Adaptive synchronization of a hyperchaotic Lü system based on extended passive control*

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(Received 20 October 2009; revised manuscript received 18 November 2009)

This paper investigates the adaptive synchronization of hyperchaotic Lü systems based on the method of extended passive control. By combining the feedback control, the extended passive control method with two output variables is developed, which can synchronize hyperchaotic Lü systems asymptotically and globally more easily without knowing the bound of state of the hyperchaotic system. Adaptive laws are introduced to estimate the unknown parameters as well. Simulation results show the effectiveness and flexibility of the proposed control scheme.

Keywords: hyperchaotic Lü system, synchronization, extended passive control, adaptive control
PACC: 0545

1. Introduction

Chaos synchronization has become one of the hot topics in nonlinear science and applications and has been extensively studied in many fields such as secure communication, information processing, biological engineering, chemical reactions, and so on.[1] Hyperchaotic systems, which hold more than one positive Lyapunov exponent and more complicated attractors, possess the characteristics of high capacity, high security and high efficiency, in comparison with the ordinary low dimensional chaotic systems. As a result, many researchers have proposed varieties of control methods to synchronize hyerchaotic systems in recent years, such as nonlinear control,[2] linear control,[3] adaptive feedback control,[4−6] the T–S fuzzy model method,[7] the neural network method,[8] and so on. As part of dissipative theory, the apassivity is introduced in detail in Refs. [9] and [10], and has aroused new interest in the field of nonlinear control. Recently, researchers have been trying to apply the passive control method to achieve the synchronization of hyperchaotic systems.[11,12] However, it is assumed that the system parameters are known in the previous studies, and furthermore, in Refs. [11] and [12], it should calculate the bound of some state of the hyperchaotic system, whose accurate value is actually very difficult to obtain; therefore, it is not easy to realize in practice.

In view of the disadvantages of the previous methods, a novel passive control method is proposed in this paper to achieve the adaptive synchronization of hyperchaotic Lü systems. The rest of this paper is organized as follows: in Section 2, the basic principle of passive control is introduced, the synchronization of hyperchaotic Lü systems via extended passive control is derived in Section 3, and the simulation results are shown in Section 4. Finally, some conclusions are given in Section 5.

2. The principle of passive control

Consider the following nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ \dot{y} &= h(x), \end{aligned} \tag{1}$$

where the states $x \in R^n$, the input $u \in R^m$, the output $y \in R^m$. $f(x)$ and $g(x)$ are smooth vector fields, $h(x)$ is a smooth mapping. Also suppose that $x = 0$ is the equilibrium point of system (1). If the equilibrium point is not at $x = 0$, it can be shifted to zero equilibrium by a coordinate transform.

Definition 1[10] System (1) is passive if there exists a real constant β such that for $\forall t \geq 0$ the following inequality is satisfied:

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta; \tag{2}$$

*Project supported by the Natural Science Foundation of Fujian Province, China (Grant No. E0710018).

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if there exists a constant β and $\rho > 0$, the following inequality is satisfied:

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau. \quad (3)$$

Definition 2[10] System (1) is a minimum phase system if $L_g h(0)$ is nonsingular and $x = 0$ is an asymptotically stable equilibrium of $f(x)$.

Letting $z = \vartheta(x)$, if system (1) is a minimum phase and the distribution spanned by the vector fields $g_1(x) \cdots g_m(x)$ is involutive, then system (1) can be represented by the equations of the following normal form:[10]

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u, \end{aligned} \quad (4)$$

where $a(z, y)$ is nonsingular for all (z, y) near $(0, 0)$.

Theorem 1[10] Suppose the system (4) is passive with a storage function V, which is positive-definite, and the system (4) is locally zero-state detectable. Let ϕ be a smooth function such that $\phi(0) = 0$ and $y^{\mathrm{T}}\phi(y) > 0$ for each nonzero y. Then the control law $u = -\phi(y)$ asymptotically stabilizes the equilibrium $x = 0$ of system (4).

3. Adaptive synchronization of hyperchaotic Lü systems

The hyperchaotic Lü system can be described as[13]

$$\begin{aligned} \dot{x}_1 &= a\,(x_2 - x_1) + x_4, \\ \dot{x}_2 &= -x_1 x_3 + c x_2, \\ \dot{x}_3 &= x_1 x_2 - b x_3, \\ \dot{x}_4 &= x_1 x_3 + r x_4, \end{aligned} \quad (5)$$

where x_i (i = 1, 2, 3, 4) are the state variables and a, b, c, r are the system parameters. When $a = 36$, $b = 3$, $c = 20$ and $-0.35 < r \leq 1.3$, system (5) will exhibit hyperchaoitc behaviour.

Assume that system (5) is the drive system, and the response system is given by

$$\begin{aligned} \dot{y}_1 &= a\,(y_2 - y_1) + y_4, \\ \dot{y}_2 &= -y_1 y_3 + c y_2 + u_1, \\ \dot{y}_3 &= y_1 y_2 - b y_3 + \psi, \\ \dot{y}_4 &= y_1 y_3 + r y_4 + u_2, \end{aligned} \quad (6)$$

where u_1, u_2 are the controllers to be designed based on passive control, modifier ψ is the feedback input which is introduced to simplify the controlled system so that the passive controllers can be realized more easily and become much simpler, in comparison to the traditional passive control method, which is shown in Refs. [11] and [12].

Define the synchronization error state as $e = y - x$; then the error dynamical system can be obtained by

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4, \\ \dot{e}_2 &= c e_2 - e_1 e_3 - e_1 x_3 - e_3 x_1 + u_1, \\ \dot{e}_3 &= -b e_3 + e_1 e_2 + e_1 x_2 + e_2 x_1 + \psi, \\ \dot{e}_4 &= e_1 e_3 + e_1 x_3 + e_3 x_1 + r e_4 + u_2. \end{aligned} \quad (7)$$

Let $z = [z_1, z_2]^{\mathrm{T}} = [e_1, e_3]^{\mathrm{T}}$ and $w = [w_1, w_2]^{\mathrm{T}} = [e_2, e_4]^{\mathrm{T}}$; then system (6) can be rewritten as

$$\begin{aligned} \dot{z}_1 &= -a z_1 + a w_1 + w_2, \\ \dot{z}_2 &= -b z_2 + z_1 w_1 + z_1 x_2 + w_1 x_1 + \psi, \\ \dot{w}_1 &= c w_1 - z_1 z_2 - z_1 x_3 - z_2 x_1 + u_1, \\ \dot{w}_2 &= z_1 z_2 + z_1 x_3 + z_2 x_1 + r w_2 + u_2. \end{aligned} \quad (8)$$

Theorem 2 If the controller $u = [u_1, u_2]^{\mathrm{T}}$ and modifier ψ is chosen as

$$\begin{aligned} \psi &= -z_1 x_2, \\ u &= \begin{bmatrix} -\hat{c} w_1 + z_1 x_3 - z_1 \hat{a} - k w_1 + \sigma_1 \\ -z_1 z_2 - z_1 x_3 - z_2 x_1 - \hat{r} w_2 - z_1 - k w_2 + \sigma_2 \end{bmatrix}, \end{aligned} \quad (9)$$

where $\hat{a}$, $\hat{c}$, $\hat{r}$ are the estimations of the parameters a, c, r, respectively, k is a positive constant and $\sigma = [\sigma_1,\ \sigma_2]^{\mathrm{T}}$ is an external signal which is connected with the reference input, and the update laws of the parameters are determined as

$$\begin{aligned} \dot{\hat{a}} &= z_1 w_1, \\ \dot{\hat{c}} &= w_1^2, \\ \dot{\hat{r}} &= w_2^2, \end{aligned} \quad (10)$$

then the system (8) will be rendered passive and will be asymptotically stable at the equilibrium point of zero, namely the system (5) and system (6) with different initial conditions will be synchronized with each other asymptotically.

Proof

Substituting $\psi = -z_1 x_2$ into Eq. (8), we find that system (8) can be represented in the following form:

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, w)w, \\ \dot{w} &= b(z, w) + a(z, w)u, \end{aligned} \quad (11)$$

where

$$f_0(z) = [-az_1, -bz_2]^{\mathrm{T}},$$
$$p(z,w) = \begin{bmatrix} a & 1 \\ z_1 + x_1 & 0 \end{bmatrix},$$
$$b(z,w) = \begin{bmatrix} cw_1 - z_1z_2 - z_1x_3 - z_2x_1 \\ rw_2 + z_1z_2 + z_1x_3 + z_2x_1 \end{bmatrix},$$
$$a(z,w) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Choose the storage function as

$$V(z,w) = W(z) + \frac{1}{2}w^{\mathrm{T}}w + \frac{1}{2}\tilde{a}^2 + \frac{1}{2}\tilde{c}^2 + \frac{1}{2}\tilde{r}^2, \quad (12)$$

where $W(z)$ is a Lyapunov function with $W(0) = 0$, $W(z) = \frac{1}{2}(z_1^2 + z_2^2)$, and $\tilde{a} = a - \hat{a}$, $\tilde{c} = c - \hat{c}$, $\tilde{r} = r - \hat{r}$.

The zero dynamics of system (11) describes those of internal dynamics which are consistent with external constraint $w = 0$, i.e., $\dot{z} = f_0(z)$, then it can be obtained that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}W(z) &= \frac{\partial W}{\partial z}f_0(z) \\ &= [z_1, z_2][-az_1, -bz_2]^{\mathrm{T}} \\ &= -az_1^2 - bz_2^2 \\ &\leq 0. \end{aligned} \quad (13)$$

So $f_0(z)$ is globally asymptotically stable. At the same time, $L_gh(0) = \mathrm{diag}(1,1)$ is nonsingular, therefore, system (11) is a minimum phase system and can be equivalent to a passive system.

The time derivative of $V(z,w)$ along Eq. (11) is as follows:

$$\begin{aligned} \dot{V}(z,w) &= \frac{\partial W(z)}{\partial z}\dot{z} + \dot{w}^{\mathrm{T}}w + \tilde{a}\dot{\tilde{a}} + \tilde{c}\dot{\tilde{c}} + \tilde{r}\dot{\tilde{r}} \\ &= \frac{\partial W(z)}{\partial z}f_0(z) + \frac{\partial W(z)}{\partial z}p(z,w)w \\ &\quad + [b(z,w) + a(z,w)u]^{\mathrm{T}}w \\ &\quad + (a - \hat{a})(-\dot{\hat{a}}) + (c - \hat{c})(-\dot{\hat{c}}) + (r - \hat{r})(-\dot{\hat{r}}) \\ &\leq \frac{\partial W(z)}{\partial z}p(z,w)w + [b(z,w) + a(z,w)u]^{\mathrm{T}}w \\ &\quad + (a - \hat{a})(-\dot{\hat{a}}) + (c - \hat{c})(-\dot{\hat{c}}) + (r - \hat{r})(-\dot{\hat{r}}). \end{aligned} \quad (14)$$

Substituting Eqs. (9) and (10) into Eq. (14), we can obtain the following inequality:

$$\dot{V}(z,w) \leq -kw^{\mathrm{T}}w + \sigma^{\mathrm{T}}w. \quad (15)$$

Integrating the inequality (15) from 0 to t, it yields

$$\begin{aligned} V(z,w) - V(z_0,w_0) \leq &-\int_0^t kw(\tau)^{\mathrm{T}}w(\tau)\mathrm{d}\tau \\ &+ \int_0^t \sigma^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau. \end{aligned} \quad (16)$$

Let $\beta = V(z_0,w_0)$; according to the definition of $V(z,w)$, it is obvious that $V(z,w) \geq 0$, so the inequality (16) can be transferred as

$$\begin{aligned} \int_0^t \sigma^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau + \beta &\geq \int_0^t kw(\tau)^{\mathrm{T}}w(\tau)\mathrm{d}\tau + V(z,w) \\ &\geq \int_0^t kw(\tau)^{\mathrm{T}}w(\tau)\mathrm{d}\tau. \end{aligned} \quad (17)$$

According to Definition 1, it can be concluded that system (11) is rendered passive. Also, letting $\sigma = [\sigma_1, \sigma_2]^{\mathrm{T}} = [0,0]^{\mathrm{T}}$, in terms of Theorem 1, system (11) will be stabilized at the equilibrium point of zero.

This completes the proof.

Remark 1 Because the controller u is only influenced by the parameters of a, c, r, it is not necessary to estimate the parameter b.

Remark 2 It can be seen from Eq. (9) that by introducing the modifier ψ, the controller u in Eq. (9) does not contain the bound of the state of the hyperchaotic system such that it can be realized more easily and is much simpler, compared to the controller presented in Refs. [11] and [12].

4. Numerical simulations

Assuming the initial conditions of drive system (5) to be $x(0) = [1,3,5,7]^{\mathrm{T}}$ and the response system (6) to be $y(0) = [8,6,4,2]^{\mathrm{T}}$, then the initial values of synchronous error system (7) are $e = [7,3,-1,-5]^{\mathrm{T}}$, and assume the initial values of the estimations of parameters to be $\hat{a}(0) = 1$, $\hat{c}(0) = 1$ and $\hat{r}(0) = 1$, respectively. Choose $k = 1$. The controllers in Eq. (9) are switched on at $t = 0$ s. It can be observed that the trajectories of the response system (6) asymptotically approach those of the drive system (5) in Fig. 1. The errors of synchronization are shown in Fig. 2, from which it can be seen that each error of the states converges to zero quickly. Figure 3 shows that the estimations of the system parameters asymptotically converge to a constant as time goes on.

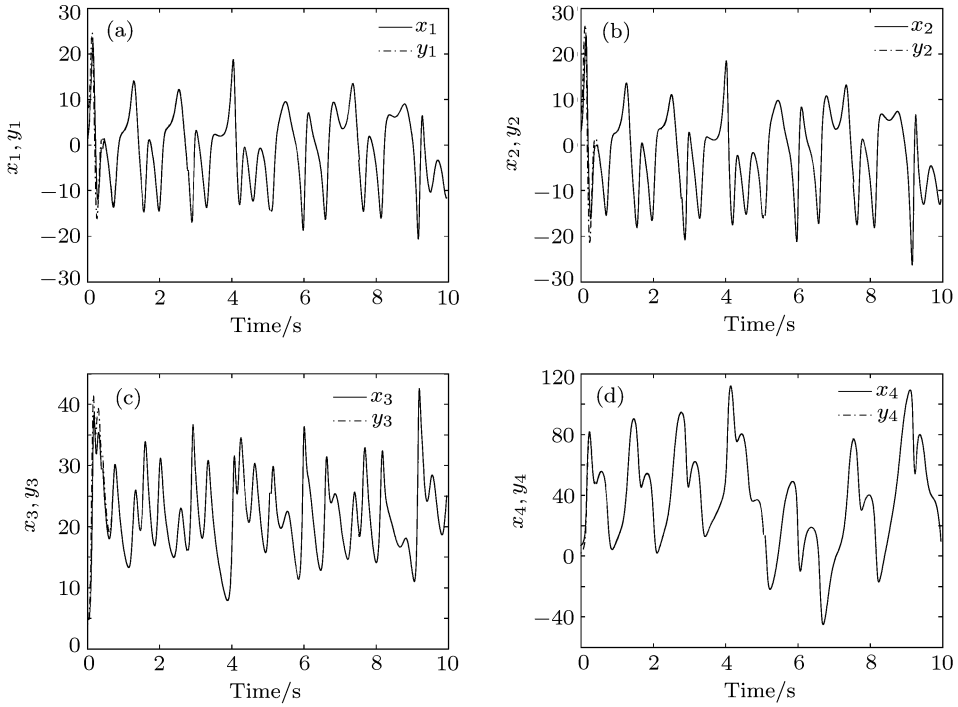


Fig. 1. Synchronization of systems (5) and (6), where (a) is the trajectory of states of x_1 and y_1, (b) states of x_2 and y_2, (c) states of x_3 and y_3, (d) states of x_4 and y_4.

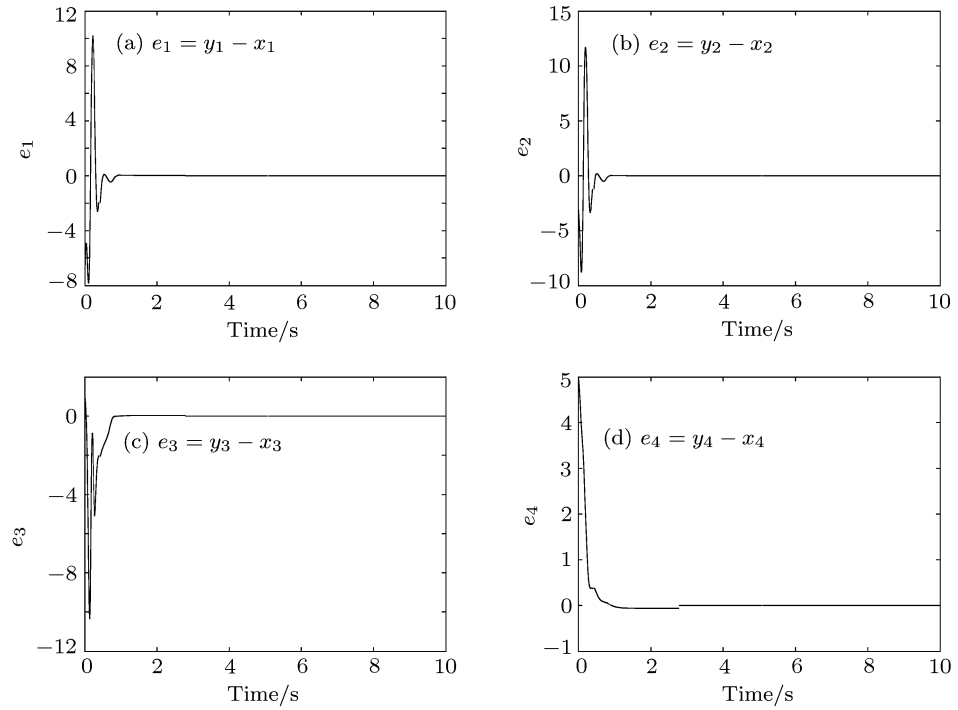


Fig. 2. Synchronization errors of systems (5) and (6), where (a) is the error of x_1 and y_1, (b) the error of x_2 and y_2, (c) the error of x_3 and y_3, (d) the error of x_4 and y_4.

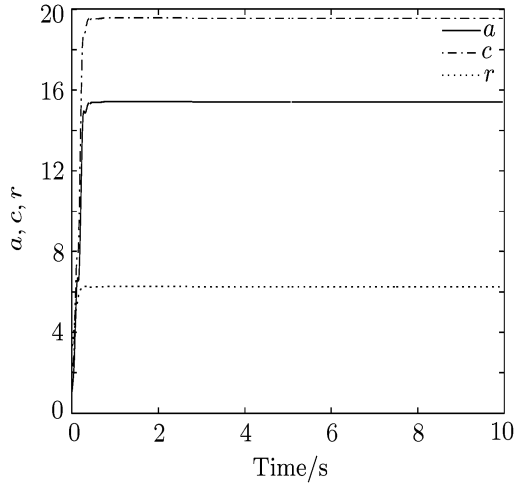


Fig. 3. The estimations of parameters a, c, r.

5. Conclusions

In this paper, an extended passive control scheme is proposed by adopting a feedback input into one state of the response system such that it can be avoided to solve the bound of the state of the hyperchaotic system, and based on the theory of passive control, novel controllers are designed to stabilize the error dynamical system at the zero equilibrium globally. Adaptive laws of the parameters are presented as well. Because the bound of the state of the hyperchaotic system is not involved, the controllers are much simpler in comparison with Refs. [11] and [12]. The proposed control scheme is easy to realize and can be applied in many other systems. Finally, numerical simulations are provided to verify the theoretical analysis and demonstrate the effectiveness of the proposed control method.

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