Sudden birth versus sudden death of entanglement for the extended Werner-like state in a dissipative environment*

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In this paper, we investigate the dynamical behaviour of entanglement in terms of concurrence in a bipartite system subjected to an external magnetic field under the action of dissipative environments in the extended Werner-like initial state. The interesting phenomenon of entanglement sudden death as well as sudden birth appears during the evolution process. We analyse in detail the effect of the purity of the initial entangled state of two qubits via Heisenberg XY interaction on the apparition time of entanglement sudden death and entanglement sudden birth. Furthermore, the conditions on the conversion of entanglement sudden death and entanglement sudden birth can be generalized when the initial entangled state is not pure. In particular, a critical purity of the initial mixed entangled state exists, above which entanglement sudden birth vanishes while entanglement sudden death appears. It is also noticed that stable entanglement, which is independent of different initial states of the qubits (pure or mixed state), occurs even in the presence of decoherence. These results arising from the combination of the extended Werner-like initial state and dissipative environments suggest an approach to control and enhance the entanglement even after purity induced sudden birth, death and revival.

Keywords: entanglement sudden death (birth), extended Werner-like state, stable entanglement
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1. Introduction

Entanglement, one of the most intriguing features in quantum mechanics, has been generally recognized to be a vital resource for the applications of quantum information and quantum computation.[1−7] The generation and the manipulation of entangled states have been extensively studied in the Heisenberg models,[8−15] including the Ising model, XY model, XXX model, XXZ model and XYZ model. In particular, the Heisenberg spin chain has been used to construct a quantum computer in many physical systems such as quantum dots,[16] nuclear spins,[17] superconductor[18] and optical lattice-based systems.[19,20] By proper encoding, the Heisenberg interaction alone can support universal quantum computation.[21] Therefore, the study of the entanglement properties of the Heisenberg spin chain has received much attention in the context of quantum information science. However, in the real world, a key question for entangled quantum states is the effect of decoherence due to the environment, which is not only a fundamental issue for quantum computation devices but also for the relation between quantum and classical physics. In most cases, this effect will introduce disentanglement and eliminate the quantum coherence in the investigated local system. There have been many investigations of decoherence in recent years, and careful investigations of well-understood model systems continue to produce surprises that add to fundamental understanding. For example, Yu and Eberly[22] have showed that entanglement may decrease abruptly to zero in a finite time due to the influence of quantum noise but not exponentially; this striking phenomenon is the so-called entanglement sudden death (ESD). Recently, as an interesting disentanglement behaviour, the phenomenon of ESD has attracted much attention; many theoretical works[23−36] have investigated the disentanglement dynamics in cavity-QED (Jaynes–Cummings

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and Tavis–Cummings models) and the spin chain. In particular, ESD has been observed recently in the lab for photonic qubits[37] and atomic ensembles.[38] Although the interaction with the environment could lead to the disentanglement of local qubits, entanglement can also be generated by coupling two qubits to a common third system. As demonstrated by Braun in Ref. [39], entanglement can also be induced when an environment plays the role of this third system. This is an important case because entanglement is generated by incoherent means which produces indirect interaction between the qubits. Common reservoirs tend to create entanglement rather than destroy it completely, since they indirectly couple the two qubits. As we will show in the following, even when the qubits are initially prepared in a factorized state, the correlation created by the environment can lead to a phenomenon which is known as entanglement sudden birth (ESB).[40,41] Opposite to the currently extensively discussed ESD, ESB is the creation of entanglement where initially unentangled qubits can be entangled after a finite evolution time. Meanwhile, a stable entangled state, in which once qubits become entangled they will never be disentangled, was also demonstrated in Ref. [42].

In those studies, the two-qubit entanglement dynamics has been previously analysed by taking pure Bell or Bell-like states as initial states. As a consequence, ESD and ESB are found to be sensitive to the initial entangled state. Namely, ESD and ESB may occur under a certain condition. In this paper, we present an exact calculation of the entanglement dynamics between two qubits coupling with a common environment at zero temperature in the case of more general initial conditions. The main purpose and motivation of the present article is to try to answer the following question: what happens to the qubit entanglement when we consider different system parameters and initial extended Werner-like states in the absence or presence of decoherence? We pay additional attention on the effects of the purity p of the initial entangled state. An important result is that ESD and ESB appear simultaneously and are sensitive to the mixedness and the degree of entanglement of the initial states as well as the system parameters. For certain classes of entangled states, entanglement sudden death occurs, while for certain initially factorized or mixed states, entanglement sudden birth takes place. Meanwhile, a critical purity of the initial mixed entangled state exists, above which ESB vanishes while ESD appears. In addition, the conditions for the conversion of ESD and ESB can be generalized when the initial entangled state is the extended Werner-like state. Thus, according to the relation between the conditions for the conversion of ESD and SDB, and the purity of the initial mixed entangled state, we can know which is the more efficient control parameter to prevent ESD from occurring. Moreover, in our two-qubit model system, for any initial state, including the common case in which the two qubits are initially unentangled and mixed, the system reaches the same steady state of pairwise entanglement in spite of population relaxation. The value of steady-state entanglement is sensitive to both the anisotropy between qubits and to the energy level separation. These parameters can be varied in some particular physical realization of our model system, so the magnitude of steady state entanglement may also be controlled.

2. Heisenberg XY model with initial extended Werner-like state

Consider an anisotropic two-qubit system interacting through an XY Heisenberg coupling under the action of dissipative environments in an external magnetic field ω along the z-axis; the corresponding Hamiltonian reads

$$H = J(S_1^+ S_2^- + S_1^- S_2^+) + \Delta(S_1^+ S_2^+ + S_1^- S_2^-) + \omega(S_1^z + S_2^z), \quad (1)$$

where $J = (J_x + J_y)/2$, $\Delta = (J_x - J_y)/2$, and $S^{\pm} = S^x \pm \mathrm{i}\, S^y$ are the spin raising and lowering operators, the parameter Δ describes the spatial anisotropy of the spin–spin interaction. The anisotropy parameter can be controlled by varying J_x and J_y, which may be achieved for an optical lattice system, and the effective external magnetic field is defined as the energy levels of our qubits. The description of the time evolution of an open system is provided by the master equation, which can be written most generally in the Lindblad form with the assumption of weak system–reservoir coupling and the Born–Markov approximation. The Lindblad equation for our case thus reads[43]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H, \rho] + \gamma \sum_{j=1,2} \left[S_j^- \rho S_j^+ - \frac{1}{2} \left\{ S_j^+ S_j^-, \rho \right\} \right], \quad (2)$$

where the Lindblad operator describes the population relaxation of the upper state of each qubit due to

its environment, and thus can be written in the form $\sqrt{\gamma}S^-$, γ is the relaxation rate which is supposed to be the same for the two qubits; this assumption is reasonable provided that the interaction does not significantly alter the energy level separations. A pair of braces {} means an anticommutator.

For the initial state of qubit-pair AB, instead of Bell-like and Werner states,[44] we shall consider the following extended Werner-like state

$$\rho_{AB}(0) = p\,|\Phi\rangle_{ABAB}\langle\Phi| + \frac{1-p}{4}I_{AB} \tag{3}$$

with p being the purity of the initial state of qubits AB, I_{AB} the 4×4 identity matrix and

$$|\Phi\rangle_{AB} = (\sin\theta\,|11\rangle + \cos\theta\,|00\rangle)_{AB}\,, \tag{4}$$

the Bell-like state. This class of mixed state arises naturally in a wide variety of physical situations.[45,46] In particular, it includes pure Bell states as well as the well-known Werner mixed state as special cases. Obviously, the state in Eq. (3) reduces to the standard Werner mixed state when $\theta = \pi/4$ and to the Bell-like pure state when $p = 1$. Only when $p > 1/3$ is the Werner state entangled. By dealing with the above extended Werner-like state, we are able to study the effect of mixedness of the initially entangled state. Both the Bell-like state and Werner state, and so the extended Werner-like state, belong to the so-called X-class state whose density matrix is of the form

$$\rho = \begin{pmatrix} \rho_{1,1} & 0 & 0 & \rho_{1,4} \\ 0 & \rho_{2,2} & \rho_{2,3} & 0 \\ 0 & \rho_{3,2} & \rho_{3,3} & 0 \\ \rho_{4,1} & 0 & 0 & \rho_{4,4} \end{pmatrix} \tag{5}$$

in the two-qubit product state basis spanned by $\{|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle\}$. The initial states are therefore more general than the Werner states since they contain the possibility that their pure part is not maximally entangled. They, as said before, thus permit us to study the effect of both the initial states' mixedness and the degree of entanglement of their pure part on the entanglement dynamics. In order to describe the entanglement dynamics of the bipartite system, we use concurrence, which was proposed by Wootters[47] as a measure of entanglement. The concurrence $C = 0$ corresponds to a separable state and $C = 1$ to a maximally entangled state. Nonzero concurrence means that the two qubits are entangled. For a system described by the above density matrix, which can denote either a pure or a mixed state, in Eq. (3), the concurrence is defined as

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \tag{6}$$

where λ_1, λ_2, λ_3, λ_4 are the eigenvalues in decreasing order of the spin-flipped density operator R defined by $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, $\tilde{\rho}$ denoting the complex conjugate of ρ; σ_y is the usual Pauli matrix. Then the concurrence can be expressed as

$$\begin{aligned} C = \max[0, 2(&\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}), \\ &2(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}})]. \end{aligned} \tag{7}$$

3. Detailed numerical analysis and results

The two-qubit entanglement dynamics has been previously analysed by taking pure Bell-like states as initial states.[23] However, as said before, it is of interest to obtain the entanglement dynamics of two independent qubits, each locally interacting with a reservoir, in the case of more general initial conditions and in particular for Werner states. The procedure we have developed is applicable to any two-qubit initial state and therefore also to Werner states. In particular, in this section we consider two-qubit initial states as an extension of the Werner states. These "extended" Werner-like (EWL) states are mixed states that may reduce to Werner mixed states or to Bell-like pure states. They will allow us to study the effects of both the mixedness and the degree of entanglement of the initial states on the entanglement dynamics. In the following, we analyse both analytically and numerically the time-dependent evolution of the entanglement of a two-qubit system for some typical initial states, including a pure, separable/entangled initial state; the pure maximally entangled state, and the mixed Werner state. Since the explicit expressions of solutions of Eq. (7) are very complicated, here we skip the details and give our results in terms of figures.

In Figs. 1 and 2, we plot the time evolution of the concurrence for various values of the degree of entanglement of their pure part θ and of the purity p without decoherence when the qubits are initially in different states. The case for $p = 1$, that is, when the initial EWL states reduce to the pure Bell-like states, has been previously treated. The concurrence time evolution has been analysed for different values of the

anisotropic parameter in an external magnetic field. So we fixed the above parameter values, and pay more attention to the effects of both the mixedness and the degree of entanglement of the initial states on the entanglement dynamics. When $\theta = 0$, the pure part of the qubits is initially in a separate state. From Fig. 1, one can find that the entanglement evolves periodically in the absence of decoherence and the initial unentangled system evolves into an entangled state. In addition, one can also observe that there are two interesting features in the entanglement evolution. One is that the pairwise entanglement oscillates with time continuously at high purity, but as the impurity increases the entanglement evolves to zero and will remain zero for a period of time before the entanglement recovers; this striking phenomenon is the so-called ESD first proposed by Yu and Eberly. The other is opposite to the feature of ESD; entanglement sudden birth (ESB) is the creation of entanglement when initially unentangled qubits can be entangled after a finite evolution time. For different initially mixed states we can see clearly that concurrence C initially maintains a constant value of zero, then when a critical point t_c is reached, C increases monotonically with the increase of scaled time t. Furthermore, we find that the sudden death time and sudden birth time are prolonged and the maximal value that the entanglement can reach decreases with increasing impurity. It can be easily understood that the impurity generates some effective interaction between any two qubits which are initially uncoupled. When the impurity is very low, the effects of impurity are not strong enough to weaken the entanglement, so continuous entanglement evolution appears. But the higher the impurity, the stronger the effect of impurity on the entanglement evolution, and as a result, ESD and ESB become more obvious with increasing impurity. When $p < 1/3$, the entanglement will remain zero, which can be seen in Fig. 1(b). That is to say, one finds that there is initially entanglement when $p > p_c$, thus entanglement exists for these states only if $p > 1/3$. In Fig. 2, we show the time dependence of the concurrence when the qubits are initially in the Werner mixed states with the degree of entanglement, represented by $\theta = \pi/4$. The concurrence presents similar behaviours to the appearance of revivals of entanglement but without the phenomenon of ESD for $p = 1$. The new aspects we wish to examine here correspond to the case when the purity p of the initial states is less than 1. As the decrease of the concurrences value at $t = 0$ with the decrease of the purity p is linked to the corresponding increase of the initial states' mixedness, we can find that ESD also appears if $p > 1/3$, and the death time depends on the degree of initial entanglement between the two qubits and is prolonged with increasing impurity. For intermediate values of purity p, entanglement revivals after finite dark periods occur for the initial Werner-like states, which differs from previous results in pure Bell-like states.

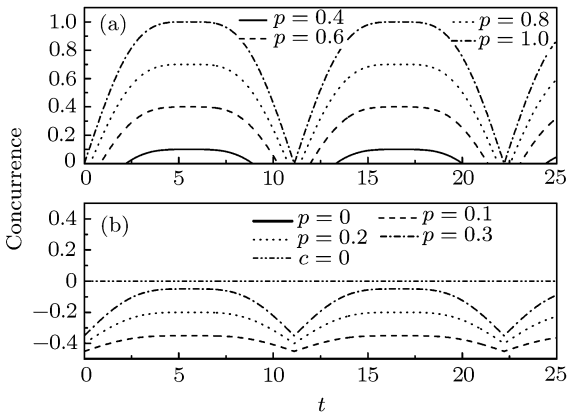


Fig. 1. The time evolution of the concurrence for various values of purity p without taking population relaxation into account when the qubits are initially in the state $\theta = 0$. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$, $\gamma = 0$.

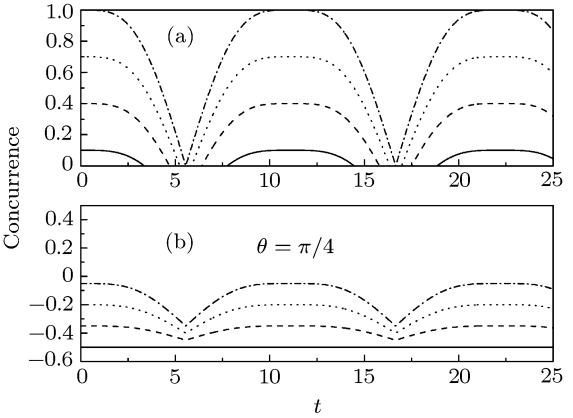


Fig. 2. The time evolution of the concurrence for various values of purity p with the initial state $\theta = \pi/4$ without decoherence. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$, $\gamma = 0$.

Another aspect of interest is how the entanglement dynamics is influenced by the value of γ, which regulates the degree of coupling strength with the environment. The above analysis can easily be extended to study entanglement dynamics to start from different initial conditions and to take into account population relaxation effects. In Figs. 3, 4 and 5, we display the concurrence as a function of t for different values

of purity p and the initial degree of entanglement θ when the qubits are in extended Werner-like states. The two-qubit state can evolve into a stationary entangled state under collective decay from the initial unentangled state. In other words, decoherence drives the qubits into a stationary entangled state instead of completely destroying the entanglement. Moreover, the stationary entanglement of two qubits is independent of the degree of purity p. Figure 3 shows that ESB occurs for both the initial mixed states but at different times, indicating a threshold value of parameter t, only above which does concurrence begin to be nonzero, i.e. the quantum correlation starts to appear, and the differences depend on the values of purity p. In particular, with the increase of the purity of the initial states, the delayed time for the appearance of ESB is retarded for both states. The ESB time is sensibly affected by slight purity changes. Compared with the case of $\gamma = 0$, it is shown that, although no interaction is present or mediated between the qubits, there is a creation of their entanglement, which means that ESB occurs for the initial Werner-like states for small values of p, which shows a rather different behaviour of concurrence for the two initial pure states in Figs. 1 and 2. The concurrences given in Fig. 5, which also shows that ESB occurs, give an evident difference in the entanglement dynamics of the two initial mixed states in Fig. 3. For mixed initial states $p < 1$, the sudden birth time is prolonged with increasing purity of the initial states ($\theta = \pi/2$), which is different from the result for $\theta = 0$, where increasing the purity can cause the length of the time intervals for nonzero entanglement to be shorter. This may be linked to the fact that the occurrence of ESB for both the initial pure or mixed states, independently from the degree of entanglement, is due to the population with time of the qubit excited states. The purity of the initial states and the mechanism of decoherence, which cause a $1 \longrightarrow 0$ transition, make the populations of the totally excited state increase more slowly than the ground state of the two-qubit system. From the viewpoint that entanglement is a resource, it would be better to have some methods to control the occurrence of ESD. In Fig. 4, we display the time dependence of the concurrence for a different initially mixed state ($\theta = \pi/4$). The phenomenon of ESB may occur for small values of p, and the purity constant between two qubits can cause the birth time for zero entanglement to be shorter. With increasing purity p, it is observed that a critical purity constant exists, above which ESB vanishes while ESD appears. Continuing to increase the value of p, the ESD disappears. It is the purity that leads to considerable difference in entanglement evolution, hence entanglement is very sensitive to any small change of purity. That is to say, by adjusting purity alone, one can also obtain both ESD and ESB. Therefore, this can provide us with a feasible way to manipulate and control the entanglement by adjusting the purity p. These results elucidate how the two-qubit entanglement dynamics in a dissipative environment depends sensibly on the

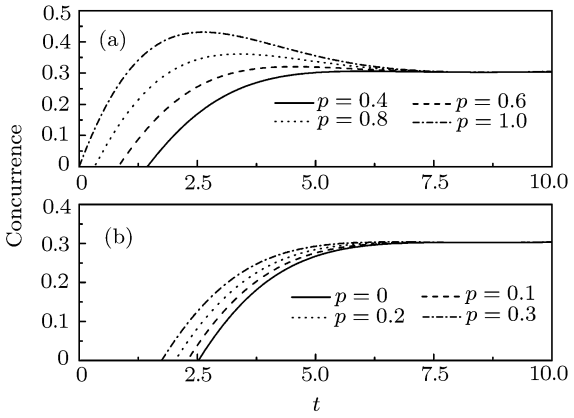


Fig. 3. The time evolution of the concurrence for various values of purity p with the initial state $\theta = 0$ and $\gamma = 0.5$. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$.

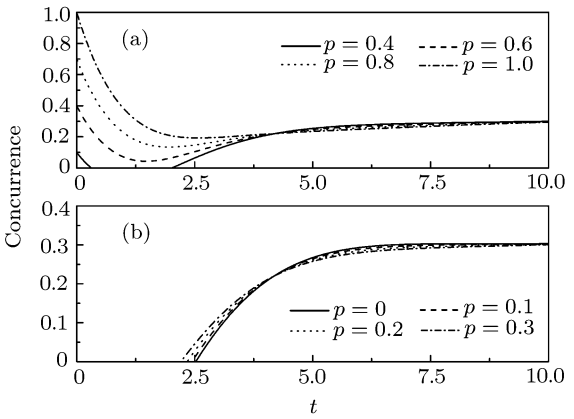


Fig. 4. The time evolution of the concurrence for various values of purity p with the initial state $\theta = \pi/4$ and $\gamma = 0.5$. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$.

values of parameters like purity or degree of entanglement of the initial state, and show that different conditions may lead to very different time behaviours

of the entanglement.

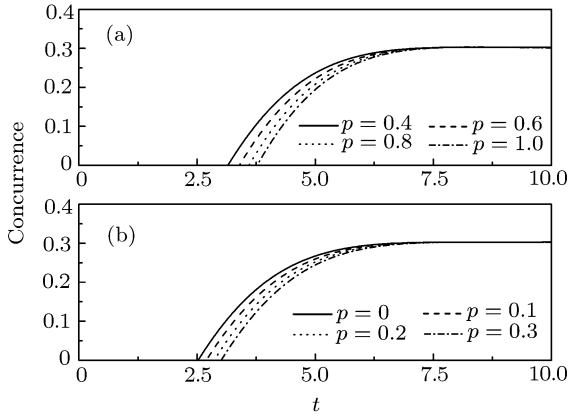


Fig. 5. The time evolution of the concurrence for various values of purity p with the initial state $\theta = \pi/2$ and $\gamma = 0.5$. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$.

From the above discussion we see that even in the presence of population relaxation, one can still obtain nonzero entanglement from the time evolution of the system. Despite the presence of decoherence and impurity, the results in Fig. 6 show that the concurrence reaches the same steady value, after some oscillatory behaviour, for a given set of system parameters regardless of the initial state of the system. Then a natural question is addressed: how much entanglement can we obtain from the time evolution of the initial extended Werner-like states? To answer this question,

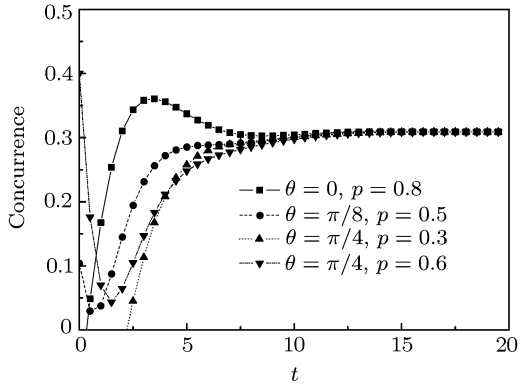


Fig. 6. The time evolution of the concurrence and steady-state concurrence for various parameter values θ and p. The parameter values for the plot are $\omega = 0.2$, $\Delta = 0.2$.

we have performed accurate calculations. As a final discussion, we consider the parameter dependence of the steady-state entanglement. To do this, we take the infinite time limit, which yields

$$\rho_{11} = \rho_{22} = \rho_{33} = \frac{\Delta^2}{4(\omega^2 + \Delta^2) + \gamma^2},$$
$$\rho_{44} = \frac{4\omega^2 + \Delta^2 + \gamma^2}{4(\omega^2 + \Delta^2) + \gamma^2},$$
$$\rho_{14} = \rho_{41}^* = -\frac{\Delta(2\omega + \mathrm{i}\,\gamma)}{4(\omega^2 + \Delta^2) + \gamma^2},$$
$$\rho_{23} = \rho_{32} = 0. \tag{8}$$

From Eq. (8), one can obtain the analytical expression of the steady-state concurrence, at $T = 0$, and the corresponding steady concurrence is found to be

$$C_{\mathrm{s}} = \frac{2\Delta\sqrt{4\omega^2 + \gamma^2} - 2\Delta^2}{4(\omega^2 + \Delta^2) + \gamma^2}. \tag{9}$$

The steady-state concurrence is seen to depend on the system parameters ω, Δ, and γ while being independent of J, the purity and the degree of entanglement of the initial states.

4. Conclusion

In conclusion, we have given an analytic solution for the evolution of entanglement in different initial system states and investigated how the decoherence, the purity and the degree of entanglement of the initial states affect the entanglement dynamics of a two-qubit system in extended Werner-like states. The dynamics of bipartite entanglement in terms of the concurrence between the qubits of our model shows interesting features. We find that ESD and ESB appear simultaneously, depending on the initial state, the anisotropic parameter, the external magnetic field, and the coupling strength with the environment. Our results also show that when initial states are not pure, the ESD time can be shorter and the occurrence of entanglement revivals can extend the time during which entanglement can be usefully exploited. A stable entanglement, controllable by the system parameters, will always be obtained for zero or a finite value. Our results will shed light on the understanding of entanglement dynamics of quantum systems with environmental effects as well as ESD and ESB in a correlated environment. We believe that our results may contribute to the explanation of the delay and avoidance of ESD of initially prepared two-qubit X-states in this open quantum system and the behaviour of quantum entanglement in realistic conditions, that is, when the effect of the environment on the quantum system is taken into account. In practice, observation of the effects we have discussed should be achievable with current experimental technologies.

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