Synchronization-based approach for parameter identification in delayed chaotic network*

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(Received 6 November 2009; revised manuscript received 19 November 2009)

This paper introduces an adaptive procedure for the problem of synchronization and parameter identification for chaotic networks with time-varying delay by combining adaptive control and linear feedback. In particular, we consider that the equations $\dot{x}_i(t)$ (for $i = r+1,\ r+2, \ldots, n$) can be expressed by the former $\dot{x}_i(t)$ (for $i = 1, 2, \ldots, r$), which is not the same as the previous equation. This approach is also able to track changes in the operating parameters of chaotic networks rapidly and the speed of synchronization and parameter estimation can be adjusted. In addition, this method is quite robust against the effect of slight noise and the estimated value of a parameter fluctuates around the correct value.

Keywords: chaotic network, parameter identification, synchronization, time-varying delay

PACC: 0545

1. Introduction

Chaos appears in the time evolution of some kinds of nonlinear equations.[1] Chaotic systems intrinsically defy synchronization due to the evolution of a chaotic system which is sensitively dependent on its initial condition. However, over the past few years, the surprising phenomenon of synchronization between coupled chaotic systems has generated much interest since the pioneering work of Pecora and Carrol (PC).[2,3] The interest in understanding the synchronization characteristics of chaotic systems stems from its potential applications in a variety of areas, such as in secure communication, chemical and biological systems, information science, optics, and so on.[1,4,5] Recently, a wide variety of approaches have been proposed for the synchronization of chaotic systems, which include the PC method,[2] the Ott, Grebagi and Yorke (OGY) method,[3] the scalar driving method, coupling control, the manifold-based method,[6] fuzzy control, the impulsive control method,[7] active control, adaptive control,[8−13] the time-delay feedback approach,[14] etc. However, the aforementioned approaches and many other synchronization methods are valid for chaotic systems only when the systems' parameters are known. But in many practical situations, the values of some systems' parameters cannot be exactly known *a priori*, and the synchronization will be destroyed and broken with the effects of these uncertainties. Therefore, this paper is devoted to the synchronization-based estimation of connection parameters by combining the adaptive scheme and dynamical linear feedback control, provided the model is known. Recently, there has been increasing interest in the study of dynamical properties of chaotic networks due to their potential applications in different fields such as combinatorial optimization, pattern recognition, signal, and image processing.[15−17] Most existing works have focused on the stability analysis and periodic oscillation of this kind of chaotic networks. In particular, the introduction of delays into chaotic networks may make their dynamical behaviours much more complicated even with a strange attractor.[18] Recently, the chaos synchronization phenomenon for chaotic networks has drawn the attention of some researchers.[19,20] In the above references the synchronization schemes are proposed based on exactly knowing the concrete values of the connection weight matrices, and there is an assumption that the parameters in all coupled chaotic networks are identical.

Some synchronization-based strategies have been

*Project supported by the National Natural Science Foundation of China (Grant Nos. 70571030 and 90610031), the Social Science Foundation from Ministry of Education of China (Grant No. 08JA790057) and the Advanced Talents' Foundation and Student's Foundation of Jiangsu University (Grant Nos. 07JDG054 and 07A075).

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devised to estimate all unknown parameters of the master chaotic system. In Refs. [10] and [21]–[25], some schemes such as auto-synchronization, random optimization, error minimization and the geometric control method have been developed to recover the unknown parameter values of a given model. By combining the adaptive scheme and dynamical linear feedback, an analytical scheme was proposed for estimating all unknown parameters from time series in Refs. [26] and [27], based on the invariance principle of differential equations. Moreover, Huang has given a more detailed proof and some interesting remarks on the adaptive-feedback control algorithm.[28] Some other authors have also used synchronization to estimate unknown parameters.[29–31] However, the estimation of parameters for delayed chaotic systems has not been explicitly considered and studied.

Motivated by the above discussions, in this paper, we consider the problem of identification of parameters for delayed chaotic networks. We extend the method proposed in Refs. [29] and [30] to chaotic networks with time-varying delay and show that this method is also effective in this case. In this paper, based on the invariance principle of functional differential equations, a dynamical feedback process is adopted, which is quite different from the conventional feedback scheme. The feedback term vanishes once the synchronization and identification of parameters have been achieved successfully.

This paper is organized as follows. In Section 2, the chaotic network model and some necessary definitions and hypotheses are given. In Section 3, by combining the dynamical feedback control and adaptive control, the synchronization-based scheme for the identification of parameters in chaotic networks with time-varying delay is described. Some numerical examples are given to verify the effectiveness of described parameter estimation scheme in Section 4. We conclude the paper in Section 5.

2. Chaotic network model and preliminaries

In this section, we consider the following chaotic network:

$$\begin{cases} \dot{x}_i(t) = f_i(x_i(t)) + \sum\limits_{j=1}^{r} a_{ij} g_j(x_j(t-\tau)) - \gamma \sum\limits_{j=1}^{r} b_{ij} x_j(t), \quad i = 1, 2, \ldots, r, \\ \dot{x}_i(t) = k_i(\bar{x}_r(t)) + \sum\limits_{j=r+1}^{n} c_{ij} x_j(t), \quad i = r+1, r+2, \ldots, n, \end{cases} \tag{1}$$

or in a compact form

$$\begin{cases} \dot{x}_r(t) = f(x_r(t)) + Ag(x_r(t-\tau)) - \gamma B x_r(t), \\ \dot{x}_{n-r}(t) = K(x_r(t)) + C x_{n-r}(t), \end{cases} \tag{2}$$

where $x_i(t)$ $(i = 1, 2, \ldots, n)$ denotes the state variable of the chaotic system,

$$\bar{x}_r(t) = (x_1(t), x_2(t), \ldots, x_r(t))^{\mathrm{T}},$$

r is an integer and $1 \leq r \leq n$, $\tau(t) > 0$. Functions $f_i(\cdot)$ and $g_i(\cdot) : R \to R$ are continuous, and $f_i(0) = g_i(0) = 0$, $\gamma > 0$ is a constant, $k_i(\cdot) \in C[R^r, R]$ and $k_i(0) = 0$, $A = (a_{ij})_{r\times r}$, $B = (b_{ij})_{r\times r}$ and $C = (c_{ij})_{(n-r)\times(n-r)}$ are real matrices and c_{ij} is negative constant, which denotes the strength of neuron interconnections.

Throughout the paper, we make the following three assumptions:

(i) There exist non-negative L_i and $\tilde{L}_i$ $(i = 1, 2, \ldots, r)$ such that

$$|f_i(x) - f_i(y)| \leq L_i |x - y|,$$
$$|g_i(x) - g_i(y)| \leq \tilde{L}_i |x - y|,$$

for $\forall x, y \in R$, and let

$$L = \max_{1\leq i\leq r} L_i, \qquad \tilde{L} = \max_{1\leq i\leq r} \tilde{L}_i.$$

(ii) $\tau(t)$ is a differential function with $0 \leq \dot{\tau}(t) < 1$. Clearly, this assumption is certainly ensured if the transmission delay $\tau(t)$ is a constant.

(iii) For $\forall x = (x_1, x_2, \ldots, x_r)^{\mathrm{T}} \in R^r$, $\|x\|$ denotes the norm of x defined by $\|x\| = (x^{\mathrm{T}}x)^{1/2}$. For $\forall Q \in R^{r\times r}$, $\|Q\|$ indicates the norm of Q induced by $\|Q\| = (\lambda_{\max}(Q^{\mathrm{T}}Q))^{1/2}$.

3. Description of the parameter identification scheme

In order to observe the synchronization behaviour of system (1), we introduce another chaotic network which is the response system of the drive system (1). The behaviour of the response system depends on the behaviour of the drive system, but the drive system is not influenced by the response system

$$\begin{cases} \dot{y}_i(t) = f_i(y_i(t)) + \sum_{j=1}^{r} \hat{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{r} \hat{b}_{ij} y_j(t) + u_i(t), \quad i = 1, 2, \ldots, r, \\ \dot{y}_i(t) = \hat{k}_i(\bar{y}_r(t)) + \sum_{j=r+1}^{n} \hat{c}_{ij} y_j(t), \quad i = r+1, r+2, \ldots, n, \end{cases} \tag{3}$$

or in a compact form

$$\begin{cases} \dot{y}_r(t) = f(y_r(t)) + \hat{A} g(y_r(t-\tau)) - \gamma \hat{B} y_r(t) - u_r(t), \\ \dot{y}_{n-r}(t) = K(y_r(t)) + \hat{C} y_{n-r}(t), \end{cases} \tag{4}$$

where $y_i(t)$ $(i = 1, 2, \ldots, n)$ denotes the state variable of the response system, $u_i(t) = \varepsilon_i \cdot e_i(t)$, $\varepsilon_i = -\alpha_i \cdot e_i^2(t)$ indicates the external control input that will be appropriately designed for a control objective, and $\bar{y}_r(t) = (y_1(t), y_2(t), \ldots, y_r(t))^{\mathrm{T}}$.

Letting $e_i(t) = x_i(t) - y_i(t)$, the error dynamical system of Eqs. (1) and (3) is

$$\begin{cases} \dot{e}_i(t) = F_i(e_i(t)) + \sum_{j=1}^{r} a_{ij} G_j(e_j(t-\tau)) + \sum_{j=1}^{r} \tilde{a}_{ij} g_j(y_j(t-\tau)) \\ \qquad\qquad - \gamma \sum_{j=1}^{r} b_{ij} e_j(t) - \gamma \sum_{j=1}^{r} \tilde{b}_{ij} y_j(t) - u_i(t), \quad i = 1, 2, \ldots, r, \\ \dot{e}_i(t) = K_i(\bar{e}_r(t)) + \sum_{j=r+1}^{n} c_{ij} e_j(t) + \sum_{j=r+1}^{n} \tilde{c}_{ij} y_j(t), \quad i = r+1, r+2, \ldots, n, \end{cases} \tag{5}$$

where $F_i(e_i(t)) = f_i(x_i(t)) - f_i(y_i(t)), G_j(e_i(t-\tau)) = g_j(x_j(t-\tau)) - g_j(y_j(t-\tau))$,

$$K_i(\bar{e}_r(t)) = k_i(\bar{x}_r(t)) - k_i(\bar{y}_r(t)).$$

Equation (5) can be rewritten in the following matrix form:

$$\begin{cases} \dot{e}_r(t) = F(e_r(t)) + AG(e_r(t-\tau)) + \tilde{A} g(y_r(t-\tau)) - \gamma B e_r(t) - \gamma \tilde{B} y_r(t) - \varepsilon * e_r(t), \\ \dot{e}_{n-r}(t) = K(e_r(t)) + C e_{n-r}(t) + \tilde{C} y_{n-r}(t), \end{cases} \tag{6}$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r)^{\mathrm{T}} \in R^r$ are the updated feedback gain, and the mark $*$ is defined as $\varepsilon * e_r(t) = (\varepsilon_1 \cdot e_1(t), \varepsilon_2 \cdot e_2(t), \ldots, \varepsilon_r \cdot e_r(t))^{\mathrm{T}}$, where $e_r(t) = (e_1(t), e_2(t), \ldots, e_r(t))^{\mathrm{T}}$, $e_{n-r}(t) = (e_{r+1}(t), e_{r+2}(t), \ldots, e_n(t))^{\mathrm{T}}$,

$$\begin{aligned} &F(e_r(t)) = (F_1(e_1(t)), F_2(e_2(t)), \ldots, F_r(e_r(t)))^{\mathrm{T}}, \\ &G(e_r(t-\tau)) = (G_1(e_1(t-\tau)), G_2(e_2(t-\tau)), \ldots, G_r(e_r(t-\tau)))^{\mathrm{T}}, \\ &K(e_r(t)) = (K_{r+1}(e_r(t)), K_{r+2}(e_r(t)), \ldots, K_n(e_r(t)))^{\mathrm{T}}. \end{aligned}$$

Theorem 1 Under the assumptions (i), (ii) and (iii), the dynamical feedback strength $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r)^{\mathrm{T}}$ and the estimated parameters $\tilde{A}, \tilde{B}$, and $\tilde{C}$ are adapted according to the following updated law, respectively, $\varepsilon_i = -\alpha_i e_i^2(t)$, $i = 1, 2, \ldots, r$, and

$$\begin{cases} \dot{\tilde{a}}_{ij} = -\eta_{ij} e_i(t) g_j(y_j(t-\tau)), & i, j = 1, 2, \ldots, r, \\ \dot{\tilde{b}}_{ij} = \gamma \delta_{ij} e_i(t) y_i(t), & i, j = 1, 2, \ldots, r, \\ \dot{\tilde{c}}_{ij} = -\gamma_{ij} e_i(t) y_i(t), & i, j = r+1, r+2, \ldots, n, \end{cases} \tag{7}$$

where $\eta_{ij} > 0$, $\delta_{ij} > 0$, $\gamma > 0$ $(i = 1, 2 \ldots, r)$ and $\gamma_{ij} > 0$ $(i = r+1, r+2, \ldots, n)$ are arbitrary constants, respectively, then the controlled response chaotic network (4), satisfies the following conditions:

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} |a_{ij} - \hat{a}_{ij}| = \lim_{t\to\infty} \left|b_{ij} - \hat{b}_{ij}\right| = 0, \quad i, j = 1, 2, \ldots, r,$$
$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} |c_{ij} - \hat{c}_{ij}| = 0, \quad i, j = r+1, r+2, \ldots, n.$$

Proof Letting $\tilde{A} = A - \hat{A}$, $\tilde{C} = C - \hat{C}$ and $\tilde{B} = B - \hat{B}$ be the estimation errors of the parameters A, C and B, and subtracting the second equation of Eq. (2) from the second equation of Eq. (3), we can obtain the error dynamical system as follows:

$$\begin{cases} \dot{e}_r(t) = F(e_r(t)) + AG(e_r(t-\tau)) + \tilde{A}g(y_r(t-\tau)) - \gamma B e_r(t) - \gamma \tilde{B} y_r(t) - \varepsilon * e_r(t), \\ \dot{e}_{n-r}(t) = K(e_r(t)) + Ce_{n-r}(t) + \tilde{C} y_{n-r}(t), \end{cases} \tag{8}$$

$\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r)^{\mathrm{T}} \in R^r$ are the updated feedback gain; and the mark $*$ is defined as $\varepsilon * e_r(t) = (\varepsilon_1 \cdot e_1(t), \varepsilon_2 \cdot e_2(t), \ldots, \varepsilon_r \cdot e_r(t))^{\mathrm{T}}$.

We design the following Lyapunov function:

$$\begin{aligned} V(e(t)) = {} & \frac{1}{2}\sum_{i=1}^{r} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{r}\frac{1}{\alpha_i}(\varepsilon_i - l)^2 + \frac{1}{2}\sum_{i=1}^{r}\left(\sum_{j=1}^{r}\frac{1}{\eta_{ij}}\tilde{a}_{ij}^2 + \sum_{j=1}^{r}\frac{1}{\delta_{ij}}\tilde{b}_{ij}^2\right) \\ & + \frac{1}{2}\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\frac{1}{\gamma_{ij}}\tilde{c}_{ij}^2 + \frac{1}{2(1-\sigma)}\int_{t-\tau}^{t} g^{\mathrm{T}}(e(s))g(e(s))\mathrm{d}s, \end{aligned} \tag{9}$$

where l is a constant to be determined.

Calculating the derivative of Eq. (9) along the trajectories of Eq. (8), we have

$$\begin{aligned} \dot{V}(e(t)) = {} & \sum_{i=1}^{r} e_i^{\mathrm{T}}(t)\dot{e}_i(t) - \sum_{i=1}^{r}(\varepsilon_i - l)\dot{\varepsilon}_i + \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{a}_{ij}\dot{\tilde{a}}_{ij} + \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{b}_{ij}\dot{\tilde{b}}_{ij} + \sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\tilde{c}_{ij}\dot{\tilde{c}}_{ij} \\ & + \frac{1}{2(1-\sigma)}g^{\mathrm{T}}(e(t))g(e(t)) - \frac{1-\dot{\tau}}{2(1-\sigma)}g^{\mathrm{T}}(e(t-\tau))g(e(t-\tau)) \\ = {} & e_r^{\mathrm{T}}(t)[F(e_r(t)) + AG(e_r(t-\tau)) + \tilde{A}g(y_r(t-\tau)) - \gamma B e_r(t) - \gamma\tilde{B}y_r(t) - \varepsilon * e_r(t)] \\ & + e_{n-r}^{\mathrm{T}}(t)[K(e_r(t)) + Ce_{n-r}(t) + \tilde{C}y_{n-r}(t)] + \sum_{i=1}^{r}(\varepsilon_i - l)e_i^2(t) - \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{a}_{ij}e_i(t)g_j(y_j(t-\tau)) \\ & + \sum_{i=1}^{r}\sum_{j=1}^{r}\gamma\tilde{b}_{ij}e_i(t)y_j(t) - \sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\tilde{c}_{ij}e_i(t)y_j(t) \\ & + \frac{1}{2(1-\sigma)}g^{\mathrm{T}}(e(t))g(e(t)) - \frac{1-\dot{\tau}}{2(1-\sigma)}g^{\mathrm{T}}(e(t-\tau))g(e(t-\tau)) \\ = {} & e_r^{\mathrm{T}}(t)[F(e_r(t)) + AG(e_r(t-\tau)) - \gamma B e_r(t)] + e_{n-r}^{\mathrm{T}}(t)[K(e_r(t)) + Ce_{n-r}(t)] \\ & + \sum_{i=1}^{r} -le_i^2(t) + \frac{1}{2(1-\sigma)}g^{\mathrm{T}}(e(t))g(e(t)) - \frac{1-\dot{\tau}}{2(1-\sigma)}g^{\mathrm{T}}(e(t-\tau))g(e(t-\tau)). \end{aligned}$$

According to the properties of assumptions (i) and (ii), then we can obtain

$$\begin{aligned} & G^{\mathrm{T}}(e_r(t-\tau))G(e_r(t-\tau)) = \sum_{i=1}^{r} G_i^2(e_i(t-\tau)) \le \sum_{i=1}^{r}\tilde{L}_i^2 e_i^2(t-\tau) = \tilde{L}^2 e_r^{\mathrm{T}}(t-\tau)e_r(t-\tau), \\ & \|F(e_r(t))\|^2 \le L^2 e_r^{\mathrm{T}}(t)e_r(t), \quad g^{\mathrm{T}}(e(t-\tau))g(e(t-\tau)) \ge 0, \quad \frac{1-\dot{\tau}}{2(1-\sigma)} \ge \frac{1}{2}, \\ & g^{\mathrm{T}}(e(t))g(e(t)) = \sum_{i=1}^{r} g_i^2(e_i(t)) \le \sum_{i=1}^{r}\tilde{L}_i^2 e_i^2(t) = \tilde{L}^2 e_r^{\mathrm{T}}(t)e_r(t), \\ & \dot{V}(t) \le \eta^{\mathrm{T}}(t)Q\eta(t), \end{aligned} \tag{10}$$

where $\eta(t) = (e_r^{\mathrm{T}}(t),\, e_{n-r}^{\mathrm{T}}(t),\, e_{n-r}^{\mathrm{T}}(t-\tau))^{\mathrm{T}}$

$$Q = \begin{pmatrix} \Im_1 & 0 & \Im_3 \\ 0 & \Im_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Im_1 = \left(L - r\lambda_{\min}(B) - l + \frac{\tilde{L}^2}{2(1-\sigma)}\right) I_r, \quad \Im_2 = \tilde{L}A + KI_r, \quad \Im_3 = \left(C - \frac{\tilde{L}^2}{2}\right) I_{n-r}.$$

The constant l can be properly chosen to make $\dot{V}(t) \leq 0$.

Therefore, based on the Lyapunov stability theory, the error vector $e(t) \to 0$, as $t \to 0$. Then Theorem 1 has been proved.

It is obvious that $M = \{\dot{V}(e(t))| = 0\} = \{e(t) = 0\}$. Therefore the set $E = \{e(t) = 0,\, \hat{C} = C,\, \hat{A} = A,\, \hat{B} = B,\, \varepsilon = \varepsilon_0\}$ is the largest invariant set contained in M for Eq. (8). In fact, if one of the following equalities cannot hold: $\hat{C} = C$, $\hat{A} = A$, $\hat{B} = B$, then $e(t) = 0$ cannot be a fixed point of Eq. (8) at all, i.e., one cannot conclude that the solution $e(t)$ is equal to 0 for $t > 0$ when the initial values $e(t) = 0$. So according to the invariant principle of functional differential equations, starting with arbitrary initial values of Eq. (8), the trajectory converges asymptotically to the set E, i.e., $e(t) = 0$, $\hat{C} = C$, $\hat{A} = A$ and $\hat{B} = B$, as $t \to \infty$. This indicates that the unknown parameters C, A and B can be successfully estimated by using updated laws (5) and (6), and synchronization is achieved at the same time. This ends the proof.

In Theorem 1, we can note that the variable feedback strength ε_i is automatically adapted to a suitable strength depending on the initial values for the synchronization of chaotic networks, which is significantly different from the usual linear feedback. On the other hand, by using an ordinary linear feedback scheme with constant feedback strength, the unknown parameters can also be estimated with certain updated laws. In the following, a corollary will be given to show this fact.

For simplicity, we assume that all notations are the same as those mentioned before.

Corollary 1 Under the assumptions (i), (ii) and (iii), the fixed feedback strength $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r)^{\mathrm{T}}$ is large enough and the estimated parameters $\hat{C}$, $\hat{A}$, and $\hat{B}$ are adapted according to the updated law (7), then the controlled response chaotic network (3) is synchronized with the drive network (1), and satisfies the following conditions:

$$\begin{aligned} &\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} |a_{ij} - \hat{a}_{ij}| = \lim_{t\to\infty} \left|b_{ij} - \hat{b}_{ij}\right| = 0, \qquad i,j = 1,2,\ldots,r, \\ &\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} |c_{ij} - \hat{c}_{ij}| = 0, \qquad i,j = r+1, r+2, \ldots, n. \end{aligned}$$

Proof By constructing another Lyapunov function:

$$\begin{aligned} V(e(t)) = {} & \frac{1}{2}\sum_{i=1}^{r} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{r}\left(\sum_{j=1}^{r}\frac{1}{\eta_{ij}}\tilde{a}_{ij}^2 + \sum_{j=1}^{r}\frac{1}{\delta_{ij}}\tilde{b}_{ij}^2\right) \\ & + \frac{1}{2}\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\frac{1}{\gamma_{ij}}\tilde{c}_{ij}^2 + \frac{1}{2(1-\sigma)}\int_{t-\tau}^{t} g^{\mathrm{T}}(e(s))g(e(s))\mathrm{d}s. \end{aligned} \tag{11}$$

This proof is similar to the proof of Theorem 1; we can easily derive the result. Its proof is straightforward and hence omitted.

Remark 1 The scheme described in this paper can be used to identify the parameters of chaotic systems but not stable systems, because for two stable systems with identical equilibrium point, synchronization can be easily obtained even with completely different parameters and structures. However, as stated in Ref. [30], the analysis of the results should be based on the LaSalle invariant principle, since the Lyapunov direct method can only guarantee stability in the Lyapunov sense and cannot guarantee asymptotic stability.

Remark 2 In practice, the linear feedback controller with large enough strength in Corollary 1 is not realizable. Although the appropriate feedback strength can be ascertained by certain calculations for concrete dynamical systems, its value is different for nonidentical systems. On the other hand, the feedback strength of different systems can be automatically enhanced to the required value for the synchronization of drive and response systems in Theorem 1.

Remark 3 Some sufficiently large adaptive gains $\alpha_i, \eta_{ij}, \delta_{ij}$ $(i, j = 1, 2, \ldots, r)$ and γ_{ij} $(i, j = r+1, \ldots, n)$ would lead to fast synchronization and quick identification of parameters, while for sufficiently small adaptive gains, the time to achieve synchronization and estimation of parameters may be quite long.

4. An example

In this section, we give a numerical example to demonstrate the effectiveness of our results.

Consider the following system:

$$\begin{cases} \dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{2} a_{ij} g_j(x_j(t-\tau)) - \gamma \sum_{j=1}^{2} b_{ij} x_j(t), \quad i = 1, 2, \\ \dot{x}_3(t) = k_3(\bar{x}_2(t)) + C x_3(t), \end{cases} \tag{12}$$

where $f_i(x_i(t)) = \tanh(x_i(t))$, $g_i(x_i(t-\tau)) = \tanh(x_i(t-\tau))$, $i = 1, 2$.

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{1}{20} \\ \frac{1}{20} & -\frac{1}{10} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = -1$, $k_3(\bar{x}_2(t)) = x_1(t)$, $\gamma = 1$ and $\tau = 1$. The system satisfies assumption (i) with $L = \tilde{L} = L_i = \tilde{L}_i = 1$, $i = 1, 2$. The initial condition $(x_1(t), x_2(t), x_3(t))^{\mathrm{T}} = (0.2, 0.1, 0.3)^{\mathrm{T}}$, for $t \in [-1, 0]$.

The response system of the first equation of Eq. (3) is

$$\begin{cases} \dot{y}_i(t) = f_i(y_i(t)) + \sum_{j=1}^{2} \hat{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{2} \hat{b}_{ij} y_j(t) + u_i(t), \quad i = 1, 2, \\ \dot{y}_3(t) = \hat{k}_i(\bar{y}_2(t)) + C y_3(t), \end{cases} \tag{13}$$

where $f_i(x_i(t)) = \tanh(x_i(t))$, $g_i(x_i(t-\tau)) = \tanh(x_i(t-\tau))$, $i = 1, 2$.

$\hat{A} = \begin{pmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{20} \\ \frac{3}{20} & -\frac{3}{10} \end{pmatrix}, \hat{B} = \begin{pmatrix} \hat{b}_{11} & \hat{b}_{12} \\ \hat{b}_{21} & \hat{b}_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = -1$, $k_3(\bar{x}_2(t)) = x_1(t)$, $\gamma = 1$ and $\tau = 1$. The system satisfies assumption (i) with $L = \tilde{L} = L_i = \tilde{L}_i = 1$, $i = 1, 2$. The initial condition $(y_1(t), y_2(t), y_3(t))^{\mathrm{T}} = (0.4, 0.2, 0.6)^{\mathrm{T}}$, for $t \in [-1, 0]$; $\varepsilon_i(0) = \hat{a}_{ii}(0) = \hat{b}_{ii}(0) = 0, i = 1, 2$, and parameter update gain $\eta_{ij} = 10$, $\delta_{ij} = 10$, $(i, j = 1, 2)$, $\gamma_{33} = 10$. Letting $e_i(t) = x_i(t) - y_i(t)$, the error dynamical system of Eqs. (12) and (13) is

$$\begin{cases} \dot{e}_i(t) = F_i(e_i(t)) + \sum_{j=1}^{2} a_{ij} G_j(e_j(t-\tau)) + \sum_{j=1}^{2} \tilde{a}_{ij} g_j(y_j(t-\tau)) \\ \qquad\qquad - \gamma \sum_{j=1}^{2} b_{ij} e_j(t) - \gamma \sum_{j=1}^{2} \tilde{b}_{ij} y_j(t) - u_i(t), \quad i = 1, 2, \\ \dot{e}_3(t) = K_3(\bar{e}_2(t)) + C e_3(t) + \tilde{C} y_3(t), \end{cases} \tag{14}$$

where $F_i(e_i(t)) = f_i(x_i(t)) - f_i(y_i(t)), G_j(e_i(t-\tau)) = g_j(x_j(t-\tau)) - g_j(y_j(t-\tau))$, $i = 1, 2$, $K_3(\bar{e}_r(t)) = k_3(\bar{x}_2(t)) - k_3(\bar{y}_2(t))$, and $\bar{e}_2(t) = (e_1(t), e_2(t))^{\mathrm{T}}$.

5. Conclusion

In summary, we have shown that a combination of synchronization based on dynamical feedback with an adaptive evolution for parameters unknown to the responser enables the estimation of the unknown parameters for uncertain delayed chaotic networks. In comparison with previous methods, time-delay is taken into account in this simple, analytical and systematic synchronization-based identification scheme of parameters. In addition, it

is quite robust against the effect of noise, and it is also able to rapidly track changes in the operating parameters of the experimental chaotic network. We also believe that this approach can be easily generalized to the case of other continuous and discrete time-delayed chaotic networks, and also the case of other chaotic dynamical systems with delay.

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