Quasinormal modes of the scalar field in five-dimensional Lovelock black hole spacetime*

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(Received 8 August 2009; revised manuscript received 21 August 2009)

In this paper, using the third-order WKB approximation, we investigate the quasinormal frequencies of the scalar field in the background of a five-dimensional Lovelock black hole. We find that the ultraviolet correction to Einstein theory in the Lovelock theory makes the scalar field decay more slowly and oscillate more quickly, and the cosmological constant makes the scalar field decay more slowly and oscillate more slowly in the Lovelock black hole background.

Keywords: quasinormal modes, third-order WKB approximation, Lovelock black hole
PACC: 0420, 0490

1. Introduction

The quasinormal modes,[1] depending only on black hole parameters, are of great importance in gravitational-wave astrophysics, and might be observed in existing or advanced gravitational-wave detectors. Furthermore, black holes are often used as a testing ground for ideas in quantum gravity, and their quasinormal modes are obvious candidates for an interpretation in terms of quantum levels.[2] Because it is so important for black hole physics and gravitational-wave astrophysics, there are a lot of authors who have focused on the quasinormal modes of matter fields in different black hole backgrounds in the past decade, such as quasinormal modes of black holes in anti-de Sitter space,[3−6] Dirac field quasinormal modes[7−10] and scalar field quasinormal modes[11−17] in different backgrounds. Recently some scholars have investigated the effects of dark energy and dark matter on quasinormal modes[18−31] and some have extended the investigation of quasinormal modes to higher dimensional spacetimes.[32−36]

Lovelock[37,38] extended the Einstein tensor, which is the only symmetric and conserved tensor depending on the metric and its derivatives up to the second order, to the most general tensor. The obtained tensor is nonlinear in the Riemann tensor and differs from the Einstein tensor only if the spacetime has more than 4 dimensions. Therefore, the Lovelock theory is the most natural extension of general relativity in higher dimensional spacetimes. On the other hand, Lovelock theory also resembles string inspired models of gravity as its action contains, among others, the quadratic Gauss–Bonnet term, which is the dimensionally extended version of the four-dimensional Euler density. This quadratic term is presented in the low energy effective action of heterotic string theory.[39] The Lovelock theory represents a very interesting scenario to study how the physics of gravity results are corrected at short distance due to the presence of higher order curvature terms in the action. C. Garraffo *et al.*[40−42] gave a black hole solutions of this theory, and discussed how short distance corrections to black hole physics substantially change the qualitative features. M. Aiello *et al.*[43] presented the exact five-dimensional charged black hole solution in Lovelock gravity coupled to Born–Infeld electrodynamics. In their paper they also investigated thermodynamical properties of Lovelock black hole spacetime. Furthermore, M. H. Dehghani and R. Pourhasan[44] focused on the temperature of the uncharged black holes of the third order Lovelock gravity and the entropy through the use of first law of thermodynamics. They analysed thermodynamical stability and found that there exists an intermediate thermodynamically unstable phase for black holes with hyperbolic horizon. R. A. Konoplya *et al.*[45,46] presented an analysis of the scalar perturbations in the background of Bauss–Bonnet black hole spacetimes and its (in)stability in

*Project supported by the National Natural Science Foundation of China (Grant No. 10873004), the Key Project of the National Natural Science Foundation of China (Grant No. 10935013), the Scientific Research Fund of the Educational Department of Hunan Province of China (Grant No. 08B051), the Program for Excellent Talents in Hunan Normal University and the State Key Development Program for Basic Research Program of China (Grant No. 2010CB832803).

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high dimensions.[47]

The aim of this paper is to study the quasinormal mode of a scalar field in five-dimensional Lovelock black hole spacetime for different angular quantum numbers l using the third-order WKB approximation, a numerical method devised by Schutz, Will and Iyer.[48−50] The paper is organized as follows: In Section 2 we will give a brief review of Lovelock black hole spacetime in five dimensions. In Section 3 a detailed analysis on the quasinormal mode of a scalar field in Lovelock black hole spacetime in five-dimensions is performed. In the last section a brief conclusion is given.

2. Lovelock black hole spacetime in five dimensions

The Lovelock–Lagrangian density in D dimensions is[37,38]

$$L = \sum_{k=0}^{N} \alpha_k \lambda^{2(k-1)} L_k, \tag{1}$$

where $N = D/2 - 1$ (for even D) and $N = (D-1)/2$ (for odd D). In Eq. (1), α_k and λ are constants which represent the coupling of the terms in the whole Lagrangian and give the proper dimensions.

In Eq. (1) L_k is

$$L_k = \frac{1}{2^k}\sqrt{-g}\delta^{i_1\cdots i_{2k}}_{j_1\cdots j_{2k}} R^{j_1 j_2}_{i_1 i_2} \cdots R^{j_{2k-1} j_{2k}}_{i_{2k-1} i_{2k}}, \tag{2}$$

where $R^{\mu}{}_{\nu\rho\gamma}$ is the Riemann tensor in D dimensions, $R^{\mu\nu}{}_{\rho\sigma} = g^{\nu\delta} R^{\mu}{}_{\delta\rho\sigma}$, g is the determinant of the metric $g_{\mu\nu}$ and $\delta^{i_1\cdots i_{2k}}_{j_1\cdots j_{2k}}$ is the generalized Kronecker delta of order $2k$.[51]

The Lagrangians up to order 2 are given by[52,53]

$$L_0 = \sqrt{-g}, \tag{3}$$

$$L_1 = \frac{1}{2}\sqrt{-g}\delta^{i_1 i_2}_{j_1 j_2} R^{j_1 j_2}_{i_1 i_2} = \sqrt{-g}R, \tag{4}$$

$$\begin{aligned} L_2 &= \frac{1}{4}\sqrt{-g}\delta^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} R^{j_1 j_2}_{i_1 i_2} R^{j_3 j_4}_{i_3 i_4} \\ &= \sqrt{-g}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2), \end{aligned} \tag{5}$$

where we recognize the usual Lagrangian for the cosmological term, the Einstein–Hilbert Lagrangian and the Lanczos Lagrangian,[52,53] respectively.

For dimensions $D = 5$ and $D = 6$ the Lovelock–Lagrangian is a linear combination of the Einstein–Hilbert and Lanczos Lagrangians.

Hence, the geometric action is written as

$$S = \int L \mathrm{d}^D x. \tag{6}$$

In this paper we only consider the spacetime in five dimensions, so the Lagrangian is a linear combination of the Einstein–Hilbert and the Lanczos ones, and the Lovelock tensor results

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} \\ &\quad - \alpha \Big\{ \frac{1}{2} g_{\mu\nu} (R_{\rho\delta\gamma\lambda} R^{\rho\delta\gamma\lambda} - 4 R_{\rho\delta} R^{\rho\delta} + R^2) \\ &\quad - 2 R R_{\mu\nu} + 4 R_{\mu\rho} R^{\rho}_{\nu} + 4 R_{\rho\delta} R^{\rho\delta}_{\mu\nu} \\ &\quad - 2 R_{\mu\rho\delta\gamma} R^{\rho\delta\gamma}_{\nu} \Big\}. \end{aligned} \tag{7}$$

The five-dimensional Lovelock theory mainly corresponds to Einstein gravity coupled to the dimensional extension of four-dimensional Euler density, that is to say, the theory referred to as Einstein–Gauss–Bonnet theory. The spherically symmetric solution in five dimensions[54] takes the following form:

$$\mathrm{d}s^2 = -N(r)\mathrm{d}t^2 + N^{-1}(r)\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_3^2, \tag{8}$$

where $\mathrm{d}\Omega_3^2$ is the metric of a unitary 3-sphere, and

$$N(r) = \frac{4\alpha - 4M + 2r^2 - \Lambda r^2/3}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}}, \tag{9}$$

where M, Λ are ADM mass, cosmological constant, respectively, and α is the coupling constant of an additional term that presents the ultraviolet correction to Einstein theory.

3. Quasinormal mode of a scalar field in Lovelock black hole spacetime

The general perturbation equation for the massless scalar field ψ in the curve spacetime is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\psi = 0. \tag{10}$$

Introducing the variables

$$\psi = \frac{\mathrm{e}^{-\mathrm{i}\,\omega t}\Phi(r)}{r}\mathrm{Y}(\theta,\varphi,\phi),$$

where $\mathrm{Y}(\theta,\varphi,\phi)$ is the three-dimensional spherical harmonic function and the tortoise coordinate

$$r_* = \int \left(\frac{4\alpha - 4M + 2r^2 - \Lambda r^2/3}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}}\right)^{-1} \mathrm{d}r.$$

Substituting Eq. (9) into Eq. (10), we obtain a radial perturbation equation

$$\frac{\mathrm{d}^2\Phi(r)}{\mathrm{d}r_*^2} + (\omega^2 - V(r))\Phi(r) = 0, \tag{11}$$

where

$$V(r) = \frac{4\alpha - 4M + 2r^2 - \Lambda r^2/3}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}}\left[\frac{l(l+2)}{r^2} + \frac{3}{4r^2}\frac{4\alpha - 4M + 2r^2 - \Lambda r^2/3}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}} + \frac{1}{4\alpha}\left(3 - \frac{3 + 4\alpha M}{\sqrt{9r^4 + 12\alpha\Lambda r^4 + 144M\alpha}}r^2\right)\right]. \tag{12}$$

It is obvious that the effective potential V depends only on the value of r, angular quantum number l, ADM mass M, cosmological constant Λ and coupling constant α, respectively.

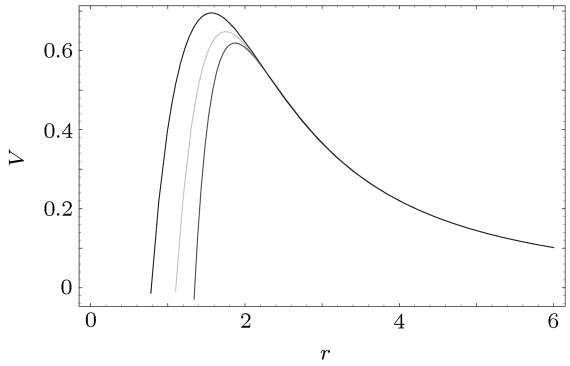


Fig. 1. The behaviour of the effective potential $V(r)$ vs. r for the Lovelock black hole by fixed parameters $l = 1$, $M = 1$, $\Lambda = 0.1$ and coupling constants $\alpha = 0.1$ (right), 0.4 (middle), 0.7 (left).

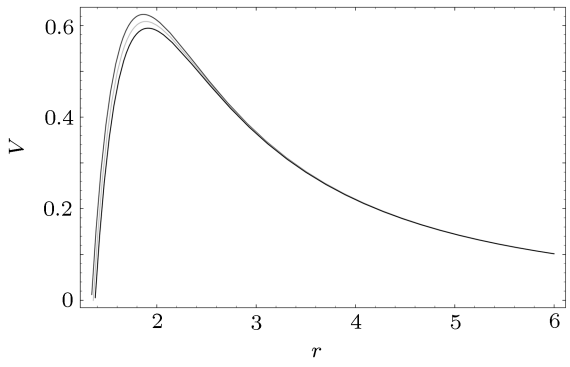


Fig. 2. The behaviour of the effective potential $V(r)$ vs. r for the Lovelock black hole by fixed parameters $l = 1$, $M = 1$, $\alpha = 0.1$ and cosmological constants $\Lambda = 0$ (top), 0.3 (middle), 0.6 (bottom).

Figure 1 and the top part of Fig. 4 show the variation of the effective potential and its peak point $r_{\rm p}$ with respect to the coupling constant α. From these two figures we can find that the peak value of the potential barrier gets lower and the location of the peak ($r = r_{\rm p}$) moves along the right when the coupling constant α decreases. In Fig. 2 and the bottom part of Fig. 4 we give the variation of the effective potential and its peak point $r_{\rm p}$ with respect to the cosmological constant Λ. On the other side, from these two figures we can find that the peak value of the potential barrier gets lower and the location of the peak ($r = r_{\rm p}$) moves along the right when the coupling constant Λ increases, which is different from the coupling constant α. But from Fig. 3 we can see that the peak value of the potential barrier gets higher and the location of the peak point ($r = r_{\rm p}$) moves along the right when the angular quantum number l increases.

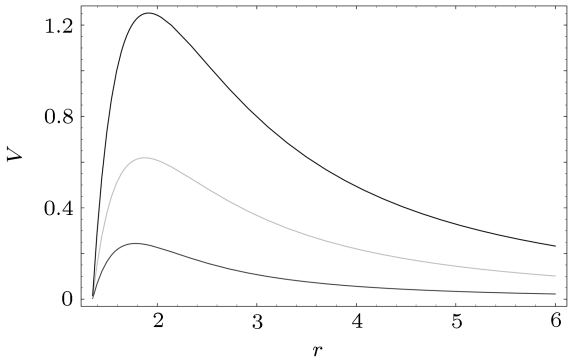


Fig. 3. The behaviour of the effective potential $V(r)$ vs. r for the Lovelock black hole by fixed parameters $M = 1$, $\Lambda = \alpha = 0.1$ and angular quantum numbers $l = 1$ (bottom), 2 (middle), 3 (top).

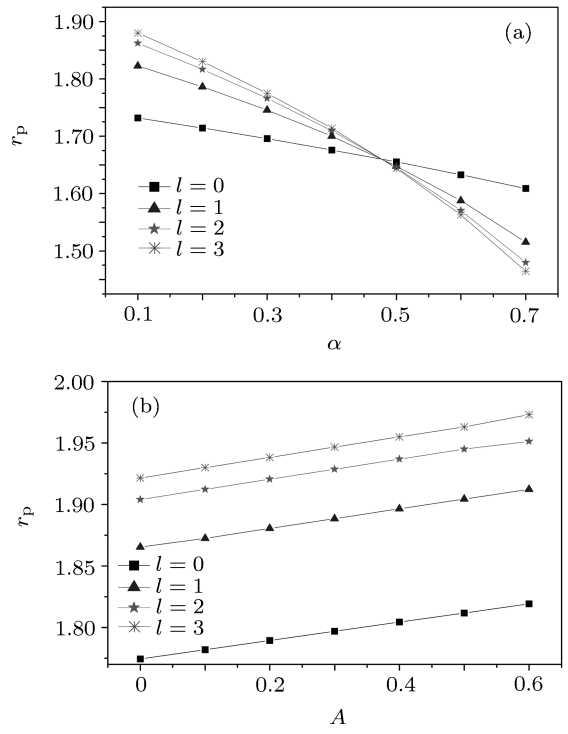


Fig. 4. The peak point ($r = r_{\rm p}$) of the effective potential vs. the parameters of the Lovelock black hole for different angular quantum numbers. Figure (a) corresponds to Fig. 1 and figure (b) corresponds to Fig. 2.

From the effective potential $V(r)$, i.e., Eq. (12) and Figs. 1 and 2, we find that the quasinormal frequencies depend on the coupling constant α and the cosmological constant Λ. In this paper, we plan to investigate the relationship between the quasinormal mode and the coupling constant α and the cosmological constant Λ, respectively. For convenience we take $M = 1$ in our calculation. In order to evaluate the quasinormal frequencies for the massless scalar field in Lovelock black hole spacetime (8), we use the third-order WKB approximation, a numerical method devised by Schutz, Will and Iyer.[48] This method has been used extensively in evaluating quasinormal frequencies of various black holes because of its considerable accuracy for lower-lying modes. In this approximate method, the formula for the complex quasinormal frequencies ω is

$$\omega^2 = [V_0 + (-2V_0'')^{1/2}\Lambda] - \mathrm{i}\left(n+\frac{1}{2}\right)(-2V_0'')^{1/2}(1+\Omega), \quad (13)$$

where

$$\Lambda = \frac{1}{(-2V_0'')^{1/2}}\left\{\frac{1}{8}\left(\frac{V_0^{(4)}}{V_0''}\right)\left(\frac{1}{4}+N^2\right) - \frac{1}{288}\left(\frac{V_0'''}{V_0''}\right)^2(7+60N^2)\right\}, \quad (14)$$

$$\Omega = \frac{1}{(-2V_0'')^{1/2}}\left\{\frac{5}{6912}\left(\frac{V_0'''}{V_0''}\right)^4(77+188N^2) - \frac{1}{384}\left(\frac{V_0'''^2V_0^{(4)}}{V_0''^3}\right)(51+100N^2) + \frac{1}{2304}\left(\frac{V_0^{(4)}}{V_0''}\right)^2(67+68N^2) + \frac{1}{288}\left(\frac{V_0'''V_0^{(5)}}{V_0''^2}\right)(19+28N^2) - \frac{1}{288}\left(\frac{V_0^{(6)}}{V_0''}\right)(5+4N^2)\right\}, \quad (15)$$

and $N = n + \frac{1}{2}$,

$$V_0^{(s)} = \left.\frac{\mathrm{d}^s V}{\mathrm{d}r_*^s}\right|_{r_*=r_*(r_\mathrm{p})}.$$

Here n is the overtone number and r_p is the value of the polar coordinate r corresponding to the peak of the effective potential (12). Substituting the effective potential (12) into the above formula, we can obtain the quasinormal frequencies of the scalar field in the background of a five-dimensional Lovelock black hole. Figure 5 and Table 1 show the real and imaginary parts of quasinormal frequencies for the scalar field with the variation of coupling constant α and angular quantum number l.

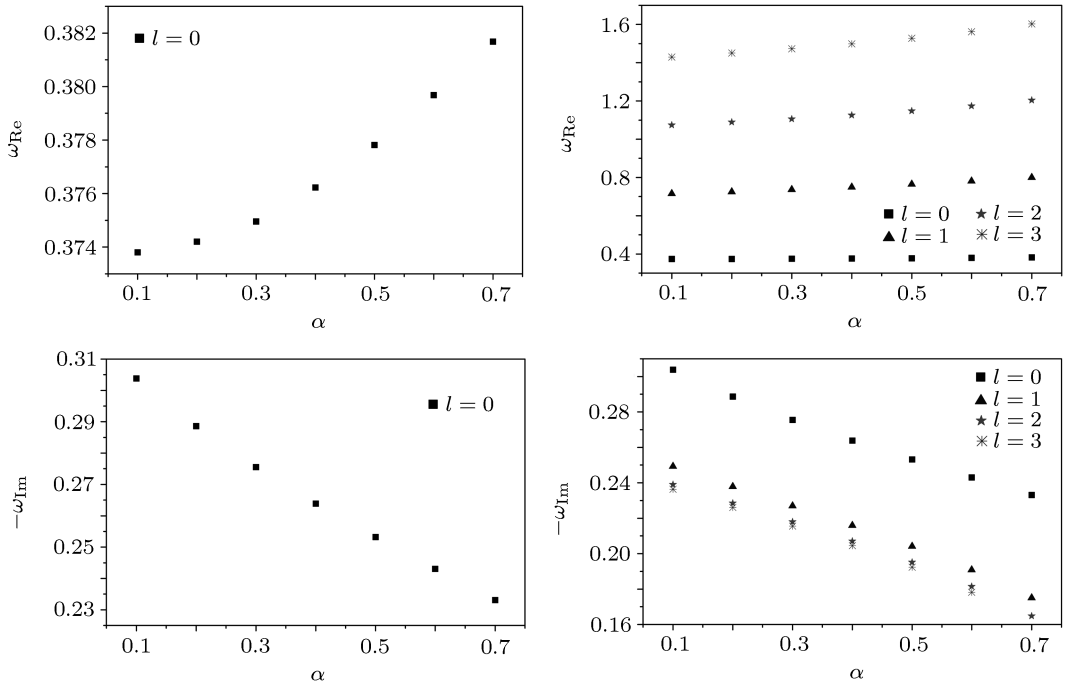


Fig. 5. Variation of the real parts (top row) and imaginary parts (bottom row) of quasinormal frequencies of the scalar field in Lovelock black hole spacetime with parameters $M = 1$, $\Lambda = 0.1$.

By analyzing these data and curves, we can find that, when the coupling constant α (i.e. the additional term presents the ultraviolet correction to Einstein theory) increases, the real part of the quasinormal frequencies

of the scalar field increases, while the imaginary part decreases, which means that the ultraviolet correction makes the scalar field decay more slowly and oscillate more quickly. Figure 6 and Table 2 show the real and imaginary parts of quasinormal frequencies for the scalar field with the variation of the cosmological constant Λ and angular quantum number l. Based on the data, we can draw the conclusion that, when the cosmological constant Λ increases, the real part and the imaginary part of the quasinormal frequencies of the scalar field decrease, which means that the cosmological constant makes the scalar field decay more slowly and oscillate more slowly.

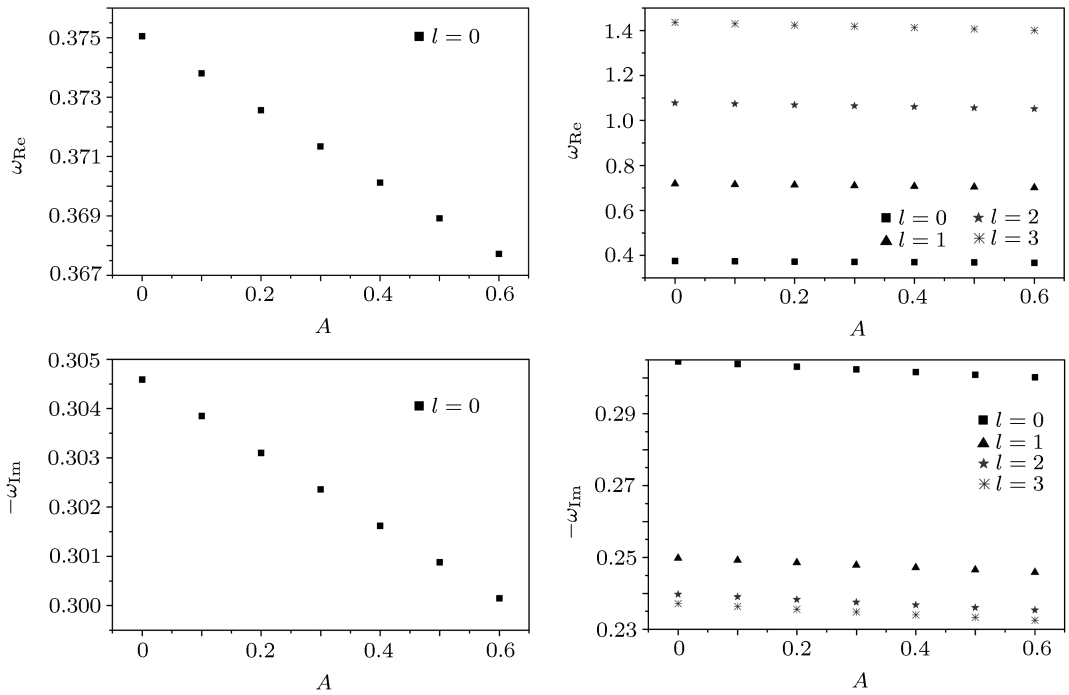


Fig. 6. Variation of the real parts (top row) and imaginary parts (bottom row) of quasinormal frequencies of the scalar field in Lovelock black hole spacetime with parameters $M = 1$, $\alpha = 0.1$.

Table 1. Quasinormal frequencies of the scalar field in Lovelock black hole spacetime with parameters $M = 1$, $\Lambda = 0.1$ and $n = 0$.

α	ω $(l = 0)$	ω $(l = 1)$	ω $(l = 2)$	ω $(l = 3)$
0.1	0.373800–0.303847i	0.715847–0.249250i	1.07354–0.238997i	1.43010–0.236352i
0.2	0.374204–0.288632i	0.725245–0.237874i	1.08845–0.228468i	1.45024–0.226095i
0.3	0.374958–0.275545i	0.736564–0.226962i	1.10557–0.217960i	1.47295–0.215606i
0.4	0.376224–0.263901i	0.749615–0.215928i	1.12509–0.207052i	1.49867–0.204530i
0.5	0.377815–0.253200i	0.764359–0.204153i	1.14737–0.195167i	1.52804–0.192338i
0.6	0.379676–0.243030i	0.780916–0.190888i	1.17301–0.181496i	1.56212–0.178243i
0.7	0.381678–0.233081i	0.799620–0.175064i	1.20317–0.164778i	1.60272–0.161014i

Table 2. Quasinormal frequencies of the scalar field in Lovelock black hole spacetime with parameters $M = 1$, $\alpha = 0.1$ and $n = 0$.

Λ	ω $(l = 0)$	ω $(l = 1)$	ω $(l = 2)$	ω $(l = 3)$
0	0.375051–0.304596i	0.718738–0.249821i	1.07808–0.239750i	1.43618–0.237140i
0.1	0.373800–0.303847i	0.715847–0.249250i	1.07354–0.238997i	1.43010–0.236352i
0.2	0.372561–0.303102i	0.712923–0.248564i	1.06905–0.238251i	1.42409–0.235571i
0.3	0.371335–0.302359i	0.710035–0.247882i	1.06462–0.237511i	1.41815–0.234797i
0.4	0.370118–0.301619i	0.707182–0.247203i	1.06024–0.236776i	1.41229–0.234030i
0.5	0.368915–0.300883i	0.704363–0.246530i	1.05591–0.236049i	1.40650–0.233271i
0.6	0.367723–0.300150i	0.701578–0.245860i	1.05164–0.235327i	1.40078–0.232518i

4. Conclusions

Using the third-order WKB approximation, a numerical method devised by Schutz, Will and Iyer, we obtained the quasinormal frequencies of the scalar field in the background of a five-dimensional Lovelock black hole in further detail. We find that the ultraviolet correction to Einstein theory in the Lovelock theory makes the scalar field decay more slowly and oscillate more quickly, and the cosmological constant makes the scalar field decay more slowly and oscillate more slowly in the Lovelock black hole background.

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