Convergent robust stabilization conditions for fuzzy chaotic systems based on the edgewise subdivision approach*

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(Received 13 October 2009; revised manuscript received 17 November 2009)

This paper concerns the problem of stabilizing fuzzy chaotic systems via the viewpoint of the edgewise subdivision approach. Firstly, a new edgewise subdivision algorithm is proposed to implement the simplex edgewise subdivision which divides the overall fuzzy chaotic systems into a lot of sub-systems by a kind of algebraic description. These sub-systems have the same volume and shape characteristics. Secondly, a novel kind of control scheme which switches by the transfer of different operating sub-systems is proposed to achieve convergent stabilization conditions for the underlying controlled fuzzy chaotic systems. Finally, a numerical example is given to demonstrate the validity of the proposed methods.

Keywords: chaotic system, Takagi–Sugeno fuzzy model, convergent stabilization conditions, edgewise subdivision

PACC: 0545

1. Introduction

Since Ott, Grebogi and Yorke (OGY) proposed the famous OGY method for chaotic control,[1] various control methods have been proposed by means of linear or nonlinear frameworks.[2−8] Among the existing literature for stabilizing nonlinear chaotic systems, the so-called Takagi–Sugeno (TS) fuzzy model method is one of the most popular. This is because of the fact that a wide class of nonlinear systems can be approximated by the TS fuzzy model,[9] which consists of locally linear models connected by IF-THEN rules. Since each sub-model is linear, the conventional linear system theory can be applied to the class of nonlinear control systems, and stability analysis and control synthesis have been widely studied.[10]

On the other hand, for the problem of stabilization for controlled TS fuzzy chaotic systems, most of the existing results are based on the single Lyapunov function approach[11−14] which is a source of conservatism. For the sake of reducing the conservatism, piecewise quadratic Lyapunov functions,[15] parameter-dependent Lyapunov functions[16] and k-sample variation Lyapunov functions[17,18] are also proposed. It is worth noting that the results in Refs. [16]–[18] are only suitable for the discrete-time case. For continuous-time TS fuzzy systems, the bounds of the time derivatives of fuzzy membership functions are required to implement the non-quadratic controller synthesis.[19] However, it is difficult to obtain the bound for most actual settings and this requirement restricts its range of application. Based on the above analysis, the attainment of the necessary and sufficient stabilization condition for continuous-time TS fuzzy chaotic systems is still an open problem and this fact motivates our research interest.

In this paper, convergent robust stabilization conditions for fuzzy chaotic systems are proposed via the viewpoint of the edgewise subdivision approach. A new edgewise subdivision algorithm is firstly proposed to implement the simplex edgewise subdivision which divides the overall fuzzy chaotic systems into a lot of sub-systems by a kind of algebraic description. These sub-systems have the same volume and shape char-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 50977008, 60774048 and 60821063), the Program for Cheung Kong Scholars and National Basic Research Program of China (Grant No. 2009CB320601).

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acteristics. Thus, a novel kind of control scheme, which is switched by the transfer of different operating sub-systems, is proposed to achieve convergent stabilization conditions for the underlying controlled fuzzy chaotic systems, as the number of obtained subsystems increases. Moreover, there is no need for the bound of the time derivative of the membership functions in the controller synthesis process which facilitates the range of the application. Furthermore, the partition of the fuzzy chaotic systems is applied to reduce the conservatism of the linear matrix inequality (LMI)-based sufficient condition. As the LMI-based stabilization conditions are necessary and sufficient for linear systems, the proposed scheme approaches this necessary and sufficient condition, in the sense that the underlying examined fuzzy chaotic system becomes increasingly smaller (i.e. the overall fuzzy chaotic system is divided into as many sub-systems as expected).

The rest of this paper is organized as follows. In Section 2, the problem under consideration is described and a brief description of the TS fuzzy model is given; in Section 3, a new edgewise subdivision algorithm and a novel kind of control scheme are proposed to stabilize the underlying fuzzy chaotic system. An illustrative example is given in Section 4 and finally some conclusions are drawn in Section 5.

2. Preliminaries

In this section, we briefly review the TS fuzzy model. Considering parametric uncertainties,[20] we propose a TS fuzzy system in which the i-th rule is formulated in the following form:

Rule i: If $z_1(t)$ is N_{i1} and ... and $z_n(t)$ is N_{in}, then

$$\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t), \qquad (1)$$

where $z_i(t)$ is the premise variable assumed to be independent of the controlled input $u(t)$, N_{ij} is a fuzzy set, $x(t) \in \mathbb{R}^n$ is the state vector, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are the system matrix and input matrix, respectively, q is the number of fuzzy rules, ΔA_i and ΔB_i represent parametric uncertainties in the system, and satisfy the following conditions:

$$\left[\begin{array}{cc} \Delta A_i & \Delta B_i \end{array}\right] = D_i F_i(t) \left[\begin{array}{cc} E_{1i} & E_{2i} \end{array}\right], \qquad (2)$$

where D_i, E_{1i} and E_{2i} are known real constant matrices of appropriate dimensions, and $F_i(t) \in \mathbb{R}^{i\times j}$ is an unknown matrix function with Lebegue-measurable elements, and satisfies:

$$F_i^{\mathrm{T}}(t)F_i(t) \le I,$$

where I is the identity matrix of appropriate dimension.

Then, applying the product-inference rule, singleton fuzzifier, and the centre of gravity defuzzifier to the above fuzzy rule base, the overall fuzzy chaotic systems can be inferred as

$$\begin{aligned}\dot{x}(t) = &\sum_{i=1}^{q} h_i(z(t))((A_i + \Delta A_i)x(t) \\ &+ (B_i + \Delta B_i)u(t)),\end{aligned} \qquad (3)$$

where

$$h(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}, \quad w_i(z(t)) = \prod_{j=1}^{n} N_{ij}(z_j(t)).$$

We end this section with a useful lemma which plays an important role in dealing with the problem of robust control.

Lemma 1[21] Given matrices P, M and N of appropriate dimensions and with P symmetrical, then $P + MF(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)M^{\mathrm{T}} < 0$ holds for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$ if and only if for some $\varepsilon > 0$, $P + \varepsilon^{-1}MM^{\mathrm{T}} + \varepsilon N^{\mathrm{T}}N < 0$.

3. Main results

In this section, the problem of stabilizing fuzzy chaotic systems (3) will be investigated via the viewpoint of the edgewise subdivision approach. As is well known, the fuzzy membership function $h_i(z(t))$ $(i = 1, \ldots, q)$ plays an important part in the fuzzy chaotic systems (3). Hence there is a chance to reduce the conservatism if we make a good use of its information in the process of controller design. However, in existing work, the bound of the time derivative of the membership functions is required, which is difficult to obtain for most actual settings and restricts its range of application. For the sake of conceiving convergent stabilization conditions, we attempt to divide the overall fuzzy chaotic systems into a lot of subsystems with the same volume and shape characteristics. Thus a novel kind of controller switched by the transfer of two different obtained subsystems will be proposed to stabilize the overall fuzzy chaotic system. Firstly, a new kind of edgewise subdivision algorithm will be proposed to implement this task.

3.1. New edgewise subdivision algorithm

Let $P_0, P_1, \ldots, P_d$ be affinely independent points spanning a simplex σ in $\mathbb{R}^d$. If $b_0, b_1, \ldots, b_d$ are non-negative real numbers whose sum is 1, then $X = \sum_{i=0}^{d} b_i P_i$ is a point in σ. The b_i are the barycentric coordinates of X. The main goal of this subsection is to divide the simplex σ into a lot of subsimplices.

In order to refine the simplicial mesh, the simplices are split with a simplex edgewise subdivision as presented in Ref. [22]. This technique allows the splitting of a d-simplex into k^d ($k \in \mathbb{Z}_+$) simplices in an algebraic description. These k^d simplices are generated from k^d matrices $M \in \mathbb{R}^{k\times(d+1)}$ of integer numbers in the range [0, d], which denote a coloured scheme:

$$M = \begin{bmatrix} \chi_{0,0} & \chi_{0,1} & \cdots & \chi_{0,d} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{k-1,0} & \chi_{k-1,1} & \cdots & \chi_{k-1,d} \end{bmatrix}. \quad (4)$$

Let $[\chi_{0,j}, \cdots, \chi_{k-1,j}]^{\mathrm{T}}$ be the column of M, then the j-th vertex of the subsimplex can be computed with the following equation:

$$P_{\chi_{0,j},\cdots,\chi_{k-1,j}} = \frac{1}{k}(P_{\chi_{0,j}} + P_{\chi_{1,j}} + \cdots + P_{\chi_{k-1,j}}). \quad (5)$$

Reference [22] only proposed the low dimensional case for conceiving coloured scheme matrices M, like $k = 2$, $d = 2$. Actually, these low dimensional ones are easy to imagine by a geometric method. When the dimension increases (for those case $k > 2$, $d > 3$), how to lead to a mechanical description of the subdivision amenable to computer implementation becomes an interesting and meaningful task. In order to implement the simplex edgewise subdivision, a new kind of algorithm for computing the coloured schemes will be developed as follows:

Algorithm Coloured Scheme
$j = 1$;
for $i_1 = 1 : k$ do
for $i_2 = 1 : k$ do
$\vdots$
for $i_d = 1 : k$ do
$G(j, 1 : d) = [i_1, i_2, \cdots, i_d]$; $j = j + 1$;
end for
$\vdots$
end for
end for
end **algorithm**

It is easy to see that $G \in \mathbb{R}^{k^d \times d}$ corresponds to k^d coloured scheme matrices M, i.e., once we get the matrix G, the k^d coloured scheme matrices M will be obtained by using the information on one of the rows of matrix G, respectively.

To clarify the application of the coloured schemes to simplex edgewise subdivision, consider the case of $k = 2$, $d = 2$ with the set of vertices $\{P_0 = [0, 0]^{\mathrm{T}}$, $P_1 = [1, 0]^{\mathrm{T}}$, $P_2 = [0, 1]^{\mathrm{T}}\}$ as presented in Fig. 1 and use the above algorithm; the matrix G is obtained as: $G = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{bmatrix}$. Then those $k^d = 2^2 = 4$ matrices M can be obtained as follows:

$$M_0 = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}.$$

Considering each column of these coloured schemes and Eq. (5), we find that the four subsimplices σ_i have the following sets of vertices:

$$\sigma_0 = \left\{ P_{02} = \frac{1}{2}(P_0 + P_2) = [0, 0.5]^{\mathrm{T}},\ P_{12} = \frac{1}{2}(P_1 + P_2) = [0.5, 0.5]^{\mathrm{T}},\ P_{22} = \frac{1}{2}(P_2 + P_2) = [0, 1]^{\mathrm{T}} \right\},$$

$$\sigma_1 = \left\{ P_{01} = \frac{1}{2}(P_0 + P_1) = [0.5, 0]^{\mathrm{T}},\ P_{11} = \frac{1}{2}(P_1 + P_1) = [0.5, 0]^{\mathrm{T}},\ P_{12} = \frac{1}{2}(P_1 + P_2) = [0.5, 0.5]^{\mathrm{T}} \right\},$$

$$\sigma_2 = \left\{ P_{01} = \frac{1}{2}(P_0 + P_1) = [0.5, 0]^{\mathrm{T}},\ P_{02} = \frac{1}{2}(P_0 + P_2) = [0, 0.5]^{\mathrm{T}},\ P_{12} = \frac{1}{2}(P_1 + P_2) = [0.5, 0.5]^{\mathrm{T}} \right\},$$

$$\sigma_3 = \left\{ P_{00} = \frac{1}{2}(P_0 + P_0) = [0, 0]^{\mathrm{T}},\ P_{01} = \frac{1}{2}(P_0 + P_1) = [0.5, 0]^{\mathrm{T}},\ P_{02} = \frac{1}{2}(P_0 + P_2) = [0, 0.5]^{\mathrm{T}} \right\}.$$

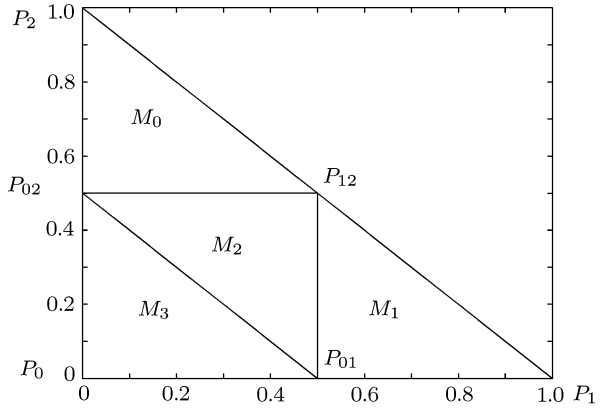


Fig. 1. Edgewise subdivision of a triangle.

Recalling Eq. (3) and using the above new algorithm, we obtain the switching fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{k^{q-1}} \sum_{j=1}^{q} \lambda_i(z(t))h_j(z(t))((A_j + \Delta A_j)x(t) + (B_j + \Delta B_j)u(t)), \tag{6}$$

where λ_i is a function satisfying:

$$\lambda_i(z(t)) = \begin{cases} 1, \ z(t) \in \sigma_i, \\ 0, \ \text{otherwise}, \end{cases}, \quad \sum_{i=1}^{k^{q-1}} \lambda_i(z(t)) = 1.$$

σ_i $(i = 1, 2, \ldots, k^{q-1})$ denotes the i-th subsystem developed by the above edgewise subdivision algorithm.

Remark 1 The partition of the overall TS fuzzy chaotic systems (3) is applied here to reduce the conservatism of the LMI-based sufficient conditions. As the LMI-based conditions are necessary and sufficient for linearizing certain systems, the proposed procedure asymptotically approaches these necessary and sufficient conditions, in the sense that the examined overall fuzzy model examined becomes increasingly smaller.

3.2. Convergent stabilization conditions based on the above edgewise subdivision algorithm

Based on the new edgewise subdivision algorithm provided in the above subsection, we will propose a novel kind of state feedback controller which is switched by the transfer of two different subsystems:

$$u = \sum_{i=1}^{k^{q-1}} \lambda_i(z(t))K_i x, \tag{7}$$

where $\lambda_i(z(t))$ has been defined in Eq. (6).

Based on Eqs. (6) and (7), the closed-loop fuzzy chaotic systems can be represented as

$$\dot{x}(t) = \sum_{i=1}^{k^{q-1}} \sum_{j}^{q} \lambda_i(z(t))h_j(z(t)) \times (\bar{A}_j x(t) + \bar{B}_j K_i x(t)), \tag{8}$$

where $\bar{A}_j = A_j + \Delta A_j$ and $\bar{B}_j = B_j + \Delta B_j$.

Remark 2 Unlike the usual parallel distributed compensation (PDC) scheme[11] and the piecewise controller,[8] there are k^{q-1} controlled gain matrices K_i to be designed while the value of k can be chosen arbitrarily. Hence it has a chance to further reduce the conservatism while the value of k increases, and the convergent stabilization condition will be attained when k is sufficiently large. This fact will also be illustrated in Section 4.

Based on the above analysis, the main theorem is developed as follows:

Theorem 1 The uncertain TS fuzzy chaotic model (3) can be stabilized with the state feedback controller (7), if there exist symmetric and positive matrix X, and matrices F_i $(i = 1, 2, \ldots, k^{q-1})$, positive scalars $\varepsilon^i_{m,j}, \eta^i_{m,j}$ $(i = 1, 2, \ldots, k^{q-1}, m = 0, 1, \ldots, k-1, j = 1, 2, \ldots, q)$, for k^{q-1} coloured scheme matrices which could be obtained by the new edgewise subdivision algorithm proposed in the above subsection,

$$M_i = \begin{bmatrix} \chi^i_{0,0} & \chi^i_{0,1} & \cdots & \chi^i_{0,q-1} \\ \vdots & \vdots & \ddots & \vdots \\ \chi^i_{k-1,0} & \chi^i_{k-1,1} & \cdots & \chi^i_{k-1,q-1} \end{bmatrix}, \tag{9}$$

such that the following LMIs hold:

$$\begin{bmatrix} \Xi^{11}_{ji} & \Xi^{12}_{ji} & \Xi^{13}_{ji} & \cdots & \Xi^{1(2k)}_{ji} & \Xi^{1(2k+1)}_{ji} \\ * & \Xi^{22}_{ji} & 0 & \ddots & 0 & 0 \\ * & * & \Xi^{33}_{ji} & \ddots & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \Xi^{(2k)(2k)}_{ji} & 0 \\ * & * & * & * & * & \Xi^{(2k+1)(2k+1)}_{ji} \end{bmatrix} < 0,$$
$$i = 1, 2, \ldots, k^{q-1}, \ j = 1, \ldots, q, \tag{10}$$

where

$$\Xi_{ji}^{11} = \sum_{m=0}^{k-1}(R^i_{\chi^i_{m,j-1}+1} + \varepsilon^i_{m,j} D_{\chi^i_{m,j-1}+1} D^{\mathrm{T}}_{\chi^i_{m,j-1}+1} + \eta^i_{m,j} D_{\chi^i_{m,j-1}+1} D^{\mathrm{T}}_{\chi^i_{m,j-1}+1},$$
$$R^i_{\chi^i_{m,j-1}+1} = A_{\chi^i_{m,j-1}+1} X + B_{\chi^i_{m,j-1}+1} F_i + (A_{\chi^i_{m,j-1}+1} X + B_{\chi^i_{m,j-1}+1} F_i)^{\mathrm{T}},$$
$$\Xi_{ji}^{12} = X E^{\mathrm{T}}_{1\chi^i_{0,j-1}+1}, \quad \Xi_{ji}^{22} = -\varepsilon^i_{0,j} I, \quad \Xi_{ji}^{13} = (E_{2\chi^i_{0,j-1}+1} F_i)^{\mathrm{T}}, \quad \Xi_{ji}^{33} = -\eta^i_{0,j} I,$$
$$\Xi_{ji}^{1(2k)} = X E^{\mathrm{T}}_{1\chi^i_{k-1,j-1}+1}, \quad \Xi_{ji}^{(2k)(2k)} = -\varepsilon^i_{k-1,j} I,$$
$$\Xi_{ji}^{1(2k+1)} = (E_{2\chi^i_{k-1,j-1}+1} F_i)^{\mathrm{T}}, \quad \Xi_{ji}^{(2k+1)(2k+1)} = -\eta^i_{k-1,j} I.$$

Moreover, the controller gain matrix could be calculated as $K_i = F_i X^{-1}$ $(i = 1, 2, \ldots, k^{q-1})$, and the conservatism will further be reduced as the value of k increases.

Proof Now, choose the Lyapunov function as

$$V(x(t)) = x^{\mathrm{T}}(t) P x(t), \tag{11}$$

where $P > 0$ is to be determined.

The time derivative of $V(x(t))$ along the trajectory of Eq. (8) is given as

$$\begin{aligned} \dot{V}(x(t)) &= 2x^{\mathrm{T}}(t) P \dot{x}(t) \\ &= x^{\mathrm{T}}(t) \sum_{i=1}^{k^{q-1}} \sum_{j=1}^{q} \lambda_i h_j \\ &\quad \times (P(\bar{A}_j + \bar{B}_j K_i) + *) x(t), \end{aligned} \tag{12}$$

where $*$ denotes the transpose of the corresponding term.

If $z(t) \in \sigma_i$ $(i = 1, 2, \ldots, k^{q-1})$, we have

$$\begin{aligned} \dot{V}(x(t)) &= x^{\mathrm{T}}(t) \sum_{j=1}^{q} h_j(z(t)) \\ &\quad \times \left(P(\bar{A}_j + \bar{B}_j K_i) + *\right) x(t), \end{aligned} \tag{13}$$

and the characteristic is achieved by the i-th coloured scheme matrix M_i.

On the other hand, the i-th coloured scheme matrix M_i can be obtained by the new algorithm proposed in the above subsection:

$$M_i = \begin{bmatrix} \chi^i_{0,0} & \chi^i_{0,1} & \cdots & \chi^i_{0,q-1} \\ \vdots & \vdots & \ddots & \vdots \\ \chi^i_{k-1,0} & \chi^i_{k-1,1} & \cdots & \chi^i_{k-1,q-1} \end{bmatrix}. \tag{14}$$

Pre- and post-multiplying the right side of Eq. (13) by P^{-1}, and using the following changes of variables: $X = P^{-1}$, $F_i = K_i X$, we find that $\dot{V}(x(t)) < 0$ holds if the following inequality holds:

$$\sum_{j=1}^{q} h_j(z(t)) Q^i_j < 0, \tag{15}$$

where $Q^i_j = \bar{A}_j X + \bar{B}_j F_i + (\bar{A}_j X + \bar{B}_j F_i)^{\mathrm{T}}$.

It is easy to see that $\dot{V}(x(t)) < 0$ is equivalent to $\sum_{j=1}^{q} h_j(z(t)) Q^i_j < 0$ $(i = 1, 2, \ldots, k^{q-1})$. Combined with the characteristic provided by M_i, we can calculate the new q matrices

$$\tilde{Q}^i_{\chi^i_{0,j}, \cdots, \chi^i_{k-1,j}} = \frac{1}{k}(Q^i_{\chi^i_{0,j-1}+1} + Q^i_{\chi^i_{1,j-1}+1} + \cdots + Q^i_{\chi^i_{k-1,j-1}+1}), \quad j = 1, 2, \ldots, q. \tag{16}$$

Thus, if we have $\tilde{Q}^i_{\chi^i_{0,j}, \cdots, \chi^i_{k-1,j}} < 0$ $(j = 1, 2, \ldots, q)$, the i-th subsystem is stabilized by the i-th controller K_i. Moreover, the overall fuzzy chaotic system is stabilized by these k^{q-1} controllers if $\tilde{Q}^i_{\chi^i_{0,j}, \cdots, \chi^i_{k-1,j}} < 0$ $(i = 1, 2, \ldots, k^{q-1};\ j = 1, 2, \ldots, q)$ are satisfied.

Due to the presence of uncertainty in $\tilde{Q}^i_{\chi^i_{0,j}, \ldots, \chi^i_{k-1,j}}$, the inequality $\tilde{Q}^i_{\chi^i_{0,j}, \ldots, \chi^i_{k-1,j}} < 0$ is not a linear matrix inequality. To overcome this obstacle, the following technique will be used.

Using Lemma 1, we have

$$\begin{aligned} Q^i_{\chi^i_{m,j-1}+1} &= \bar{A}_{\chi^i_{m,j-1}+1} X + \bar{B}_{\chi^i_{m,j-1}+1} F_i + * \\ &\le A_{\chi^i_{m,j-1}+1} X + B_{\chi^i_{m,j-1}+1} F_i + * \\ &\quad + \varepsilon^i_{m,j} D_{\chi^i_{m,j-1}+1} D^{\mathrm{T}}_{\chi^i_{m,j-1}+1} + {\varepsilon^i_{m,j}}^{-1} X E^{\mathrm{T}}_{1\chi^i_{m,j-1}+1} E_{1\chi^i_{m,j-1}+1} X \\ &\quad + \eta^i_{m,j} D_{\chi^i_{m,j-1}+1} D^{\mathrm{T}}_{\chi^i_{m,j-1}+1} + {\eta^i_{m,j}}^{-1} (E_{2\chi^i_{m,j-1}+1} F_i)^{\mathrm{T}} E_{2\chi^i_{m,j-1}+1} F_i, \\ &\quad m = 0, 1, \ldots, k-1. \end{aligned} \tag{17}$$

Thus, it is easy to see that

$$k\tilde{Q}^i_{\chi^i_{0,j},\cdots,\chi^i_{k-1,j}} \leq \sum_{m=0}^{k-1}(A_{\chi^i_{m,j-1}+1}X + B_{\chi^i_{m,j-1}+1}F_i + * + \varepsilon^i_{m,j}D_{\chi^i_{m,j-1}+1}D^{\mathrm{T}}_{\chi^i_{m,j-1}+1} + {\varepsilon^i_{m,j}}^{-1}E^{\mathrm{T}}_{1\chi^i_{m,j-1}+1}E_{1\chi^i_{m,j-1}+1} + \eta^i_{m,j}D_{\chi^i_{m,j-1}+1}D^{\mathrm{T}}_{\chi^i_{m,j-1}+1} + {\eta^i_{m,j}}^{-1}(E_{2\chi^i_{m,j-1}+1}F_i)^{\mathrm{T}}E_{2\chi^i_{m,j-1}+1}F_i). \quad (18)$$

Applying the Schur complement lemma, we find that equation (18) is equivalent to Eq. (10); in other words, the underlying fuzzy chaotic systems could be stabilized by the state feedback controller Eq. (7) if those LMIs conditions (10) are satisfied. Moreover, more controlled gain matrices K_i $(i = 1, \ldots, k^{q-1})$ will be obtained when a larger value of k is selected. In other words, more information on the membership functions is considered and hence the conservatism will be gradually reduced as the value of k increases.

This completes the proof.

Remark 3 In the above theorem, convergent stabilization conditions are obtained by using the new edgewise subdivision algorithm. The obtained convergent conditions are in terms of LMIs which are easy to solve with the well-known Matlab LMI tools. Furthermore, the asymptotically necessary and sufficient condition will be obtained if we further divide the overall system by either increasing the value of k or subdividing the achieved subsystems. This fact will also be illustrated by numerical experiments.

4. Illustrative examples

In this section, to demonstrate the effectiveness of the proposed controller design scheme, we simulate the control of the Lorenz system with uncertainties. The control objective is to drive its chaotic trajectory to the origin. The Lorenz equations are as follows:

$$\begin{aligned}\dot{x}_1(t) &= -a(x_1(t) - x_2(t)),\\ \dot{x}_2(t) &= bx_1(t) - x_2(t) - x_1(t)x_3(t),\\ \dot{x}_3(t) &= x_1(t)x_2(t) - cx_3(t). \end{aligned} \quad (19)$$

Figure 2 shows the phase portrait of its attractor with system parameters $(a, b, c) = (10, 28, 8/3)$ from time step $t_0 = 0$ to $t_f = 20$, while the initial condition is $(x_1, x_2, x_3) = (10, -10, -10)$.

To construct a TS fuzzy model for the Lorenz system, the quadratic nonlinear terms x_1x_2 and $-x_1x_3$ in Eq. (1) must be expressed as linearly weighted sums of some linear functions and we can construct an exact TS fuzzy model of the system (19) as follows:[7]

Plant Rules:

Rule 1: IF $x_1(t)$ is $N_{\min}$, THEN $\dot{x}(t) = A_1x(t)$;

Rule 2: IF $x_1(t)$ is $N_{\max}$, THEN $\dot{x}(t) = A_2x(t)$;

where

$$A_1 = \begin{bmatrix} -a & a & 0 \\ b & -1 & -x_{\min} \\ 0 & x_{\min} & -c \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -a & a & 0 \\ b & -1 & -x_{\max} \\ 0 & x_{\max} & -c \end{bmatrix}.$$

Setting $[x_{\min},\ x_{\max}]$ as [–20, 30], then the membership functions are

$$N_{x_{\min}} = \frac{-x_1 + x_{\max}}{x_{\max} - x_{\min}}, \quad N_{x_{\max}} = \frac{x_1 - x_{\min}}{x_{\max} - x_{\min}}. \quad (20)$$

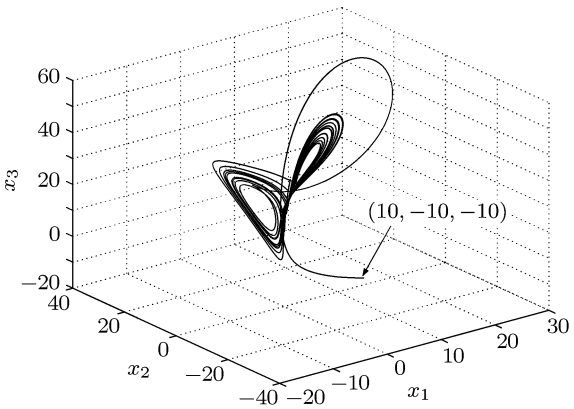


Fig. 2. Phase portrait of Lorenz chaotic attractor under initial condition $x(0) = (10, -10, -10)$.

Thus, the TS fuzzy model of the controlled system with an uncertain parameter value is:

Rule 1: IF $x_1(t)$ is $N_{\min}$,

THEN $\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u(t)$;

Rule 2: IF $x_1(t)$ is $N_{\max}$,

THEN $\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u(t)$.

The uncertainty matrices ΔA_i and ΔB_i are presented

as $\Delta A_i = D_i F_i(t) E_{1i}$ and $\Delta B_i = D_i F_i(t) E_{2i}, i = 1, 2$, where $D_1 = D_2 = \mathrm{diag}[-m, m, m]$,

$$E_{11} = E_{12} = \begin{bmatrix} a & -a & 0 \\ b & 0 & 0 \\ 0 & 0 & c \end{bmatrix}, \quad E_{12} = E_{22} = 0.$$

The system parameters a, b, c are randomly varied within $100m\%$ of their nominal values and the feasible interval of m could be calculated by solving LMI Eq. (10) with some fixed value of k.

To show the convergence characteristic of the obtained stabilization conditions, several feasible intervals about m will be calculated by using Theorem 1 with different values of k, which are listed in Table 1. From Table 1, it is easy to see that the conservatism is gradually reduced as the value of k increases.

Table 1. Feasible intervals of m with different values of k.

k	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
m	[–0.21, 0.21]	[–0.34, 0.34]	[–0.42, 0.42]	[–0.46, 0.46]	[–0.47 ,0.47]

Next, choosing $k = 6$, $m = 0.4$ and using the algorithm provided in Section 3, we can calculate the following $k^{q-1} = 6^{2-1} = 6$ coloured scheme matrices:

$$M_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$M_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad M_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus, by using Theorem 1 the controller gain matrices K_i, $(i = 1, 2, 3, 4, 5, 6)$ can be obtained

$$K_1 = \begin{bmatrix} 9.7547 & 2.5290 & -473.4642 \\ -40.5290 & 0.7544 & 32.9347 \\ 473.4642 & -32.9347 & 2.4210 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 9.8270 & 277.5778 & -50.1571 \\ -315.5779 & 0.8267 & -184.2423 \\ 50.1571 & 184.2423 & 2.4934 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 9.7840 & -153.2789 & -59.0809 \\ 115.2788 & 0.7837 & -118.7344 \\ 59.0809 & 118.7344 & 2.4503 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 9.7840 & -50.5149 & -24.0461 \\ 12.5149 & 0.7837 & -27.3951 \\ 24.0461 & 27.3951 & 2.4503 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 9.8270 & -29.4298 & 199.2488 \\ -8.5702 & 0.8267 & -4.2944 \\ -199.2488 & 4.2944 & 2.4934 \end{bmatrix},$$

$$K_6 = \begin{bmatrix} 9.7547 & -18.9420 & 3.0950 \\ -19.0580 & 0.7544 & 0.1577 \\ -3.0950 & -0.1577 & 2.4210 \end{bmatrix}.$$

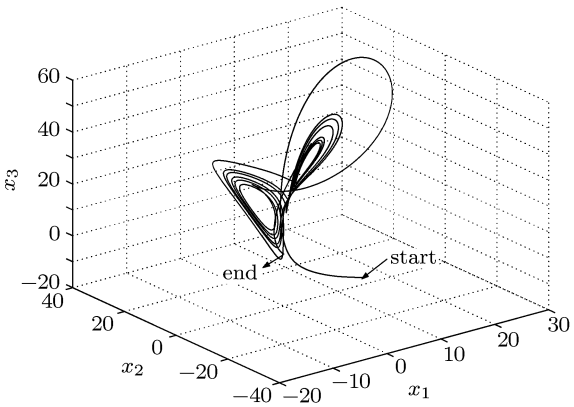


Fig. 3. The phase trajectory of the Lorenz system with uncertainties via switching controller (7).

The initial values are $x(0) = (-10, -10, -10)$. The simulation time is 20 s. Using these obtained controlled gain matrices, we show the controlled result in Fig. 3. The controlled input is activated at $t = 15$ s for comparison. Before the controlled input is added, the phase trajectory of the Lorenz system is chaotic. However, after the controlled input is activated, the chaotic state is quickly directed to the origin.

5. Conclusions

In this work, convergent stabilization conditions are proposed based on the edgewise subdivision approach. The overall chaotic fuzzy space is divided into k^{q-1} subspaces via a new edgewise subdivision algorithm. Then, a novel controlled law which switches with the transfer between two different subsystems is proposed to stabilize the underlying chaotic systems. Finally, the effectiveness of the proposed results is demonstrated by numerical simulation. Since applications of chaos have enormous potential in many fields, such as life systems, biological systems, medical systems, etc., the method shown in this paper will be valuable in future theoretical research[23−25] and engineering applications.[26]

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