Noisy teleportation of qubit states via the Greenberger–Horne–Zeilinger state or the W state*

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The effects of distributing entanglement through the amplitude damping channel or the phase damping channel on the teleportation of a single-qubit state via the Greenberger–Horne–Zeilinger state and the W state are discussed. It is found that the average fidelity of teleportation depends on the type and rate of the damping in the channel. For the one-qubit affected case, the Greenberger–Horne–Zeilinger state is as robust as the W state, i.e., the same quantum information is preserved through teleportation. For the two-qubit affected case, the W state is more robust when the entanglement is distributed via the amplitude damping channel; if the entanglement is distributed via the phase damping channel, the W state is more robust when the noisy parameter is small while the Greenberger–Horne–Zeilinger state becomes more robust when it is large. For the three-qubit affected case, the Greenberger–Horne–Zeilinger state is more robust than the W state.

Keywords: noisy teleportation, amplitude damping channel, phase damping channel

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1. Introduction

Quantum teleportation, which allows us to transfer an unknown quantum state from a sender (Alice) to a receiver (Bob) by using previously shared entanglement and classical communication, is a fundamental and important protocol for quantum information science and technology.[1] The original protocol, using an Einstein–Podolsky–Rosen (EPR) state

$$|\psi\rangle_{\rm EPR} = (|00\rangle + |11\rangle)/\sqrt{2}, \tag{1}$$

which is the maximal two-qubit entangled state as the quantum channel, was proposed in Ref. [2]. Quantum teleportation allows us to transfer quantum information through a quantum entangled state which is used as a quantum channel with better fidelity than by means of a classical channel.[2−4] However, entanglement is susceptible to interaction with the environment, which results in disentanglement.[5−9] The effects of different types and strengths of noise acting on the two-qubit quantum channel on the fidelity of quantum single-qubit teleportation have been studied.[10−13]

In addition, one can teleport an unknown qubit state by using the multi-qubit entangled state.[14−18] Recently, it has been shown that both the Greenberger–Horne–Zeilinger (GHZ) state

$$|\psi\rangle_{\rm GHZ} = (|000\rangle_{123} + |111\rangle_{123})/\sqrt{2} \tag{2}$$

and the W state

$$|\psi\rangle_{\rm W} = (|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}\,|001\rangle_{123})/2 \tag{3}$$

can be used as the quantum channel for the perfect teleportation of an unknown qubit state.[18] The quantum circuits for teleportation using these two states as quantum channels are illustrated in Fig. 1[19] and the effects of noise on the fidelity of teleportation have been discussed by the authors. It is found that the issue of robustness, as compared between the GHZ state and W state, i.e., which state loses less quantum information in the noisy channels, is completely dependent on the type of noisy channel. For the ($L_{2,x}$, $L_{3,x}$, $L_{4,x}$) noisy channel, where the L's denote Lindblad operators, the GHZ state is always robust as compared to the W state, while the reverse situation occurs in the ($L_{2,y}$, $L_{3,y}$, $L_{4,y}$) noisy channel. In the ($L_{2,z}$, $L_{3,z}$, $L_{4,z}$) noisy channel, the W state is more robust than the GHZ state when the noisy parameter (κ) is comparatively small, while the GHZ state becomes more

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robust when κ is large. In the isotropic noisy channel, both states preserve an equal amount of quantum information.[19] We have not read a report on research into the effects of the amplitude damping (AD) channel and the phase damping (PD) channel on the teleportation of a single-qubit state via the GHZ state or W state. As we all know, the AD channel and PD channel are two typical kinds of noisy channel in consideration of the model of the interaction between the qubit and the environment.[1,5] Therefore, we will concentrate on this problem in the following.

However, it is essential to establish the distant entanglement for the implementation of quantum teleportation. This task can be accomplished by producing a local entangled state and transmitting one component of it to a distant location via a quantum channel. There are three methods to establish the quantum channel used in our paper. **Case 1** (one qubit affected): the entangled state is prepared by Alice and the third qubit of the state is delivered to Bob through a noisy channel. **Case 2** (two qubits affected): the entangled state is prepared by Bob and the first two qubits are delivered to Alice through a noisy channel. **Case 3** (three qubits affected): the entangled state is prepared by a third party and delivers the first two qubits to Alice and the third qubit to Bob through noisy channels. Here, we discuss the effects of distributing entanglement through the AD channels and the PD channels on the teleportation of a single-qubit state for these three cases. It is found that the fidelity of teleportation depends on the input state and the damping rate of the quantum channel except that only two qubits of the GHZ state are delivered through the AD channel, while the average fidelities of teleportation always depend on the type and rate of the damping in the channel. For the one-qubit affected case, the GHZ state is as robust as the W state, i.e., the same quantum information is preserved. For the two-qubit affected case, the W state is more robust when the entanglement is distributed via the AD channel; if the entanglement is distributed via the PD channel, the W state is more robust when the noisy parameter (κ) is small while the GHZ state becomes more robust when κ is large. For the three-qubit affected case, the GHZ state is more robust than the W state.

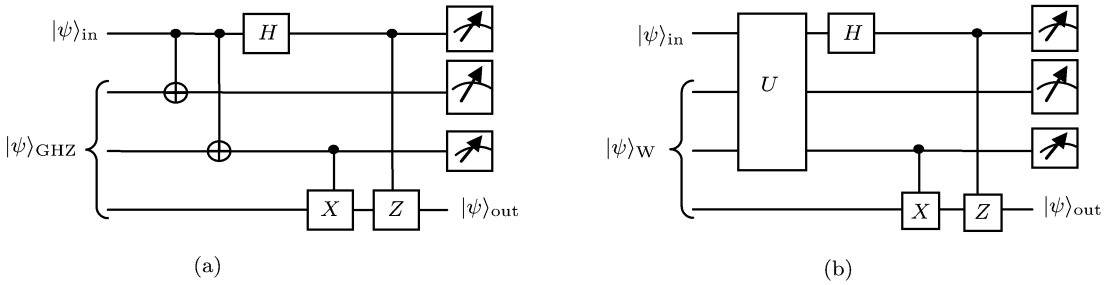


Fig. 1. A quantum circuit for teleportation of an unknown qubit state via the GHZ state (a) and the W state (b).

2. Amplitude damping channel

In this section, we explore the effects of the AD channels on quantum teleportation. A qubit state can be represented on a Bloch sphere as

$$|\psi\rangle_{\rm in} = \cos\frac{\theta}{2}|0\rangle + {\rm e}^{{\rm i}\varphi}\sin\frac{\theta}{2}|1\rangle\,, \tag{4}$$

where θ and ψ are the polar and azimuthal angles, respectively. Suppose that Alice wants to transmit an unknown qubit state expressed as Eq. (4) to Bob while the entangled state is distributed through a noisy channel. The quantum circuits for teleportation via the GHZ state and W state are shown in Fig. 1. The three top lines belong to Alice, while the bottom one belongs to Bob. The explicit expression for the U gate[19] is

$$U = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -\sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} & -\sqrt{2} & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}. \tag{5}$$

For an AD channel, the effect of noise on one qubit

can be described by the following Kraus operators[1]

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix} \tag{6}$$

with $p = 1 - \mathrm{e}^{-\kappa t}$ where κ is the decay rate of the qubit due to the AD channel. Throughout this paper, we assume that the decay rates κ of different qubits are the same. The fidelity of a state used as the quantum channel can be written as

$$F_{\mathrm{c}} = \langle\psi|\, \varepsilon(|\psi\rangle\langle\psi|)\, |\psi\rangle\,, \tag{7}$$

where ε denotes a quantum operation due to the noisy channel. Fidelity of the teleportation, which measures how much information is preserved, can be given by

$$F = \langle\psi_{\mathrm{in}}|\, \rho_{\mathrm{out}}\, |\psi_{\mathrm{in}}\rangle\,, \tag{8}$$

which measures the overlap between a state $|\psi_{\mathrm{in}}\rangle$ to be teleported and the output state ρ_{out} for a teleported state. In general, a state to be teleported is unknown, hence it is more useful to calculate the average fidelity given by

$$F_{\mathrm{av}} = \frac{1}{4\pi}\int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} F(\theta,\varphi)\sin\theta \mathrm{d}\theta. \tag{9}$$

The quantity $F_{\mathrm{av}} = 2/3$ corresponds to the average fidelity obtained only by classical communication.[20]

Case 1 Bob's qubit is influenced by AD noise. Then the initial GHZ state evolves into

$$\begin{aligned}\varepsilon(\rho_{\mathrm{GHZ}}) &= \sum_{i=0}^{1} (I\otimes I\otimes E_i)\rho_{\mathrm{GHZ}}(I\otimes I\otimes E_i)^+ \\ &= \frac{1}{2}(|000\rangle\langle 000| + (1-p)\,|111\rangle\langle 111| \\ &\quad + p\,|110\rangle\langle 110| + \sqrt{1-p}\,|000\rangle\langle 111| \\ &\quad + \sqrt{1-p}\,|111\rangle\langle 000|),\end{aligned} \tag{10}$$

which is used as the quantum channel. With Eq. (7) we can obtain the fidelity of the channel

$$F_{\mathrm{c}} = (1+\sqrt{1-p})^2/4. \tag{11}$$

According to Fig. 1(a), the density ρ_{out} for the output state can be given by

$$\begin{aligned}\rho_{\mathrm{out}} &= \frac{1}{2}\left[(2-p)\cos^2\left(\frac{\theta}{2}\right) + p\sin^2\left(\frac{\theta}{2}\right)\right]|0\rangle\langle 0| + \sqrt{1-p}\,\mathrm{e}^{-\mathrm{i}\varphi}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|0\rangle\langle 1| \\ &\quad + \sqrt{1-p}\,\mathrm{e}^{\mathrm{i}\varphi}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|1\rangle\langle 0| + \frac{1}{2}\left[(2-p)\sin^2\left(\frac{\theta}{2}\right) + p\cos^2\left(\frac{\theta}{2}\right)\right]|1\rangle\langle 1|\,.\end{aligned} \tag{12}$$

Then the quantity defined in Eq. (8) is

$$F = (2-2p-2\sqrt{1-p})\cos^4(\theta/2) + (2p+2\sqrt{1-p}-2)\cos^2(\theta/2) + 1 - \frac{p}{2}. \tag{13}$$

The average fidelity over all possible input unknown states is

$$F_{\mathrm{av}} = \frac{2}{3} - \frac{p}{6} + \frac{\sqrt{1-p}}{3}. \tag{14}$$

When the W state is prepared by Alice and the third qubit is transmitted through the AD channel, it becomes

$$\begin{aligned}\varepsilon(\rho_{\mathrm{W}}) &= \sum_{i=0}^{1} (I\otimes I\otimes E_i)\rho_{\mathrm{W}}(I\otimes I\otimes E_i)^+ \\ &= \frac{1}{4}[|100\rangle\langle 100| + |100\rangle\langle 010| + |010\rangle\langle 100| + |010\rangle\langle 010| + 2(1-p)\,|001\rangle\langle 001| \\ &\quad + 2p\,|000\rangle\langle 000| + \sqrt{2(1-p)}\,|100\rangle\langle 001| + \sqrt{2(1-p)}\,|010\rangle\langle 001| \\ &\quad + \sqrt{2(1-p)}\,|001\rangle\langle 100| + \sqrt{2(1-p)}\,|001\rangle\langle 010|],\end{aligned} \tag{15}$$

which is used as the quantum channel. The fidelity of the channel is the same as the case in which the GHZ state is used as the quantum channel. According to Fig. 1(b), we can obtain the density operator ρ_{out} for the output state and calculate the fidelity F of teleportation and the average fidelity F_{av}. It is found that they are the same as that of the case when the GHZ state is used as the quantum channel.

Case 2 The AD noise acts on Alice's two qubits of the entangled state used as the quantum channel. When the first two qubits of the GHZ state are transmitted through the AD channels, it evolves into

$$\begin{aligned}\varepsilon(\rho_{\rm GHZ}) &= \sum_{i,j=0}^{1}(E_i\otimes E_j\otimes I)\rho_{\rm GHZ}(E_i\otimes E_j\otimes I)^+\\ &= \frac{1}{2}[|000\rangle\langle 000| + (1-p)|000\rangle\langle 111| + (1-p)|111\rangle\langle 000| + (1-p)^2|111\rangle\langle 111|\\ &\quad + p^2|001\rangle\langle 001| + p(1-p)|101\rangle\langle 101| + p(1-p)|011\rangle\langle 011|],\end{aligned} \tag{16}$$

which is used as the quantum channel. The fidelity of the state used as the quantum channel is

$$F_{\rm c} = (2-p)^2/4. \tag{17}$$

The fidelity of the teleported state

$$F = 1 - \frac{p}{2}, \tag{18}$$

is independent of the input state. Then the average fidelity is the same as the fidelity of teleportation.

When the first two qubits of the W state are transmitted through AD channels, it evolves into

$$\begin{aligned}\varepsilon(\rho_{\rm W}) &= \sum_{i,j=0}^{1}(E_i\otimes E_j\otimes I)\rho_{\rm W}(E_i\otimes E_j\otimes I)^+\\ &= \frac{1}{4}[(1-p)|100\rangle\langle 100| + (1-p)|100\rangle\langle 010| + \sqrt{2(1-p)}|100\rangle\langle 001|\\ &\quad + (1-p)|010\rangle\langle 100| + (1-p)|010\rangle\langle 010| + \sqrt{2(1-p)}|010\rangle\langle 001|\\ &\quad + \sqrt{2(1-p)}|001\rangle\langle 100| + \sqrt{2(1-p)}|001\rangle\langle 010| + 2|001\rangle\langle 001| + 2p|000\rangle\langle 000|],\end{aligned} \tag{19}$$

which is used as the quantum channel. The fidelity of the state used as the quantum channel, the fidelity of teleportation and the average fidelity are the same as those of **Case 1**.

Case 3 The AD noise acts on all three qubits of the entangled state used as the quantum channel. Under the influence of AD noise, the GHZ state evolves into

$$\begin{aligned}\varepsilon(\rho_{\rm GHZ}) &= \sum_{i,j,k=0}^{1}(E_i\otimes E_j\otimes E_k)\rho_{\rm GHZ}(E_i\otimes E_j\otimes E_k)^+\\ &= \frac{1}{2}[|000\rangle\langle 000| + (1-p)^{3/2}|000\rangle\langle 111| + (1-p)^{3/2}|111\rangle\langle 000| + (1-p)^3|111\rangle\langle 111|\\ &\quad + (1-p)p^2|010\rangle\langle 010| + (1-p)^2p|110\rangle\langle 110| + (1-p)^2p|101\rangle\langle 101|\\ &\quad + (1-p)^2p|011\rangle\langle 011| + (1-p)p^2|100\rangle\langle 100| + (1-p)p^2|001\rangle\langle 001| + p^3|000\rangle\langle 000|],\end{aligned} \tag{20}$$

which is used as the quantum channel. The fidelity of the state used as the quantum channel is

$$F_{\rm c} = \frac{1}{4}[2-3p+3p^2+2(1-p)^{3/2}]. \tag{21}$$

The fidelity of the teleported state is

$$F = 1-p+p^2+2[2p-2p^2+(1-p)^{3/2}-1]\cos^2(\theta/2)(1-\cos^2(\theta/2)), \tag{22}$$

and the average fidelity of teleportation is

$$F_{\rm av} = \frac{2}{3} - \frac{1}{3}p + \frac{1}{3}p^2 + \frac{1}{3}(1-p)^{3/2}. \tag{23}$$

Under the influence of AD noise, the W state evolves into

$$\varepsilon(\rho_{\rm W}) = \sum_{i,j,k=0}^{1}(E_i\otimes E_j\otimes E_k)\rho_{\rm W}(E_i\otimes E_j\otimes E_k)^+ = (1-p)|W\rangle\langle W| + p|000\rangle\langle 000|, \tag{24}$$

which is used as the quantum channel. The fidelity of the channel state can be written as

$$F_{\rm c} = 1 - p. \tag{25}$$

The fidelity of the teleportation is

$$F = -2p\cos^4(\theta/2) + 2p\cos^2(\theta/2) + 1 - p. \tag{26}$$

And the average fidelity is

$$F_{\rm av} = 1 - \frac{2}{3}p. \tag{27}$$

Table 1. Summary of fidelity of the channel ($F_{\rm c}$), fidelity of teleportation (F) and the average fidelity ($F_{\rm av}$) when the entanglement is distributed via the AD channel.

		fidelity of the channel ($F_{\rm c}$)	fidelity of teleportation (F)	average fidelity ($F_{\rm av}$)
Case 1	GHZ	$(1+\sqrt{1-p})^2/4$	$1-\frac{p}{2}+(2-2p-2\sqrt{1-p})\cos^4(\theta/2)$ $+(2p+2\sqrt{1-p}-2)\cos^2(\theta/2)$	$\frac{2}{3}-\frac{p}{6}+\frac{\sqrt{1-p}}{3}$
	W	$(1+\sqrt{1-p})^2/4$	$1-\frac{p}{2}+(2-2p-2\sqrt{1-p})\cos^4(\theta/2)$ $+(2p+2\sqrt{1-p}-2)\cos^2(\theta/2)$	$\frac{2}{3}-\frac{p}{6}+\frac{\sqrt{1-p}}{3}$
Case 2	GHZ	$(2-p)^2/4$	$1-\frac{p}{2}$	$1-\frac{p}{2}$
	W	$(1+\sqrt{1-p})^2/4$	$1-\frac{p}{2}+(2-2p-2\sqrt{1-p})\cos^4(\theta/2)$ $+(2p+2\sqrt{1-p}-2)\cos^2(\theta/2)$	$\frac{2}{3}-\frac{p}{6}+\frac{\sqrt{1-p}}{3}$
Case 3	GHZ	$(2-3p+3p^2+$ $2\sqrt{(1-p)^3})/4$	$1-p+p^2+2[2p-2p^2+(1-p)^{3/2}$ $-1]\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$[2-p+p^2$ $+(1-p)^{3/2}]/3$
	W	$1-p$	$-2p\cos^4(\theta/2)+2p\cos^2(\theta/2)+1-p$	$1-\frac{2}{3}p$

The fidelity of the channel ($F_{\rm c}$), fidelity of teleportation (F) and the average fidelity ($F_{\rm av}$) for the AD channel are summarized in Table 1 when the affected GHZ and W states are used as the quantum channels. It can be noticed that the fidelity of teleportation is dependent on the input state and the rate of damping in the channel except that two qubits of the GHZ state are delivered through the AD channel. For **Case 1**, the average fidelity of teleportation and the fidelity of the channel as functions of scaled time (κt) are plotted in Figs. 2(a) and 2(d). We note that the GHZ state is as robust as the W state, i.e. the same quantum information is preserved, and the average fidelity is larger than 2/3 when $\kappa t \leq \ln(3+2\sqrt{2})$, no matter whether the GHZ state or the W state is used. By calculation, we note that $F_{\rm av} = (2F_{\rm c}+1)/3$. This relation between the fidelity of teleportation and the fidelity of the quantum channel is the same as that of the case in which two-qubit entangled states are used as the quantum channel.[21] The reason for this is that the GHZ state and the W state can be expressed as

$$|\psi\rangle_{\rm GHZ} = (|00\rangle_{12}|0\rangle_3 + |11\rangle_{12}|1\rangle_3)/\sqrt{2}$$

and

$$|\psi\rangle_{\rm W} = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|10\rangle_{12}+|01\rangle_{12})|0\rangle_3 + |00\rangle_{12}|1\rangle_3\right].$$

For the GHZ state, we can code $|0\rangle = |00\rangle_{12}$, $|1\rangle = |11\rangle_{12}$, while for the W state, we have $|0\rangle = (|01\rangle_{12} + |10\rangle_{12})/\sqrt{2}$, $|1\rangle = |00\rangle_{12}$. Then they can be regarded as two-qubit entangled states when only Bob's qubit is under the influence of AD noise. For **Case 2**, the average fidelity of teleportation and the fidelity of the channel are plotted in Figs. 2(b) and 2(e). It can be noticed that the W state is more robust than the GHZ state. If the GHZ state is prepared by Bob and distributed though the AD channel, the average fidelity is larger than 2/3 when $\kappa t \leq \ln 3$, while the average fidelity is larger than 2/3 when $\kappa t \leq \ln(3+2\sqrt{2})$ if the affected W state is used as the quantum channel. The relation $F_{\rm av} = (2F_{\rm c}+1)/3$ is not satisfied

when the affected GHZ state is used as the quantum channel. The reason for this is that the state $|11\rangle$ would evolve into the states $|01\rangle$, $|10\rangle$ or $|00\rangle$ under the influence of AD noise. Then it cannot be regarded as a two-qubit state. For the W state, the relation $F_{\rm av} = (2F_{\rm c} + 1)/3$ is satisfied. For **Case 3**, the average fidelity of teleportation and the fidelity of the channel are plotted in Figs. 2(c) and 2(f). It can be seen that the GHZ state is robust compared to the W state. The average fidelity of teleportation is larger than 2/3 when $\kappa t \leq \ln((\sqrt{5}+3)/2)$ if the affected GHZ state is used as the quantum channel. Meanwhile, the relation $F_{\rm av} = (2F_{\rm c} + 1)/3$ is not satisfied for the same reason as for **Case 2**. The fidelity of teleportation is larger than 2/3 when $\kappa t \leq \ln 2$ if the affected W state is used as the quantum channel, and the relation $F_{\rm av} = (2F_{\rm c} + 1)/3$ is satisfied.

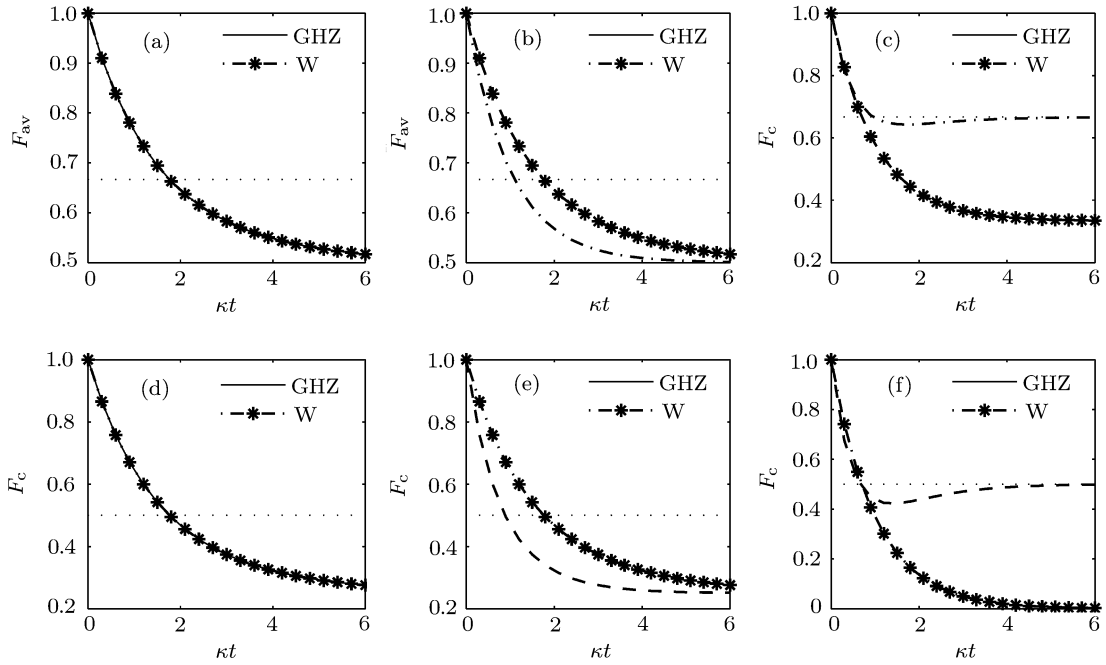


Fig. 2. The average fidelity of teleportation with the AD channel is plotted as a function of scaled time for **Case 1** (a), **Case 2** (b) and **Case 3** (c). The fidelity of the state used as the quantum channel is plotted for **Case 1** (d), **Case 2** (e) and **Case 3** (f).

3. Phase damping channel

In this section, we explore the effects of phase damping channels on quantum teleportation. For a phase damping channel, the effect of noise on one qubit can be described by the following Kraus operators[1]

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix} \tag{28}$$

with $p = 1 - {\rm e}^{-\kappa t}$ where κ is the decay rate of the qubit due to the PD channel. With the similar calculations above, we can obtain the following results.

From Table 2, it can be noticed that the fidelity of teleportation is dependent on the input state and the rate of damping in the channel. The maximal fidelity of teleportation can be obtained at a fixed scaled time for $\theta = 0$ and $\theta = \pi$ corresponding to $|\psi\rangle_{\rm in} = |0\rangle$ and $|\psi\rangle_{\rm in} = |1\rangle$, respectively; and the relation $F_{\rm av} = (2F_{\rm c} + 1)/3$ is satisfied. The reason for this is that under the influence of PD noise the GHZ and W states can always be regarded as two-qubit entangled states. The average fidelity of teleportation and the fidelity of the quantum channel as functions of scaled time are plotted in Fig. 3. For **Case 1**, the GHZ state is as robust as the W state when they are used as the quantum channel. The average of fidelity is always larger than 2/3. For **Case**

2, the W state is more robust when $\kappa t < 2\ln 3$ while the GHZ state becomes more robust when $\kappa t > 2\ln 3$, and the average of fidelity is always larger than 2/3 when the GHZ state is prepared to be used as the quantum channel. For **Case 3**, the GHZ state is more robust. The average fidelity of teleportation decays to 2/3 when $\kappa t = \ln 5$ if the W state is used as the quantum channel, while for the GHZ state, the average fidelity is always larger than 2/3.

Table 2. Summary of fidelity of the channel (F_c), fidelity of teleportation (F) and the average fidelity (F_{av}) when the entanglement is distributed via the PD channel.

		fidelity of the channel (F_c)	fidelity of teleportation (F)	average fidelity (F_{av})
Case 1	GHZ	$(1+\sqrt{1-p})/2$	$1+2(\sqrt{1-p}-1)\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$\frac{2}{3}+\frac{1}{3}\sqrt{1-p}$
	W	$(1+\sqrt{1-p})/2$	$1+2(\sqrt{1-p}-1)\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$\frac{2}{3}+\frac{1}{3}\sqrt{1-p}$
Case 2	GHZ	$(2-p)/2$	$1-2p\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$1-\frac{p}{3}$
	W	$(4+4\sqrt{1-p}-p)/8$	$1-\frac{p}{4}+(2\sqrt{1-p}+p-2)\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$\frac{2}{3}-\frac{p}{12}+\frac{1}{3}\sqrt{1-p}$
Case 3	GHZ	$(1+\sqrt{(1-p)^3})/2$	$1+2(\sqrt{(1-p)^3}-1)\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$\frac{2}{3}+\frac{1}{3}\sqrt{(1-p)^3}$
	W	$(8-5p)/8$	$1-\frac{p}{4}-p\cos^2(\theta/2)(1-\cos^2(\theta/2))$	$1-\frac{5}{12}p$

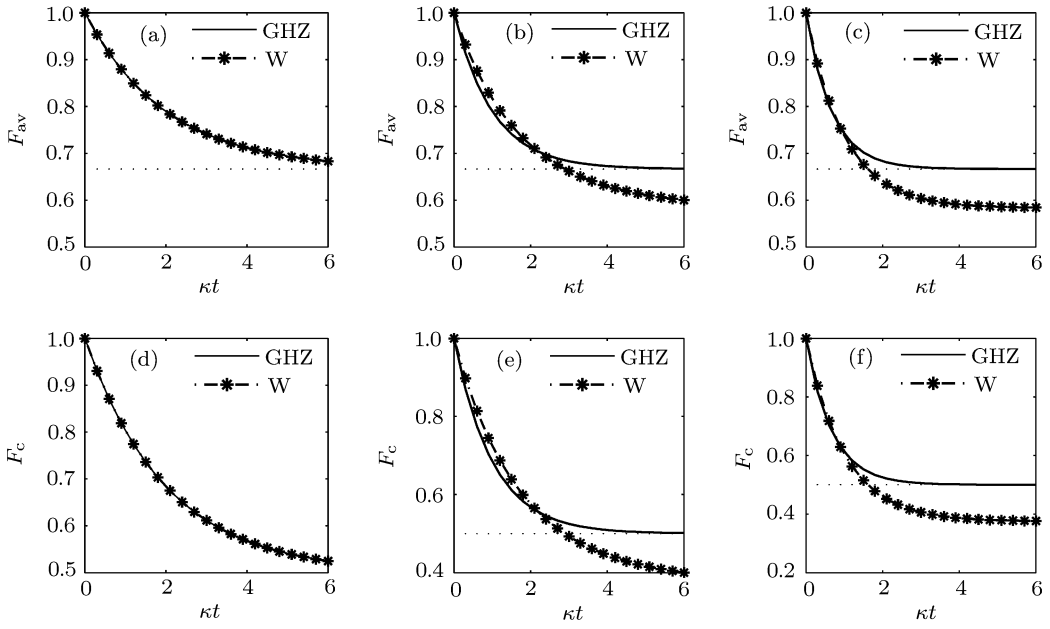


Fig. 3. The average fidelity of teleportation with a PD channel is plotted as a function of scaled time for **Case 1** (a), **Case 2** (b) and **Case 3** (c). The fidelity of the state used as the quantum channel is plotted for **Case 1** (d), **Case 2** (e) and **Case 3** (f).

4. Conclusions

We have examined the problem of teleportation fidelity in the presence of AD noise and PD noise during the entanglement distribution in the teleportation process. For the one-qubit affected case, the GHZ state is as robust as the W state, i.e. the same quantum information is preserved through teleportation. The average fidelity of teleportation decays to 2/3 when $\kappa t \leq \ln(3+2\sqrt{2})$ if the entanglement is distributed through the AD channel, while the average fidelity of teleportation is always larger than 2/3 if the entanglement is distributed through the PD channel. For the two-qubit affected case, the W state is more robust than the GHZ state when

the two qubits are delivered through the AD channels. For the PD channels, the W state is more robust when $\kappa t < 2\ln 3$, while the GHZ state becomes more robust when $\kappa t > 2\ln 3$. For three-qubit affected noise, the GHZ state is more robust than the W state. When the GHZ state affected by the PD noise is used as the quantum channel, the average fidelity of teleportation is always larger than 2/3. In conclusion, the issue of which state preserves more information in teleportation when using the GHZ state or the W state is dependent on the type of noise channel and the method of quantum distribution.

These results are different from those obtained when the teleportation of a single qubit is accomplished through the ($L_{2,x}$, $L_{3,x}$, $L_{4,x}$), ($L_{2,y}$, $L_{3,y}$, $L_{4,y}$), and ($L_{2,z}$, $L_{3,z}$, $L_{4,z}$) types of noisy channel. These will develop our knowledge of noise teleportation of single-qubit states via the GHZ and W states. According to the method of entanglement distribution and the type of noise affected, we can choose the GHZ state or the W state as the quantum channel to obtain larger fidelity.

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