Traffic dynamics of an on-ramp system with a cellular automaton model*

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This paper uses the cellular automaton model to study the dynamics of traffic flow around an on-ramp with an acceleration lane. It adopts a parameter, which can reflect different lane-changing behaviour, to represent the diversity of driving behaviour. The refined cellular automaton model is used to describe the lower acceleration rate of a vehicle. The phase diagram and the capacity of the on-ramp system are investigated. The simulation results show that in the single cell model, the capacity of the on-ramp system will stay at the highest flow of a one lane system when the driver is moderate and careful; it will be reduced when the driver is aggressive. In the refined cellular automaton model, the capacity is always reduced even when the driver is careful. It proposes that the capacity drop of the on-ramp system is caused by aggressive lane-changing behaviour and lower acceleration rate.

Keywords: lane-changing behaviour, acceleration rate, cellular automaton model, on-ramp system
PACC: 0520, 0550

1. Introduction

In recent years, much attention has been paid to modelling the dynamics of traffic flow.[1−14] Many theoretical models have been proposed to explain the evolution mechanism of traffic flow. They can be conceptually classified into macroscopic and microscopic models. In the so-called microscopic vehicular traffic flow models, individual vehicles interact with each other and the properties of traffic are reflected by the collecting behaviours of those vehicles. Car-following models and cellular automaton (CA) models are the two main kinds of microscopic models. Due to efficient and fast performance in computer simulation, the CA model has become an excellent tool for simulating real traffic flow. In 1992, Nagel and Schreckenberg proposed the well-known Nagel–Schareckenberg (NaSch) model.[15] Although it is very simple, the NaSch model can reproduce some real traffic phenomena, such as the occurrence of phantom traffic jams and the realistic flow-density relation (fundamental diagram). Later, various generalizations and extensions of the NaSch model were proposed to simulate the behaviour of traffic flow.[16−18] Nowadays, realistic driving behaviours are introduced into the NaSch model, then complex traffic behaviours are simulated.[19−23]

A traffic bottleneck is a section of road with a carrying capacity substantially below that characterizing other sections of the same road. So capacity drop is a significant property of bottlenecks. Along with capacity drop, complex traffic flow patterns also emerge, such as synchronized flow. As an important factor in real traffic, bottlenecks have attracted the interest of many researchers. Bottlenecks include on-ramps, off-ramps, lane closings, uphill gradients, narrow road sections, etc. Among the various types of bottlenecks, the on-ramp and the off-ramp have been widely studied by using CA models.[24−32] For the on-ramp system, there are several methods to describe the behaviour of a vehicle entering a main road from an on-ramp. Some previous work[24,33] implied a simple manner, where vehicles are inserted directly onto the highway with a frequency. But those models are not very realistic because the ramps themselves are not modelled. In order to describe the on-ramp system more realistically, Jia *et al.*[27] established the CA

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model for the on-ramp system and the acceleration lane is adopted. As the vehicles from the on-ramp lane have to change from the acceleration lane to the main road, appropriate lane-changing rules are needed to model the realistic traffic behaviours at the on-ramp. In this paper, a parameter is introduced to represent the different lane-changing behaviours of drivers and the traffic dynamics are analysed in detail.

In order to simulate more realistic traffic behaviours, finer discretization in space, that is to say a vehicle is allowed to span a number of consecutive cells in the longitudinal direction, is considered in the CA models for traffic flow.[34−36] Here we call these models refined CA models. In the refined CA models, the acceleration rate is low because the cell has a small length. In addition, more realistic driving behaviours can be well described, such as slow-to-start, velocity adoption, expectation, etc. Complex traffic flow patterns, typically synchronized flow, can be reproduced. But the dynamics of traffic flow at an on-ramp have not been systematically studied with the refined CA model. This is also what we are going to do in this paper.

In Section 2, a model for an on-ramp system is proposed. In Section 3, the simulation results are analysed in detail. Lastly, the conclusion is given.

2. Model

The on-ramp system with an accelerating lane has been well studied by using a single-cell model in our previous work.[27] The schematic diagram of the on-ramp system is shown in Fig. 1. One can see that the road is divided into four sections: region A (main road upstream of point M), B (on-ramp upstream of point M), C (main road downstream of point N) and D (the accelerating section). The single lane refined NaSch model is used to model the movement of a vehicle. In section D, the road has two lanes, thus some additional lane-changing rules are needed to simulate the merging behaviours of the vehicle in the right lane.

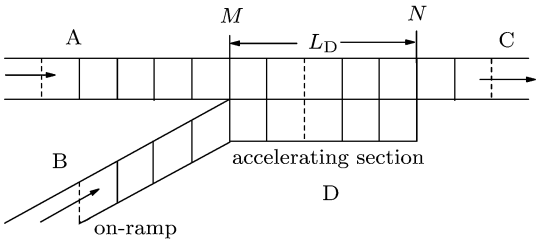


Fig. 1. Schematic illustration of the on-ramp system.

Before going to the consideration of lane-changing rules in section D, we first introduce the CA model for vehicle movement. In a cellular automaton traffic flow model, the road is divided into L cells, and a vehicle has a length of l cell(s). It is usually assumed that the length of a vehicle is 7.5 m, then the length of a cell corresponds to $7.5/l$ m. So as l increases, the space discretization is finer. Furthermore, the acceleration rate is usually set as 1 cell/s^2, thus the value of l also determines the acceleration rate. In the original NaSch model, $l = 1$ is selected. Here the refined NaSch model, in which l can be larger than 1, is used to describe the forward movement of vehicles. Next, we briefly recall the updating rules of the refined NaSch model. There are 4 sub-steps: (i) acceleration: $v_n(t + 1/3) \to \min(v_n(t) + 1, v_{\max})$; (ii) deceleration: $v_n(t+2/3) \to \min(v_n(t+1/3), d_n)$; (iii) randomization: $v_n(t + 1) \to \max(v_n(t + 2/3) - 1, 0)$ with probability p; (iv) position update: $x_n(t+1) \to x_n(t) + v_n(t+1)$. Here $v_n(t)$ and $x_n(t)$ denote the velocity and position of the vehicle n respectively; $v_{\max}$ is the maximum velocity; $d_n = x_{n+1}(t) - x_n(t) - l$ denotes the number of empty cells in front of the vehicle n, and l is the length of the vehicle, i.e., the number of cells taken up by a vehicle; p is the randomization probability. In the original NaSch model, $l = 1$ is selected, while in the refined NaSch model, $l > 1$.

The above rules control the forward motion of vehicles. In the case of two-lane traffic one has to introduce an additional set of lane-changing rules. In section D, the vehicle in the right lane must change to the left lane before it reaches the end of section D, and the vehicle in the left lane is not allowed to change to the right lane. Unlike the lane-changing rules in Ref. [27], a simpler rule is adopted in this paper. If the condition

$$d_{n,\text{front}} \geq l \quad \text{and} \quad d_{n,\text{back}} \geq \max(0, v_{\text{ob}} + kl) \quad (1)$$

is met, the lane-changing is performed by vehicle n. Here $d_{n,\text{front}}$ ($d_{n,\text{back}}$) denotes the gap to the front (back) vehicle in the destination lane at time t. Similarly, with d_n, $d_{n,\text{front}} = x_{n,\text{front}} - x_n - l$ and $d_{n,\text{back}} = x_n - x_{n,\text{back}} - l$, where $x_{n,\text{front}}$ and $x_{n,\text{back}}$ are the positions of the front and back vehicles on the destination lane. We should point out that the vehicle in the destination lane driving side by side with vehicle n is deemed to be the back vehicle of vehicle n. The v_{ob} represents the velocity of the following vehicle in the destination lane at time t. Condition $d_{n,\text{front}} \geq l$ means "I can move into the destination lane in the next time step"; condition $d_{n,\text{back}} \geq \max(0, v_{\text{ob}} + kl)$

indicates that the gap to the back vehicle is acceptable, and k is an integer. Higher values of k mean that the driver needs a larger gap to complete the lane change, that is to say, he is more careful. Three values of k are selected to reflect different lane-changing behaviours. $k = -1$ denotes that the driver is aggressive. The aggressive driver believes that the following vehicle will decelerate by l cells when the lane-changing is performing. $k = 0$ indicates the driver is moderate. The moderate driver believes that the following vehicle will maintain speed when the lane-changing is performing. $k = 1$ means the driver is careful. The careful driver thinks that the following vehicle may accelerate by l cells when the lane-changing is performing. According to the above analysis, we can obtain that the minimum acceptable gaps to the back vehicle are $\max(0, v_{\rm ob} - l)$, $v_{\rm ob}$ and $v_{\rm ob} + l$ for aggressive, moderate and careful drivers, respectively.

The boundary conditions are adopted as follows. Assume that the first cells on sections A and B correspond to x=1, and the entrance regions of roads A and B include $v_{\max}$ cells, i.e., the vehicles can enter roads A and B from the cells $1, 2, \ldots, v_{\max}$. In one time step, when the update of the vehicles on the road is completed, the positions of the last vehicles on roads A and B and that of the first vehicle on road C, which are denoted as $x_{\rm A}^{\rm last}$, $x_{\rm B}^{\rm last}$ and $x_{\rm C}^{\rm first}$, respectively, are checked. If $x_{\rm A}^{\rm last}$ ($x_{\rm B}^{\rm last}$) $> v_{\max}$, a vehicle with velocity $v_{\max}$ is injected with probability $\alpha_{\rm A}$ ($\alpha_{\rm B}$) at the cell $\min[x_{\rm A}^{\rm last}(x_{\rm B}^{\rm last}) - v_{\max},\ v_{\max}]$. Near the exit of road C, the leading vehicle is removed if $x_{\rm C}^{\rm last}$ is larger than the length of lane C and the following vehicle becomes the new leading vehicle and it moves without any hindrance.

3. Simulations and discussions

In this section, the simulation results are presented. In the simulations, section C is divided into $(1000 \times l)$ cells, section D into $L_D \times l$ cells, sections A and B into $(2000 \times l)$ cells. The length of a vehicle corresponds to 7.5 m. One time step corresponds to 1 s. The model parameters $v_{\max} = 5 \times l$, $p = 0.1$, $L_{\rm D} = 20$ are used. The first 50000 time steps are discarded to let the transient time die out. The flux is obtained by counting the vehicles which pass a virtual detector in 50000 time steps. The fluxes on roads A, B and C are $q_{\rm A}$, $q_{\rm B}$ and $q_{\rm C}$, respectively.

3.1. Effect of lane-changing behaviour

First, the effects of lane-changing behaviours on the traffic flow are investigated, and the single cell model with $l = 1$ is used. In Fig. 2 a phase diagram in $(\alpha_{\rm A}, \alpha_{\rm B})$ space with different k is shown. Consistent with the results in Ref. [27], one can see that the phase diagram is categorized into four regions. In region I, the traffic flows on both roads A and B are free flows; in region II, the traffic is free on road A and it is congested on road B; in region III, the traffic is congested on road A and is free on road B; and in region IV, the traffic flows are congested on both roads A and B.

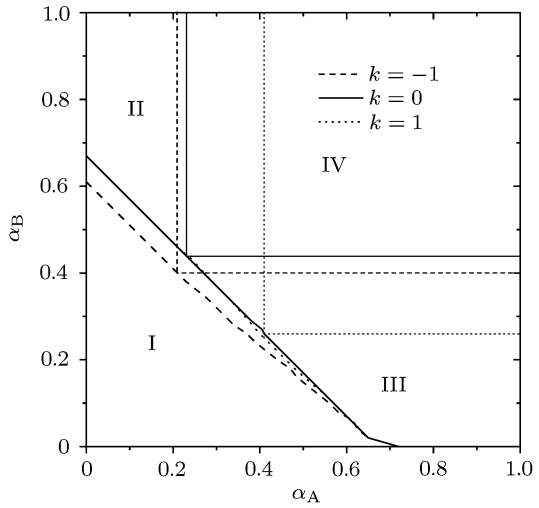


Fig. 2. The phase diagrams in ($\alpha_{\rm A}$, $\alpha_{\rm B}$) space of the on-ramp system with different k. Here $l = 1$ is used.

Compared with the case of $k = 0$, regions I, II and III shrink while region IV enlarges in the case of $k = -1$, but region I almost does not change, region II enlarges and region III shrinks in the case of $k = 1$. Such results indicate that the traffic condition on roads A and B is deteriorated by aggressive lane-changing behaviour. So aggressive lane-changing behaviour cannot make the vehicle from the ramp lane drive faster. If the driver is careful, the driving condition on road A is improved and high flux can be obtained, although the flux on road B is slightly reduced.

Figure 3 shows the flux on each road as a function of $\alpha_{\rm A}$ when $\alpha_{\rm B} = 0.3$. One can see that $q_{\rm A}$ and q_B first increase linearly with $\alpha_{\rm A}$, which indicates that the traffic on roads A and B is free. After a critical value $\alpha_{\rm A}^{\rm c}$ is reached, $q_{\rm A}$ and $q_{\rm B}$ will not change as $\alpha_{\rm A}$ increases, thus the traffic is saturated. Since $q_{\rm C} = q_{\rm A} + q_{\rm B}$, the saturated flux $q_{\rm C}^{\rm s}$ on road C is deemed to be the capacity of the on-ramp system. $q_{\rm C}^{\rm s}$

in the cases of $k = 0$ and $k = 1$ is equal to $q_{\max}$, which is the maximum flux of the single lane system, and larger than that in the case of $k = -1$. Aggressive lane-changing behaviour depresses the capacity of the on-ramp system. In reality, driving behaviour is diverse, so there must be aggressive lane-changing and the capacity drop phenomenon happens. The saturated flux on road A $q_{\rm A}^{\rm s}$ increases as the lane-changing behaviour becomes more careful. In the cases of $k = -1$ and $k = 0$, $q_{\rm B}$ does not change as $\alpha_{\rm A}$ increases, because the traffic on road B is free when that on road C is saturated. But in the case of $k = 1$, $q_{\rm B}$ decreases to a low value when the traffic on road C is saturated. This indicates that the traffic on road B is congested. This is consistent with the results in the phase diagram (Fig. 2).

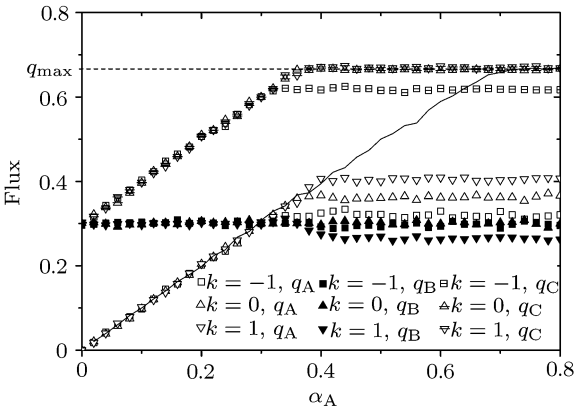


Fig. 3. The flux on each road as a function of $\alpha_{\rm A}$ when $\alpha_{\rm B} = 0.3$. The solid line represents $q_{\rm A}$ in the case of $\alpha_{\rm B} = 0.0$.

Figure 4 shows the flux on each road as a function of $\alpha_{\rm B}$ when $\alpha_{\rm A} = 0.5$. When $\alpha_{\rm B}$ is not very large, $q_{\rm B}$ and $q_{\rm C}$ increase linearly while $q_{\rm A}$ stays unchanged as $\alpha_{\rm B}$ grows. This corresponds to region I in the phase diagram, and the traffic on roads A and B is free. With further increase of $\alpha_{\rm B}$, $q_{\rm C}$ does not increase any longer; however, $q_{\rm A}$ decreases while $q_{\rm B}$ still increases linearly. This corresponds to region III in the phase diagram, and the traffic on road A is saturated, but that on road B is free. When $\alpha_{\rm B}$ is also large, the traffic on road B is also saturated. We note that in the saturated state, the saturated flux $q_{\rm A}^{\rm s}$ and $q_{\rm B}^{\rm s}$ in the case of $k = -1$ is lower than that in the case of $k = 0$. Aggressive lane-changing cannot increase the flux on road B but decreases it. As a result, $q_{\rm C}$ will decrease as $\alpha_{\rm B}$ grows if the lane-changing behaviour is aggressive and capacity drop happens. Figure 4 also shows that $q_{\rm A}^{\rm s}$ is lower than $q_{\rm B}^{\rm s}$ in the cases of $k = -1$ and $k = 0$, while the converse result is obtained in the case of $k = 1$. Careful lane-changing behaviour brings less disturbance to the traffic on road A.

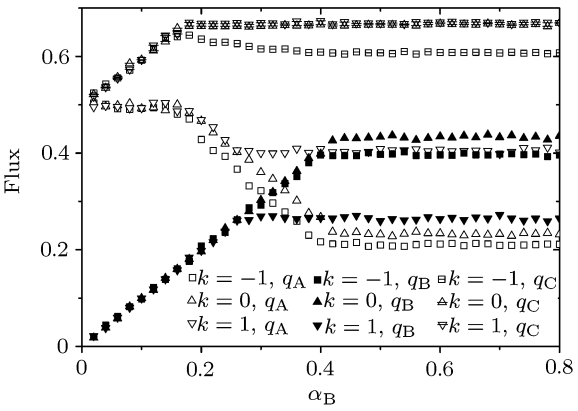


Fig. 4. The flux on each road as a function of $\alpha_{\rm B}$ when $\alpha_{\rm A} = 0.5$.

3.2. Effect of lower acceleration rate

Now, we study the effect of lower acceleration rate and fix the value of $k = 0$. In Fig. 5 a phase diagram in $(\alpha_{\rm A}, \alpha_{\rm B})$ space with different l is shown. With the increase of l, regions I and III shrink apparently while regions II and IV are enlarged. So with finer discretization in space, the traffic on road A is improved while that on road B is deteriorated. It is harder for the vehicle in the right lane within section D to change to the left lane when $l > 2$. The lane-changing

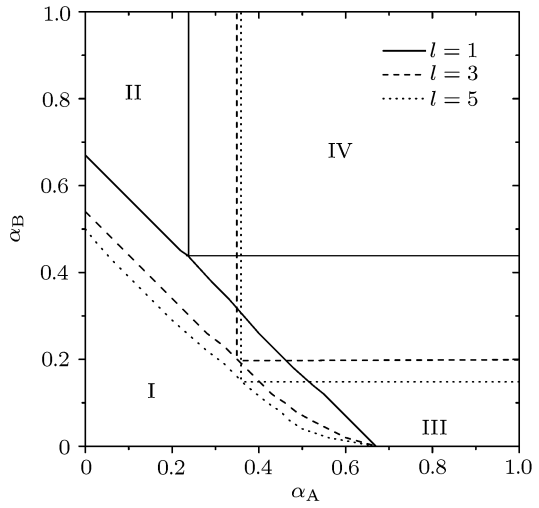


Fig. 5. The phase diagrams in ($\alpha_{\rm A}$, $\alpha_{\rm B}$) space of the on-ramp system with different l. Here $k = 0$ is used.

behaviours in the cases of $l = 1$ and $l = 3$ are shown in Fig. 6. Although the initial configurations are the same at time step t, the vehicle B2 can easily change

to the left lane in the case of $l = 1$, while it cannot change to the left lane in the case of $l = 3$. One can see that at time step $t + 2$, the vehicle B2 reaches the point N, but $d_{n,\mathrm{back}}$ is not large enough to make a lane change in the case of $l = 3$.

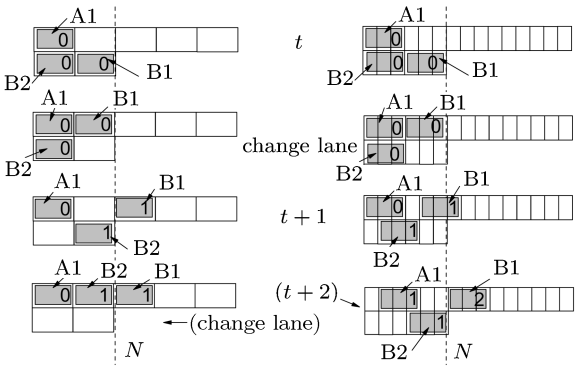


Fig. 6. Step-by-step example for the evolution process. The left part has $l = 1$ and the right part has $l = 3$. Because lane-changing does not happen at time step $t + 1$ in the case of $l = 3$, the configuration at $t + 2$ is plotted. The numbers denote the velocity of the vehicle at time step t. For simplicity, the velocity is updated under the deterministic case.

Figure 7 shows the flux on each road as a function of α_{A} when α_{B}=0.4. As l increases, the saturated flux of road C $q_{\mathrm{C}}^{\mathrm{s}}$ decreases. Thus the capacity of the on-ramp system will be reduced. In the case of $l = 1$, q_{B} will not change as α_{A} increases, because the traffic on road B is free as shown in Fig. 5. But in the cases of $l = 3$ and 5, q_{B} will decrease to saturated flux $q_{\mathrm{B}}^{\mathrm{s}}$ as α_{A} increases. The $q_{\mathrm{B}}^{\mathrm{s}}$ becomes smaller as l increases. So the traffic condition on road B is worsened. The q_{A} increases to saturated flux $q_{\mathrm{A}}^{\mathrm{s}}$ as α_{A} increases. We note that $q_{\mathrm{A}}^{\mathrm{s}}$ in the two cases of $l = 3$ and $l = 5$ has the same value and is larger than that in the case of $l = 1$. This indicates that the traffic condition of road A is improved when l is larger than 1, but it cannot be improved further when $l \geq 3$. We know that as l increases, the acceleration rate becomes small. Lane-changing usually happens at low speed. After lane-changing, the vehicle has to accelerate to high speed for driving as fast as possible. If the acceleration rate is low, more time steps are needed for the vehicle to get back to high speed. The flux of the on-ramp system is reduced. Thus the capacity drop phenomenon happens with larger l, i.e., lower acceleration rate.

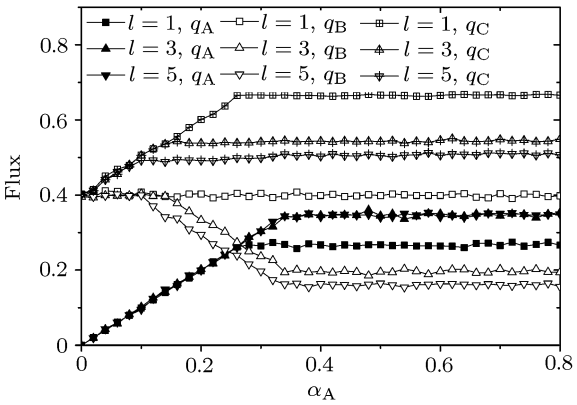


Fig. 7. The flux on each road as a function of α_{A} when $\alpha_{\mathrm{B}} = 0.4$. Here $k = 0$ is used.

Figure 8 shows the flux on each road as a function of α_{B} when $\alpha_{\mathrm{A}} = 0.4$. We also note that the saturated fluxes $q_{\mathrm{C}}^{\mathrm{s}}$ and $q_{\mathrm{B}}^{\mathrm{s}}$ decrease as l increases, while the saturated flux $q_{\mathrm{A}}^{\mathrm{s}}$ in the two cases of $l = 3$ and $l = 5$ has almost the same value and it is higher than that in the case of $l = 1$.

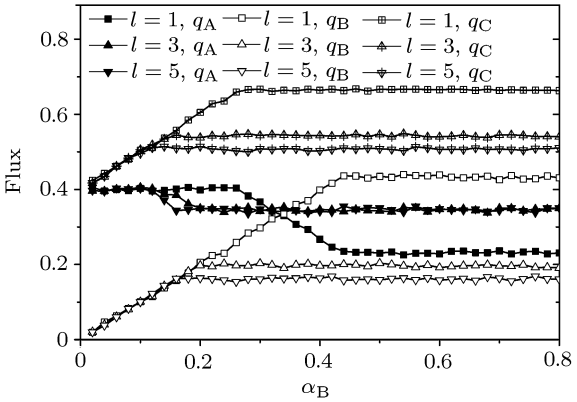


Fig. 8. The flux on each road as a function of α_{B} when $\alpha_{\mathrm{A}} = 0.4$. Here $k = 0$ is used.

With larger l, the acceleration rate becomes small and the capacity of the on-ramp system is reduced, even if there is no aggressive lane-changing behaviour.

4. Conclusion

In this paper, the traffic dynamics of the on-ramp system is investigated by a CA model. The phase diagram in $(\alpha_{\mathrm{A}}, \alpha_{\mathrm{B}})$ space and the flux on each road as a function of α_{A} and α_{B} are analysed in detail.

With aggressive lane-changing behaviour, the traffic conditions of not only road A but also road B are deteriorated. With more careful lane-changing behaviour, the traffic condition of road A is improved while that of road B is worsened. The capacity of

the on-ramp system is reduced by aggressive lane-changing behaviour.

As l becomes larger, the acceleration rate decreases, and the lane-changing ability of the vehicle from road B to road A is restricted. Thus the traffic condition of road A can be slightly improved. At the same time, the traffic condition of road B is seriously deteriorated. In all, the flux of the on-ramp system is reduced.

According to the simulation results, we can conclude that both aggressive lane-changing behaviour and a lower acceleration rate can cause capacity drop. We know that in realistic traffic, aggressive lane-changing behaviour commonly exists, and some vehicles have lower acceleration rates. So the capacity drop phenomenon widely happens at on-ramp bottlenecks.

In our future work, the effects of aggressive lane-changing behaviour and lower acceleration rate on traffic flow dynamics will be studied in other traffic conditions. We are going to explore some measures to restrain aggressive lane-changing behaviour. Then the capacity of the on-ramp can be improved.

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