Chaotic behaviours and control of chaos in the p–Ge photoconductor*

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The chaotic behaviours in the p–Ge photoconductor system are studied by changing the photo-excitation coefficient and the routes and parameter conditions are given for chaos generation in this system. A scheme for controlling chaos in the p–Ge photoconductor is presented by adding an ac bias current. Numerical simulations show that this scheme can be effectively used to control chaotic states into stable period states for this system. Moreover, the different period states with different period numbers can be obtained by appropriately adjusting the amplitude, frequency, and initial phase of the additional ac current.

Keywords: chaotic behaviours, chaos control, p–Ge photoconductor

PACC: 0545, 7230, 7240

1. Introduction

With the rapid development in fabrication technology and extrinsic semiconductor materials, investigations on extrinsic photoconductors have attracted much attention due to their extensive applications, such as photo-detectors and sensors. Previously, Teitsworth *et al.*[1−4] studied nonlinear oscillations and chaos in extrinsic Ge photoconductors experimentally and numerically; they also[5,6] reported frequency locking, quasi-periodicity, and chaos in extrinsic Ge photoconductors. Shiau *et al.*[7−10] studied numerically chaotic behaviours in n–GaAs. Kyritsi *et al.*[11,12] reported chaotic photoconductivity and the period doubling route to chaos in excited photoconductors. Recently, Feng Yu-Ling *et al.*[13] reported anti-control of chaos in the p–Ge photoconductor. On the other hand, chaos control has become a main research subject for some researchers working in this field. Many experts have successfully completed chaos control in different nonlinear systems by using different methods.[14,15] Thus, the control of chaos in photoconductors is also important because the stable periodic states in detectors made from the p–Ge photoconductor are preferred in many applications. In this paper, we will study chaos behaviours in the p–Ge photoconductor by changing the photo-excitation coefficient, then a scheme for chaos control in this system is presented by using an added ac bias current, and the validity of this scheme is demonstrated numerically.

2. Chaotic behaviours in p–Ge photoconductor

The dynamical equation of the cooled p–Ge photoconductor used here is given as follows:[2]

$$\begin{aligned}
&\frac{\mathrm{d}p}{\mathrm{d}t} = \gamma(a - a_*) + pk(a - a_*) - pra_*,\\
&\varepsilon\frac{\mathrm{d}E}{\mathrm{d}t} = J_{\mathrm{ext}} - ep\upsilon_{\mathrm{d}},\\
&p = a_* - d,\\
&J_{\mathrm{ext}} = J_0 + \Delta J\sin(2\pi f_{\mathrm{dr}}t),
\end{aligned} \tag{1}$$

where p is the hole density, γ is the hole generation rate induced by photo-excitation, i.e., the photo-excitation coefficient; a is the total acceptor density; a_* is the time-dependent ionized acceptor density; k and r are the field-dependent impact-ionization rate and recombination rate, respectively; $\varepsilon(\mathrm{d}E/\mathrm{d}t)$ is the displacement current and ε is the dielectric constant; J_{ext} is the external current density and its dc and ac components are J_0 and $\Delta J\sin(2\pi f_{\mathrm{dr}}t)$, respectively; υ_{d} is the field-dependent drift velocity, and d is the compensating donor density. Here we assume that the space charge is neutrality. So k, r, and υ_{d} are given as follows:[3,16]

$$\begin{aligned}
k \;=\; & k_0\frac{[(1/2)m_*\upsilon_{\mathrm{d}}^2/B]^{1/2}}{1 + (1/2)m_*\upsilon_{\mathrm{d}}^2/B}\\
& \times \frac{1}{1 + \exp\{[B - (1/2)m_*\upsilon_{\mathrm{d}}^2]/k_{\mathrm{B}}T_{\mathrm{h}}\}},
\end{aligned} \tag{2}$$

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$$r = r_0[(3/2)k_{\rm B}T/U]^{3/2}, \quad (3)$$

$$\upsilon_{\rm d} = \upsilon_{\rm s}(2/\pi)\tan^{-1}[(\pi/2)\mu E/\upsilon_{\rm s}], \quad (4)$$

where k_0 and r_0 are the impact ionization and recombination coefficients, respectively; B is the impurity ionization energy; $\upsilon_{\rm s}$, μ, and m_* are the saturated drift velocity, the mobility, and the hole effective mass, respectively; U is the average kinetic energy per hole; m_*, $T_{\rm h}$, and U are expressed as follows:[3,16]

$$m_* = 0.35m_0, \quad (5)$$

$$(3/2)k_{\rm B}T_{\rm h} = (3/2)k_{\rm B}T + \eta(1/2)m_*\upsilon_{\rm d}^2, \quad (6)$$

$$U = (3/2)k_{\rm B}T + (\eta+1)(1/2)m_*\upsilon_{\rm d}^2, \quad (7)$$

where m_0 is the electron mass, the coefficient η models the balance between spreading and displacement in the drifted Maxwellian; here we take $\eta \approx 1.0$.[3,16]

To study chaotic behaviours in the p–Ge photoconductor system, we solve Eqs. (1)–(7) numerically. In this work, the differential equations are solved by using a variable time-step fourth-order Runge–Kutta algorithm with total steps $N_{\rm step} = 1.0 \times 10^5$, and the bifurcation diagrams are plotted by recording the local maxima in the time series.[17] The parameter values used here are chosen according to the requirement of Ref. [1] and listed as follows: $a = 10^{11}/{\rm cm}^3$, $d = 3.0 \times 10^{10}/{\rm cm}^3$, $r_0 = 1.0 \times 10^{-5}$ cm^3/s, $\Delta J = 4.22J_0$, $k_0 = 2.0 \times 10^{-6}$ cm^3/s, $\upsilon_{\rm s} = 1.0 \times 10^7$ cm/s, $\mu = 1.0 \times 10^6$ cm^2/Vs, $J_0 = 1.3 \times 10^{-7}$ A/cm^2, $B = 10$ meV, $f_{\rm dr} = 0.32\ f_0$, f_0 is the character oscillation frequency of this p–Ge system with these parameter values and at $\Delta J = 0$, here $f_0 \approx 10000$ Hz. Then, the bifurcation diagram is presented in Fig. 1, where one can find that this p–Ge photoconductor system can enter into chaotic states through the intermittency route and period-doubling bifurcations by increasing and decreasing the γ value, respectively. One can also see that for $1.75 \times 10^{-4} < \gamma < 4.4 \times 10^{-4}$ this photoconductor system is in periodic states; for $5.0 \times 10^{-5} < \gamma < 1.75 \times 10^{-4}$ this system is in chaotic states, which are occasionally interrupted by periodic windows. Thus, the state of this photoconductor system can be changed by adjusting the photo-excitation coefficient γ. In order to further show chaotic and periodic attractors, according to Fig. 1, we choose $\gamma = 4.2 \times 10^{-4}$, $\gamma = 2.2 \times 10^{-4}$, and $\gamma = 1.0 \times 10^{-4}$. Then, the corresponding attractors, power spectra and time series of the hole density p are presented in Fig. 2, where figures (a1), (a2), (a3), and (b1), (b2), (b3) show that this system is in a 1-period (1P) state and a 4-period (4P) state, respectively; while figures (c1), (c2), and (c3) show that this system is in a chaotic state.

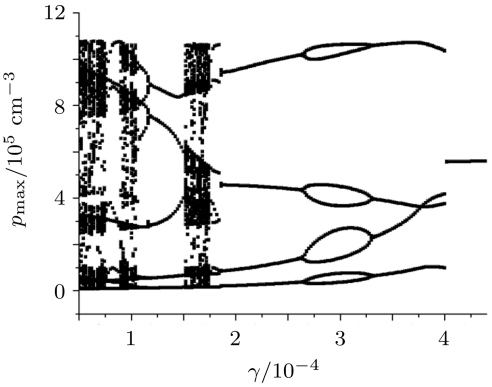
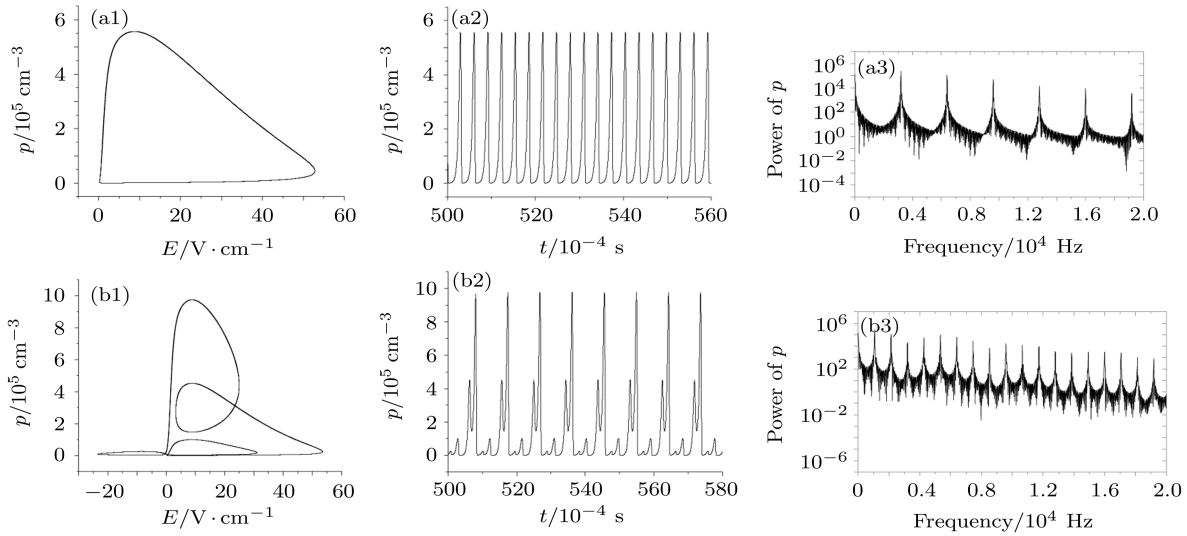


Fig. 1. Bifurcation diagram of the p–Ge system versus γ.



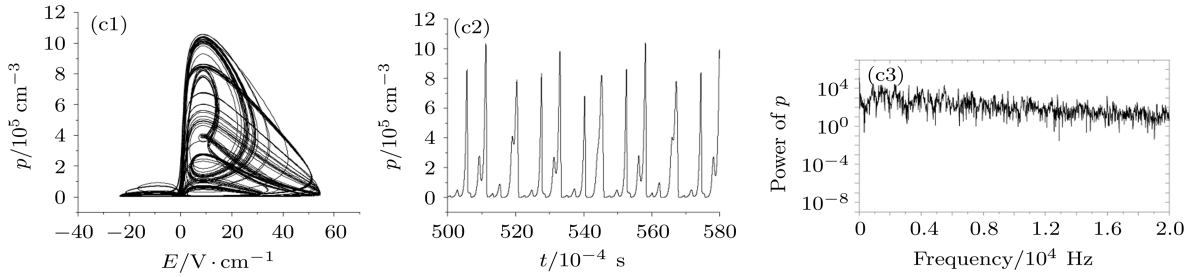


Fig. 2. The attractors on the plane E–p, time series and power spectra of p, at different γ values; figures (a1), (a2) and (a3) are for the 1P state, $\gamma = 4.2 \times 10^{-4}$; (b1), (b2) and (b3) are for the 4P state, $\gamma = 2.2 \times 10^{-4}$; (c1), (c2) and (c3) are for the chaos state, $\gamma = 1.0 \times 10^{-4}$, respectively.

3. Scheme and numerical simulations

3.1. Scheme for chaos control

Here we propose a scheme for controlling chaos in the p–Ge photoconductor based on the periodic signal perturbation theory[18,19] by adding an ac current to the original bias current. In this case equation (1) can be expressed as follows:

$$\begin{aligned} \frac{\mathrm{d}p}{\mathrm{d}t} &= \gamma(a-a_*) + pk(a-a_*) - pra_*, \\ \varepsilon\frac{\mathrm{d}E}{\mathrm{d}t} &= J_0 + \Delta J\sin(2\pi f_{\mathrm{dr}}t) \\ &\quad + mJ_0\sin(2\pi n f_{\mathrm{dr}}t + \alpha) - epv_{\mathrm{d}}, \\ p &= a_* - d, \end{aligned} \tag{8}$$

where mJ_0, nf_{dr}, and α are the amplitude, frequency, and initial phase of the additional ac current, respectively; m and n are the amplitude coefficient and frequency coefficient, respectively. Here m, n, and α can be taken as the control parameters, so as to achieve chaos control.

3.2. Numerical simulations and analyses

To illustrate the process of chaos control in this scheme, firstly the p–Ge photoconductor system is kept in the chaotic state, as shown in Fig. 2(c1). Secondly, numerical simulations are carried out by solving Eqs. (2)–(8) in two cases, that is: in case I the amplitude coefficient m is used as the control parameter; and in case II the initial phase α is used as the control parameter.

In case I, we considered two different conditions: one is for the initial phase $\alpha = 0$; the other is for $\alpha \neq 0$.

We first solve Eqs. (2)–(8) by changing the amplitude coefficient m at $\alpha = 0$, $n = 0.5$, and other parametric values are the same as those in Fig. 2(c1). Then the bifurcation diagram is presented in Fig. 3, where one can see that for $0.19 < m < 0.96$ and $1.28 < m \leq 2.0$ this photoconductor system is in periodic states. In order to further clarify that this system can be in different periodic states with different m values, according to Fig. 3 we choose $m = 0.7$ and $m = 1.6$. The corresponding attractors and time series of p are presented in Fig. 4, where subfigures (a1), (a2), and (b1), (b2) show that this system is in the 10-period (10P) and 4-period (4P) states, respectively.

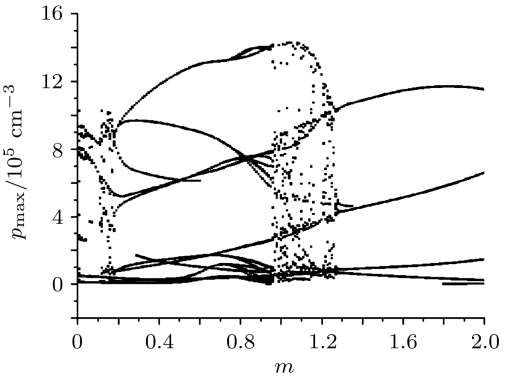


Fig. 3. Bifurcation diagram of the p–Ge system versus m for $\alpha = 0$, $n = 0.5$; the other parameter values are the same as those in Fig. 2(c1).

Now we solve Eqs. (2)–(8) by changing the amplitude coefficient m at $\alpha = 2.0$, $n = 0.5$, and the other parameter values are the same as those in Fig. 2(c1). Then, the bifurcation diagram is presented in Fig. 5, where one can see that for $0.04 < m < 0.22$, $0.27 < m < 0.42$, $0.67 < m < 0.76$, and $1.17 < m < 1.42$, this p–Ge photoconductor system is in periodic states. Similarly, to further indicate that different period

states can exist in this controlled system, according to Fig. 5, we choose $m = 0.08$ and $m = 0.19$. The corresponding attractors and time series of p are presented in Fig. 6, where subfigures (a1), (a2), and (b1), (b2) show that this system is in the 9P and 4P states, respectively. Therefore, for both $\alpha = 0$ and $\alpha \neq 0$, the chaotic state as shown in Fig. 2(c1) can be controlled into different stable period states as shown in Figs. 4 and 6 by choosing proper m values.

In case II, we change the initial phase α within the range $0 < \alpha \leq 6.0$ and hold $n = 0.5$, $m = 0.05$; other parameter values remain the same as those in Fig. 2(c1). The numerical results of Eqs. (2)–(8) are shown in Fig. 7, where one can observe that for three regions, i.e., $0 < \alpha < 2.1$, $2.4 < \alpha \leq 5.2$, and $5.05 < \alpha \leq 6.0$, this p–Ge system is in periodic states. Similarly, according to Fig. 7, the 5P and 4P states appear for $\alpha = 2.8$ and $\alpha = 4.0$, respectively. The corresponding 5P attractor and time series of p are shown in Fig. 8, the 4P attractor and time series of p are similar to Figs. 6(b1) and 6(b2), respectively.

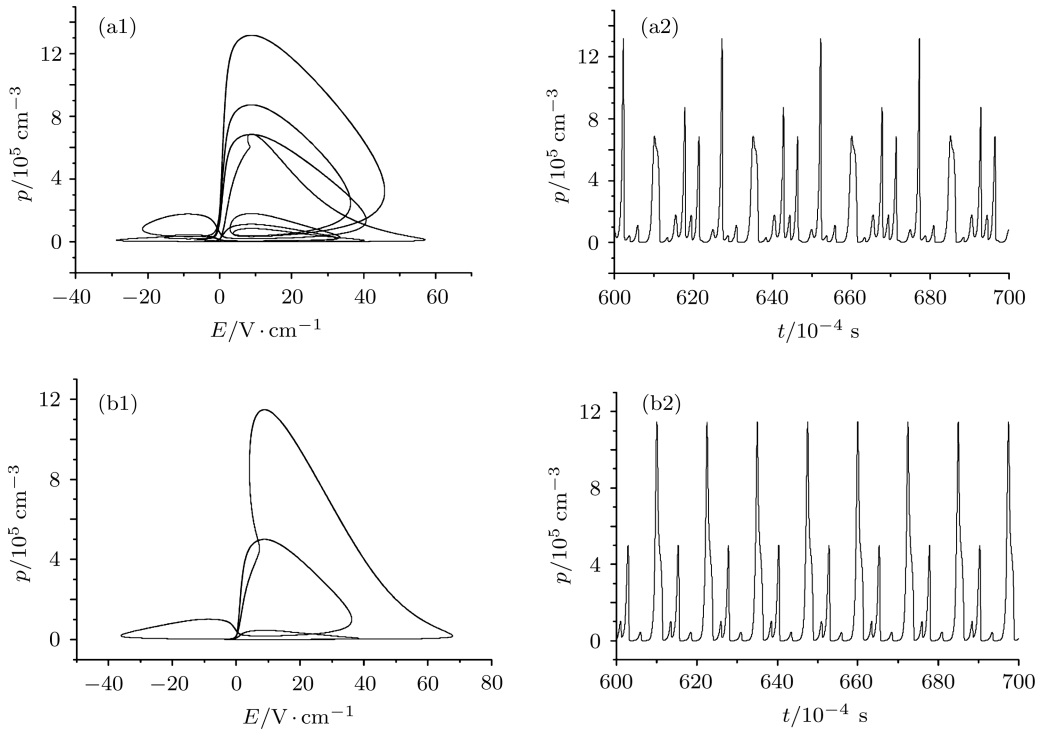


Fig. 4. The attractors on the plane E–p and time series of p for different m values. Subfigures (a1) and (a2) are for the 10P state, $m = 0.7$, while (b1) and (b2) are for the 4P state, $m = 1.6$.

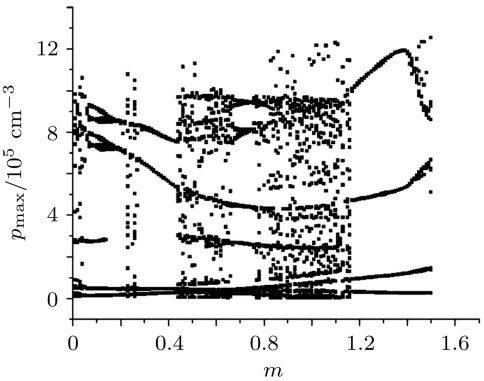


Fig. 5. Bifurcation diagram of the p–Ge photoconductor system versus m for $\alpha = 2.0$, $n = 0.5$; the other parameter values are the same as those in Fig. 2(c1).

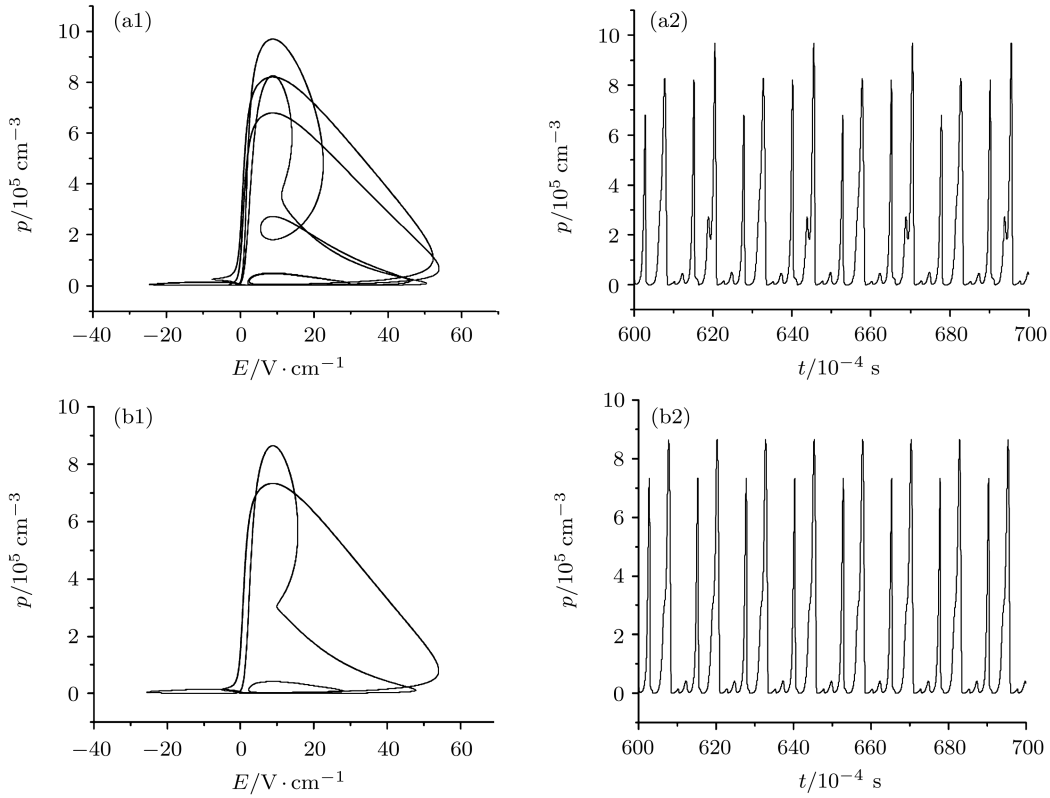


Fig. 6. The attractors on the plane E–p and time series of p at different m values. Subfigures (a1) and (a2) are for the 9P attractor and time series, $m = 0.08$, while (b1) and (b2) are for the 4P attractor and time series, $m = 0.19$.

In the chaos control discussed above, we have obtained stable periodic orbits, as shown in Figs. 4(a1), 4(b1), 6(a1), 6(b1), and 8(a), which are original unstable periodic orbits (OUPO) embedded in the chaotic attractor as shown in Fig. 2(c1). The physical mechanism is that the control method used by us forces chaotic states in the p–Ge photoconductor into periodic states via the frequency resonance[20] of the additional control signal and OUPO.

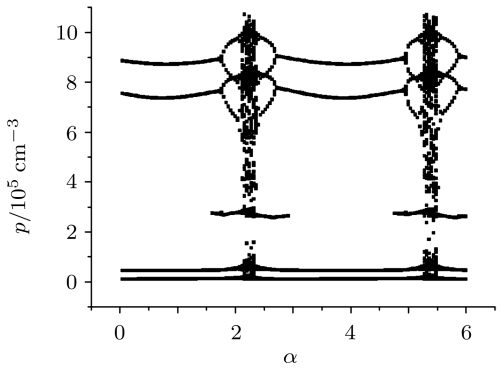


Fig. 7. Bifurcation diagram of the p–Ge system versus α for $m = 0.05$, $n = 0.5$; the other parameter values are the same as those in Fig. 2(c1).

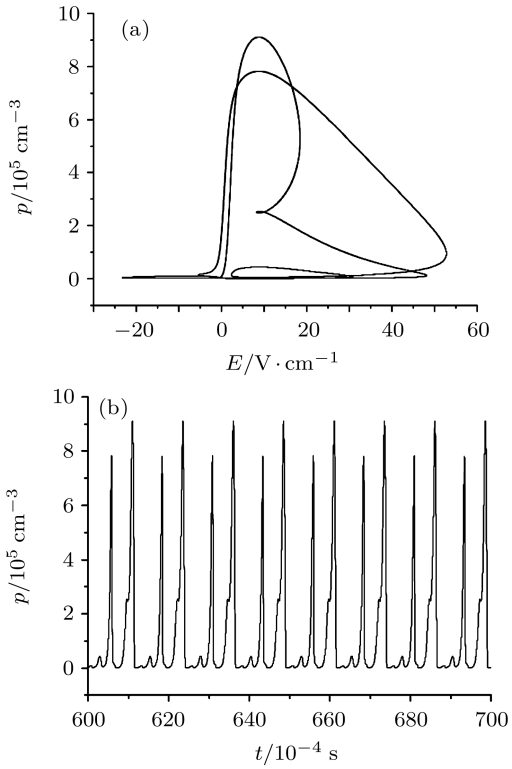


Fig. 8. The 5P attractor on the plane E–p (a) and time series of p (b), at $\alpha = 2.8$.

4. Conclusions

We have studied chaotic behaviours in the p–Ge photoconductor by changing the photo-excitation coefficient. The intermittency route and period-doubling bifurcation to chaos were illustrated, and the parameter conditions for chaos generation were given for this system. Then, we proposed a scheme for chaos control in the p–Ge system by using a periodic perturbation signal, i.e., an additional ac current. Numerical results show that our scheme can be effectively used to control chaotic states into stable period states for this system. Moreover, different periodic states, such as the 4P, 5P, 9P, and 10P states, can be obtained by properly adjusting the amplitude, frequency, and initial phase of the added ac current. For the p–Ge photoconductor, we have achieved chaos control. This may be useful for eliminating chaos in detectors made from the p–Ge photoconductor.

References

[1] Teitsworth S W, Westervelt R M and Haller E E 1983 *Phys. Rev. Lett.* **51** 825

[2] Teitsworth S W and Westervelt R M 1984 *Phys. Rev. Lett.* **53** 2587

[3] Westervelt R M and Teitsworth S W 1985 *J. Appl. Phys.* **57** 5457

[4] Westervelt R M and Teitsworth S W 1986 *Physica* D **23** 187

[5] Gwinn E G and Westervelt R M 1986 *Phys. Rev. Lett.* **57** 1060

[6] Teitsworth S W 1989 *Appl. Phys.* A **48** 127

[7] Shiau Y H and Cheng Y C 1995 *Phys. Rev.* B **52** 1698

[8] Shiau Y H and Cheng Y C 1997 *Phys. Rev.* B **56** 9247

[9] Tzeng S T and Cheng Y C 2003 *Phys. Rev.* B **68** 035211-1

[10] Tzeng S T and Tzeng Y 2005 *Phys. Rev.* B **72** 205201-1

[11] Kyritsi K G, Gallos L, Anagnostopoulos A N, Cenys A and Bleris G L 1999 *Nonlinear Phenomena in Complex Syst.* **2** 41

[12] Papazoglou E N, Kyritsi K G, Anagnostopoulos A N and Antoniou I 2002 *Chaos, Solitons and Fractals* **13** 989

[13] Feng Y L and Zhang X H 2009 *Chin. Phys.* B **18** 5212

[14] Gao J H, Xie L L and Peng J H 2009 *Acta Phys. Sin.* **58** 5218 (in Chinese)

[15] Li C L 2009 *Acta Phys. Sin.* **58** 8134 (in Chinese)

[16] Sun C W 2002 *Laser Radiation Effects* (Beijing: National Defence Industry Press) pp. 391–394 (in Chinese)

[17] Sprott J C 1997 *Phys. Lett.* A **228** 271

[18] Chacon R and Bejarano J D 1993 *Phys. Rev. Lett.* **71** 3103

[19] Braiman Y and Goldhirsch I 1991 *Phys. Rev. Lett.* **66** 2545

[20] Hu G, Xiao J H and Zheng Z G 2000 *Chaos Control* (Shanghai: Shanghai Scientific and Technological Education Press) pp. 63, 64 (in Chinese)