can be turned to for checking the stability of system (18).

4. Illustrative example

In this section, we present an example to illustrate the effectiveness of our results.

Example Consider the delayed uncertain recurrent neural networks with Markovian jumping param-

eters as follows:

$$\dot{u}(t) = \Big[- D_i(t)u(t) + A_i(t)g(u(t)) + B_i(t)g(u(t-h(t))) + C_i(t)\int_{t-\tau(t)}^{t} g(u(s))\mathrm{d}s \Big], \qquad (24)$$

where

$$D_1 = \begin{bmatrix} 2.2 & 0 \\ 0 & 2.5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.4 & 0.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.3 & 0.8 \\ 0.5 & 0.4 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.5 & -0.5 \\ 0.2 & 0.7 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad \aleph = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 2.3 & 0 \\ 0 & 2.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & 0.4 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.1 & 0.9 \\ -0.8 & 1.2 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.3 & -0.2 \\ -0.5 & 0.4 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$E_{1i} = E_{2i} = E_{3i} = E_{4i} \quad (i = 1, 2) = L = I$$

with $g(x) = \frac{1}{2}(|u+1| - |u-1|)$. Obviously, assumption (i) is satisfied with $L = \mathrm{diag}\{1, 1\}$. Applying Matlab Toolbox, by Theorem 2, for $h_2 = 0.5561$, $h_1 = 0$, $r = 0.4484$, $h = 0$, we can get the solution

$$P = \begin{bmatrix} 0.3902 & -76.4278 \\ 76.3476 & 0.4482 \end{bmatrix},$$
$$P_1 = \begin{bmatrix} 0.2767 & 107.2501 \\ -107.3998 & 0.1952 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.2759 & 198.6769 \\ -198.8337 & 0.1903 \end{bmatrix},$$
$$P_3 = \begin{bmatrix} 1.0486 & -91.3152 \\ 91.1290 & 0.9457 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.6803 & -0.1368 \\ -0.1100 & 0.5334 \end{bmatrix},$$
$$S_1 = \begin{bmatrix} 0.4139 & 0 \\ 0 & 0.4139 \end{bmatrix},$$
$$S_2 = \begin{bmatrix} 0.3272 & 258.7515 \\ -258.8581 & 0.2580 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 0.1645 & 182.7446 \\ -182.8878 & 0.0879 \end{bmatrix},$$
$$W = \begin{bmatrix} 0.2248 & -0.0394 \\ -0.0344 & 0.1608 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.2551 & -0.0364 \\ -0.0208 & 0.2129 \end{bmatrix},$$
$$b_1 = 0.3271, \quad b_2 = 0.3667.$$

Therefore, it follows from Theorem 2 that the Markovian jumping delayed neural network (24) is globally asymptotically stable in the mean square.

5. Conclusions

In this paper, delay-dependent stability of delayed recurrent neural networks with Markovian jumping parameters is considered. With a different Lyapunov functional proposed for the derivative of it, simplified delay-dependent criteria are derived. It is established that the criteria, though involved with fewer matrix variables, are less conservative than some existing ones. An example is given to verify the theoretical results.

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