Entanglement and decoherence of coupled superconductor qubits in a non-Markovian environment*

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The sudden death of entanglement is investigated for the non-Markovian dynamic process of a pair of interacting flux qubits under a thermal bath. The results show that, for initially two-qubit entangled states, entanglement sudden death (ESD) always happens in the thermal reservoir, where its appearance strongly depends on the environment. In particular, ESD of the qubits occurs more easily for the non-Markovian process than for the Markovian one.

Keywords: entanglement, coupled qubits, non-Markovian process, concurrence
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1. Introduction

One of the most striking features of quantum mechanics is the concept of entanglement. An entangled state is the state of a composite system that cannot be separated into product states in terms of subsystems. Entanglement is a quintessential property of quantum mechanics that sets it apart from any classical physical theory. An important feature of entanglement is that it gives rise to correlations that cannot be explained by any local realistic description of quantum mechanics. Therefore, quantum entanglement is a non-local quantum connection. It is not only a powerful tool to test the basic problems of quantum mechanics, but also a physical foundation on which to build future quantum information science.

It is well known that one needs sustained entanglement among the qubits as they dynamically evolve in time for many applications of interest.[1−6] In recent years, indeed, quantum entanglement has become an important physical resource for quantum communication and information processing like universal geometric quantum gates,[7−11] quantum teleportation,[12−14] superdense coding,[15] quantum key distribution and telecoloning.[14−17]

Quantum properties, however, are very fragile. In the real physical world, the interaction between a quantum system and its surrounding environment may lead to an irreversible loss of information on the system. The interacting effect is that quantum superpositions decay into statistical mixtures so as to result in a relatively short coherence time, which is known as decoherence and leads to the degradation of entanglement. Under the influence of a pure dissipative environment in particular, a mixed state of an initially entangled two-qubit system becomes completely disentangled in a finite time. This is called entanglement sudden death (ESD) and has been recently observed in two elegantly designed experiments with a photonic qubit and atomic ensemble.[18−21] In order to ensure the realisation of correct computing of quantum logic and quantum computation, people have undertaken a lot of study on ESD phenomena, including not only how to avoid ESD but also how to restore entanglement in the two-qubit system.[22−26]

One of the approaches describing open quantum systems is to separate the total system into two parts, i.e., the quantum system of interest and the surrounding environment, where the environment is modelled as an infinite collection of quantum harmonic oscillators at thermal equilibrium. In many cases of physical situations the system-reservoir coupling strength is small in comparison with the frequency of the reservoir oscillators. Thus the reservoir spectral density can be conveniently approximated to a flat spectrum.[27,28] Such systems are typically referred to as Markovian

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open quantum systems, a prominent feature of which is the irreversible flow of energy and information from the system to the environment. In the real physical world, however, the quantum system of interest interacts with "structured" reservoirs, whose spectral density may strongly vary with frequency. In this case, the systems are referred to as non-Markovian open quantum systems.[29,30] It is known that non-Markovian dynamics is characterized by the existence of a memory time scale, during which some energy or information that has been transferred from the system to the environment feeds back into the system.

Among the qubit schemes in view at present, solid state systems that make use of the Josephson effect are available candidates. Presently, three prototypes of superconducting qubits have been studied experimentally.[31−33] Therefore, abundant investigations have been done, which involve the implementation and the behaviour of superconductor qubits as well as decoherence arising from the influence of the environment. Thus, significant progress has been made in the physical implementation of quantum computation based on superconducting qubits. In the Born–Markov approximation, theoretical investigations on entanglement of superconducting qubits have typically relied on the widely used spin-boson model which postulates a purely two-level dynamics so as to neglect leakage effects. As a basic model describing a quantum system, the spin-boson model provides a simple and effective way to study the quantum dissipation characteristics.[34,35]

Up to now, numerous investigations on ESD in a variety of systems have been done. However, the questions of ESD in the interaction of qubits remain open, especially in the superconducting quantum bit system which is very sensitive to the environment. Due to the memory effect, the effect of the environment on the dynamic evolution of a quantum system is closely related to the evolutionary history of the system.[36−39] Thus, it is necessary to study in-depth the non-Markov dynamic evolution of the superconducting qubit system. It is interesting to investigate ESD in a system of interacting superconducting flux qubits coupled with the environment in the non-Markovian process.

2. Superconducting flux qubits in non-Markovian dynamics

Our physical system consists of a pair of initially entangled interacting superconducting flux qubits. Based on the spin-boson model, the total Hamiltonian for the system is then given by

$$H = H_0 + H_{\rm I}, \tag{1}$$

where

$$H_0 = \frac{\varepsilon}{2}\sigma_1^z + \frac{\varepsilon}{2}\sigma_2^z + \sum_n \omega_n b_n^\dagger b_n \tag{2}$$

contains the Hamiltonian of the two qubits characterized by the same energy bias ε and the bosonic bath characterized by the annihilation and creation operators b_n and $b_n^\dagger$. The notations σ_j^z, $\sigma_j^\pm$ $(j = 1, 2)$ are the Pauli operators for two different qubits, respectively. The interaction Hamiltonian $H_{\rm I}$ is given by

$$\begin{aligned} H_{\rm I} =\ & K(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+) + (\sigma_1^+ + \sigma_2^+)\sum_n g_n b_n \\ & + (\sigma_1^- + \sigma_2^-)\sum_n g_n^* b_n^\dagger, \end{aligned} \tag{3}$$

where K is the strength of the qubit–qubit interaction and g_n is the coupling constant of the interaction between the qubit and the local reservoir. In the interaction picture, the Hamiltonian of the system has the following form in units of $k_{\rm B} = \hbar = 1$,

$$H_{\rm I}(t) \equiv H_{\rm I}^{(s)}(t) + H_{\rm I}^{(d)}(t), \tag{4}$$

where

$$\begin{aligned} H_{\rm I}^{(s)}(t) &= K(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \\ H_{\rm I}^{(d)}(t) &= (\sigma_1^+ + \sigma_2^+)\sum_n g_n b_n {\rm e}^{{\rm i}(\varepsilon-\omega_n)t} \\ &\quad + (\sigma_1^- + \sigma_2^-)\sum_n \bar{g}_n b_n^\dagger {\rm e}^{-{\rm i}(\varepsilon-\omega_n)t}. \end{aligned}$$

It is known that the density matrix was introduced as a way to describe a quantum open system because the state of the open system is not completely known. The time evolution of the density operator ρ, which gives us information about the dynamics of the system, can then be evaluated from the integro-differential quantum-Liouville equation of motion,

$$\begin{aligned} \frac{\rm d}{{\rm d}t}\rho(t) =\ & -{\rm i}[H_{\rm I}^{(s)}(t), \rho(0)] \\ & - \int_0^t {\rm d}\tau\, {\rm Tr}_{\rm B}([H_{\rm I}(t), [H_{\rm I}(\tau), \rho(t)\otimes\rho_{\rm B}]]). \end{aligned} \tag{5}$$

Because the capacity of the heat storage R is very large, one may assume that during the process of dynamic evolution the quantum system has little effect on the heat storage. Thus the heat storage has been in its own thermal equilibrium state all the time, i.e.,

$$\rho_T(t) = \rho(t)\otimes\rho_R.$$

In addition, at the initial time $t = 0$, the heat storage only has a diagonal entry in the photon number representation and the $H_{\rm I}(t)$ will absorb or release a photon, leading to

$$\mathrm{Tr_B}([H_{\rm I}^{d}(t), \rho(0)\otimes\rho_{\rm B}]) = 0;$$

in terms of Eqs. (4) and (5), the equation of dynamics involved for the density operator may be rewritten as

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} =& -\mathrm{i}[H_{\rm I}^{(s)}(t), \rho(0)] - t\,[H_{\rm I}^{(s)}(t), [H_{\rm I}^{(s)}(t), \rho(t)]] \\ &+ B(t)\left\{[\sigma_1^-\rho(t), \sigma_1^+] + [\sigma_2^-\rho(t), \sigma_2^+]\right\} \\ &+ B^*(t)\left\{[\sigma_1^-, \rho(t)\sigma_1^+] + [\sigma_2^-, \rho(t)\sigma_2^+]\right\} \\ &+ A(t)\left\{[\sigma_1^+, \rho(t)\sigma_1^-] + [\sigma_2^+, \rho(t)\sigma_2^-]\right\} \\ &+ A^*(t)\left\{[\sigma_1^+\rho(t), \sigma_1^-] + [\sigma_2^+\rho(t), \sigma_2^-]\right\}, \quad (6)\end{aligned}$$

where

$$A(t) = \mathrm{i}\sum_n |g_n|^2 n(\omega_n)\frac{1-\mathrm{e}^{\mathrm{i}(\varepsilon-\omega_n)t}}{\varepsilon-\omega_n},$$

$$B(t) = \mathrm{i}\sum_n |g_n|^2 [n(\omega_n)+1]\frac{1-\mathrm{e}^{\mathrm{i}(\varepsilon-\omega_n)t}}{\varepsilon-\omega_n}.$$

In order to simplify the calculation but without loss of generality, the bosonic bath is assumed to be in a thermal state at $T = 0$ K. Thus the correlation functions is reduced to

$$A(t) = 0, \quad B(t) = \mathrm{i}\sum_n |g_n|^2 \frac{1-\mathrm{e}^{\mathrm{i}(\varepsilon-\omega_n)t}}{\varepsilon-\omega_n}.$$

With the two-qubit product basis,

$$|00\rangle = |0\rangle_1|0\rangle_2, \quad |01\rangle = |0\rangle_1|1\rangle_2,$$
$$|10\rangle = |1\rangle_1|0\rangle_2, \quad |11\rangle = |1\rangle_1|1\rangle_2,$$

the initially entangled qubits are considered to be in a mixed state described by the density matrix,[40]

$$\rho(0) = \begin{pmatrix} \rho_{11}(0) & 0 & 0 & \rho_{14}(0) \\ 0 & \rho_{22}(0) & \rho_{23}(0) & 0 \\ 0 & \rho_{32}(0) & \rho_{33}(0) & 0 \\ \rho_{41}(0) & 0 & 0 & \rho_{44}(0) \end{pmatrix}, \quad (7)$$

where the initial values of the matrix elements are taken as real numbers. The solution is direct, thus one has[41]

$$\begin{aligned}\rho_{11}(t) &= \mathrm{e}^{-2\gamma(t)}\rho_{11}(0), \\ \rho_{22}(t) &= \mathrm{e}^{-\gamma(t)}\left(1-\mathrm{e}^{-\gamma(t)}\right)\rho_{11}(0) \\ &\quad + \frac{1}{2}\mathrm{e}^{-\gamma(t)}\left(1+\mathrm{e}^{-2K^2t^2}\right)\rho_{22}(0) \\ &\quad + \frac{1}{2}\mathrm{e}^{-\gamma(t)}\left(1-\mathrm{e}^{-2K^2t^2}\right)\rho_{33}(0), \\ \rho_{33}(t) &= \mathrm{e}^{-\gamma(t)}\left(1-\mathrm{e}^{-\gamma(t)}\right)\rho_{11}(0) \\ &\quad + \frac{1}{2}\mathrm{e}^{-\gamma(t)}\left(1-\mathrm{e}^{-2K^2t^2}\right)\rho_{22}(0) \\ &\quad + \frac{1}{2}\mathrm{e}^{-\gamma(t)}\left(1+\mathrm{e}^{-2K^2t^2}\right)\rho_{33}(0), \\ \rho_{44}(t) &= 1-\rho_{11}(t)-\rho_{22}(t)-\rho_{33}(t), \\ \rho_{14}(t) &= \mathrm{e}^{-2\int_0^t \mathrm{d}\tau B(\tau)}\rho_{14}(0), \\ \rho_{23}(t) &= \mathrm{e}^{-\gamma(t)}\rho_{23}(0), \quad \rho_{32}(t) = \rho_{23}^*(t), \\ \rho_{41}(t) &= \rho_{14}^*(t).\end{aligned}$$

All the rest are zero. It is noted that the decay rate is given by

$$\gamma(t) = 2\,\mathrm{Re}\left\{\int_0^t \mathrm{d}\tau B(\omega,\tau)\right\}.$$

3. Entanglement dynamics

Characterisation of entanglement is one of the key features related to quantum information theory. From the point of view of a possible application, it is not only important to determine whether a given state is entangled, but also to quantify the degree of entanglement. Among several such quantities, the concurrence introduced by Bennet *et al.* is often used for this purpose.[42] It is known that the change of concurrence can be from 0 to 1; the symbiotic entanglement will also monotonously change from 0 to 1. Therefore, the symbiotic entanglement degree can be used very well to study the characters of the symbiotic entanglement as a relative amount of the symbiotic entanglement. The concurrence function for two qubits is defined as

$$C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (8)$$

where λ_j are the eigenvalues of the non-Hermitian $W = \rho(t)\bar{\rho}(t)$ arranged in descending order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$. The spin-flip operator $\bar{\rho}(t)$ is defined as

$$\bar{\rho}(t) = \left(\sigma_y^{(1)}\otimes\sigma_y^{(2)}\right)\rho^*(t)\left(\sigma_y^{(1)}\otimes\sigma_y^{(2)}\right), \quad (9)$$

where the asterisk $\rho^*(t)$ is the complex conjugate of density matrix $\rho(t)$. For the normal basis vector $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, one has

$$\sigma_y^{(1)}\otimes\sigma_y^{(2)} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

and the matrix W has the explicit form[43]

$$W = \begin{pmatrix} |\rho_{14}|^2 + \rho_{11}\rho_{44} & 0 & 0 & 2\rho_{11}\rho_{14} \\ 0 & |\rho_{23}|^2 + \rho_{22}\rho_{33} & 2\rho_{22}\rho_{23} & 0 \\ 0 & 2\rho_{32}\rho_{33} & |\rho_{23}|^2 + \rho_{22}\rho_{33} & 0 \\ 2\rho_{41}\rho_{44} & 0 & 0 & |\rho_{14}|^2 + \rho_{11}\rho_{44} \end{pmatrix}. \tag{10}$$

Eigenvalues of the matrix W are easily found to be

$$\begin{aligned} \lambda_1 &= \left(\sqrt{\rho_{22}\rho_{33}} + |\rho_{23}|\right)^2, \\ \lambda_2 &= \left(\sqrt{\rho_{22}\rho_{33}} - |\rho_{23}|\right)^2, \\ \lambda_3 &= \left(\sqrt{\rho_{11}\rho_{44}} + |\rho_{14}|\right)^2, \\ \lambda_4 &= \left(\sqrt{\rho_{11}\rho_{44}} - |\rho_{14}|\right)^2. \end{aligned}$$

In terms of these eigenvalues, we obtain the time-dependent concurrence for the interacting superconducting flux qubits. Considering the initial condition

$$\begin{aligned} \rho(0) = & \frac{1}{2}(1-b)\left(|01\rangle + \langle 10|\right)\left(\langle 01| + \langle 10|\right) \\ & + b\,|00\rangle\langle 00| \quad (0 \le b < 1), \end{aligned} \tag{11}$$

the eigenvalues of the matrix W are given by

$$\lambda_1 = (1-b)^2 \mathrm{e}^{-2\gamma(t)}, \quad \lambda_2 = \lambda_3 = \lambda_4 = 0.$$

Thus the concurrence function is given by

$$C(t) = \max\{0, \tilde{C}(t)\},$$

where

$$\tilde{C}(t) = (1-b)\mathrm{e}^{-\gamma(t)}. \tag{12}$$

4. Entanglement sudden death

In the following, we will investigate how the entanglement evolution is affected by the reservoir. For a Lorentzian distribution, the spectral density is expressed as

$$J(\omega) = \frac{\alpha}{2\pi}\frac{\beta^2}{(\omega_1 - \omega)^2 + \beta^2}, \tag{13}$$

where α is the system-environment coupling strength and β is the width of the distribution also describing the inverse of the reservoir memory time. For $\alpha \gg \beta$, the influence of the environment is strong. This implies that the evolution of the system is a non-Markovian process. In the case of $\alpha \ll \beta$, the Markovian approximation may be used to describe the evolution of the system. Using the spectral density (13), the correlation function $B(t)$ becomes

$$B(t) = \int_0^t \mathrm{d}\tau \int_{-\infty}^{+\infty} \mathrm{d}\omega' \mathrm{e}^{\mathrm{i}(\omega-\omega')\tau} J(\omega'). \tag{14}$$

By first performing the integral with respect to τ, one obtains

$$\int_0^t \mathrm{d}\tau \mathrm{e}^{\mathrm{i}(\omega-\omega')\tau} = \frac{\sin(\omega'-\omega)t}{\omega'-\omega} - \mathrm{i}\,\frac{1-\cos(\omega'-\omega)t}{\omega'-\omega}.$$

Using the above relation, equation (14) is rewritten as

$$\begin{aligned} B(t) = & \int_{-\infty}^{+\infty} \mathrm{d}\omega' J(\omega') \frac{\sin(\omega'-\omega)t}{\omega'-\omega} \\ & - \mathrm{i}\int_{-\infty}^{+\infty} \mathrm{d}\omega' J(\omega') \frac{1-\cos(\omega'-\omega)t}{\omega'-\omega}. \end{aligned} \tag{15}$$

The second term in Eq. (15) is a time-dependent Lamb shift, which in the following will be neglected. Specializing to the Lorentzian spectral density given in Eq. (15), the real part of $B(t)$ is

$$\mathrm{Re}\, B(t) = \frac{\alpha\beta^2}{(\omega_1-\omega)^2+\beta^2}\left\{\frac{1}{2} + \frac{1}{2}\left[\frac{\omega_1-\omega}{\beta}\sin(\omega_1-\omega)t - \cos(\omega_1-\omega)t\right]\mathrm{e}^{-\beta t}\right\}.$$

Then

$$\begin{aligned} \gamma(t) = & \frac{\alpha\beta^2}{(\omega_1-\omega)^2+\beta^2}\left\{t - \frac{1}{\beta}\frac{\beta^2-(\omega_1-\omega)^2}{\beta^2+(\omega_1-\omega)^2}\right\} \\ & + \frac{\alpha\beta^2(\omega_1-\omega)^2}{[\beta^2+(\omega_1-\omega)^2]^2}\left\{\frac{\beta^2-(\omega_1-\omega)^2}{\beta(\omega_1-\omega)^2}\cos(\omega_1-\omega)t - \frac{2}{(\omega_1-\omega)}\sin(\omega_1-\omega)t\right\}\mathrm{e}^{-\beta t}. \end{aligned} \tag{16}$$

From Eqs. (11) and (16), we find that for a given initial state, the concurrence is irrelevant to the interaction between the qubits, but closely related to the environmental parameters (α, β). To understand how the concurrence of the qubits affects their entanglement, we study the numerical result of Eq. (12) for different values of the parameters (α, β).

By setting $\Omega = \omega_1 - \omega$, $\beta = 2(\omega_1 - \omega)$, $\alpha = m\beta$, the concurrence $\tilde{C}(t)$ as a function Ωt is shown in Figs. 1 and 2 for different values of the ratio $m = \alpha/\beta$.

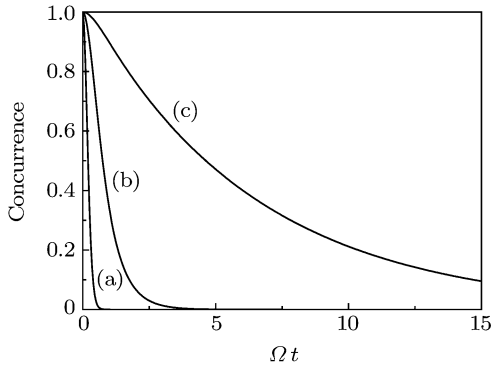


Fig. 1. Concurrence as a function of Ωt for different ratios α/β with $b = 0$, $\beta = 2\Omega$. (a) $m = 10$, (b) $m = 1$, (c) $m = 0.1$, where a Lorentzian bath distribution is used.

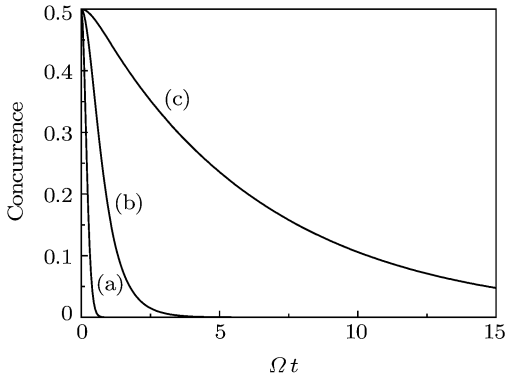


Fig. 2. Same as Fig. 1 but with $b = 0.5$, $\beta = 2\Omega$. (a) $m = 10$, (b) $m = 1$, (c) $m = 0.1$.

From Figs. 1 and 2, we find that, for the parameters $m = 1$ and 10, the entanglement measured by concurrence may disappear during the process of dynamic evolution, i.e., entanglement sudden death happens in the thermal reservoir. It is obvious that the entangled degree is dependent on the ratio $m = \alpha/\beta$. With increasing ratio, entanglement sudden death becomes faster. It is known that a large ratio corresponds to a strong correlation between the qubits and environment, which means that the non-Markovian process the system undergoes is important for entanglement sudden death. For a small ratio $m = 0.1$ corresponding to the Markovian process, entanglement sudden death is very slow during the process of dynamic evolution as shown in Figs. 1 and 2.

Comparing Fig. 1 with Fig. 2, the results show that the parameter b merely contributes to the amplitude of the concurrence function. Moreover, it has no effect on the sudden death of entangled state. If $b = 1$, however, the system has been unentangled with the concurrence being equal to zero in the evolving process. This may be understood because the initial state is unentangled for the case of $b = 1$ in terms of Eq. (11).

Note that for the parameter $\alpha = 0$ so that $\gamma(t) = 0$ and $\tilde{C}(t) = 1 - b$, ESD does not exist in this case and the quantum system dynamics is only determined by the initial condition of the entangled qubits.

In addition, the spectral width is a correlative parameter for the environmental dissipation, where the larger β implies a larger dissipation. In Fig. 3, using the parameters $\Omega = \omega_1 - \omega$, $\beta = (\omega_1 - \omega)/2$, $\alpha = m\beta$, we plot $\tilde{C}(t)$ as a function Ωt for different ratios α/β.

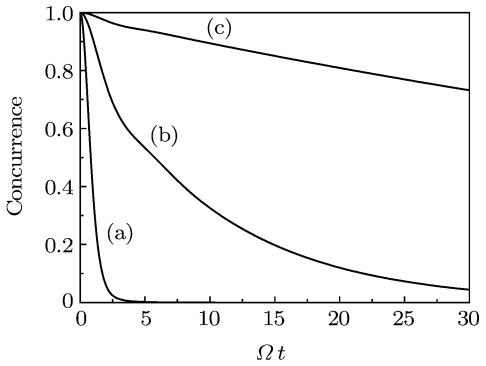


Fig. 3. Same as Figs. 1 and 2 with $b = 0$, $\beta = 0.5\Omega$. (a) $m = 10$, (b) $m = 1$, (c) $m = 0.1$.

Comparing Figs. 1 and 3, we may investigate the influence of environmental dissipation on the concurrence. Due to the feedback effect of the environment memory, the entanglement life time is shortened with the increase of environment dissipation (β), the reason for which is that the variation of environment per unit time accelerates with the increase of environment dissipation so as to influence the state of the qubit system by non-Markovian correlation. Thus the entanglement degree of the quantum system is reduced. We also find that from the corresponding curves in Figs. 1 and 3, with the increase of environment dissipation, the ESD occurring in the quantum system varies faster in the region of strong correlation (non-Markovian) than that of poor correlation (Markovian). Therefore, it is easier for entanglement to die for the non-Markovian process than the Markovian one.

5. Conclusions

In summary, entanglement sudden death has been investigated in detail for the state of initially entangled qubits varying with environment. The results show that the appearance of entanglement sudden death strongly depends on the environment, while the magnitude of entanglement relies on the initial coherences of the initially entangled qubits. Moreover, we find that the entangled life time is shorter for the non-Markovian process than for the Markovian one. Therefore, it is more important to include the non-Markovian effect than the Markovian one in ESD phenomena.

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