The symmetries of wave equations on new lattices*

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This paper focuses on studying the symmetry of a practical wave equation on new lattices. It is a new step in that the new lattice equation is applied to reduce the discrete problem of motion of an elastic thin homogeneous bar. The equation of motion of the bar can be changed into a discrete wave equation. With the new lattice equation, the translational and scaling invariant, not only is the infinitesimal transformation given, but the symmetry and Lie algebras are also calculated. We also give a new form of invariant called the ratio invariant, which can reduce the process of the computing invariant with the characteristic equation.

Keywords: new lattice equation, symmetry, discrete wave equation, invariant
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1. Introduction

It is well known that continuous symmetry is a hot topic; aspects of Lie group theory have been described in many books and lecture notes.[1−19] In recent papers, many methods have appeared to solve discrete differential equations. The lattice equation has long been used to reduce the differential equations.[6,14−18] It has developed into a powerful tool to solve differential equations. In fact, it is unnecessary to know its complete continuous solution. We can turn a continuous problem into a discrete problem and look for part of its solution. The lattice equation method has long been used to study wave equations. Not all difficult wave equations can be solved by the lattice equation only. However, a suitable lattice equation will make the problem simpler; otherwise, the issue will be hard. At the beginning of this paper, a new lattice equation will be introduced which has been developed in spectral physics. For two independent variables, even though we are not able to solve it by the lattice equation only, we can use the translational invariant and scaling invariant to make the discrete wave equation involve one variable rather than two.

In this paper, we make an attempt to adapt the ideas of invariance to study the wave equation on the new lattice. The construction of the invariant is dependent on neither the Lagrangian nor the Hamiltonian. It is based on the infinitesimal. In view of the fact of that invariant, we can also construct the ratio invariant. So in addition to the method of the characteristic equation, we obtain the ratio invariant method. While we can put the same wave equation into different lattice equations, the invariant will be different. So the new lattice will lead to a new form of the solution which has never appeared before. It is necessary to go beyond the symmetry of the continuous wave equation and to consider Lie group actions at different points of the lattice simultaneously.

The structure of this paper is arranged as follows: In Section 2, the new lattice equation[19] is given. In Section 3, we discuss the symmetry and Lie algebras of the discrete wave equation. Furthermore we give a practical example to illustrate these results.

2. The new lattice equation

Let f be a lattice function. The shift operator E is defined as follows:

$$Ef(n) = f(n+1), \tag{1}$$

$$E\varphi_n = U_n(u_n, \lambda)\varphi_n, \tag{2}$$

where

$$\varphi_n = \begin{pmatrix} \varphi_n^1 \\ \varphi_n^2 \end{pmatrix}, \tag{3}$$

$$u_n = \begin{pmatrix} v_n \\ \omega_n \end{pmatrix}, \tag{4}$$

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$$U_n = \begin{pmatrix} 0 & \alpha v_n^2 + \beta v_n \\ -1 & \lambda - \omega_n \end{pmatrix}. \quad (5)$$

We suppose that

$$\varphi_n = \begin{pmatrix} x_n \\ t_n \end{pmatrix}, \quad (6)$$

and substituting Eqs. (6) and (5) into Eq. (2) leads to

$$\begin{pmatrix} x_{n+1} \\ t_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & \alpha v_n^2 + \beta v_n \\ -1 & \lambda - \omega_n \end{pmatrix} \begin{pmatrix} x_n \\ t_n \end{pmatrix}. \quad (7)$$

Following this, we get

$$x_{n+1} = \alpha v_n^2 t_n + \beta v_n t_n, \ \ t_{n+1} = -x_n + \lambda t_n - \omega_n t_n. \quad (8)$$

When $\alpha = 1, \ \beta = 0$, we obtain the Toda lattice equation

$$x_{n+1} = v_n^2 t_n, \ \ \ t_{n+1} = -x_n + \lambda t_n - \omega_n t_n. \quad (9)$$

3. Symmetry of the discrete wave equation and Lie algebras

3.1. A practical physical case

The equation of motion of a hyperelastic homogeneous bar, whose cross-sectional area is variable along the bar,[20] is

$$\rho u_{xx} = T_x + \frac{S'(x)}{S(x)} T, \quad (10)$$

where ρ is the constant mass density, $u = x - y$ is the displacement, y is the coordinate of the point P in the present reference frame, $T(u_x)$ is the tension and $S(x)$ is the cross-sectional area which varies with an exponential law

$$S = S_0 \, \mathrm{e}^{\mu x}, \quad (11)$$

when S_0 and μ are constants, it takes the following form:

$$u_{tt} = \frac{1}{\rho} \frac{\mathrm{d}T}{\mathrm{d}u_x} u_{xx} + \frac{\mu}{\rho} T. \quad (12)$$

We take into consideration the following form for the tension function

$$T = T_0 \log(1 + u_x), \quad (13)$$

then equation (12) reads

$$u_{tt} = \frac{\mu T_0}{\rho(1 + u_x)} u_{xx} + \frac{\mu T_0}{\rho} \log(1 + u_x). \quad (14)$$

The discrete derivatives are defined as[21]

$$\Delta_t = \frac{T_t - 1}{\sigma_t}, \ \ \Delta_{xx} = \frac{T_x^2 - 2T_x + 1}{\sigma_x^2}, \quad (15)$$

where σ_t and σ_x are steps in the t and x directions, respectively.

By Eq. (15), equation (14) will be changed to

$$\begin{aligned} &\frac{u_{n+2} - 2u_{n+1} + u_n}{(t_{n+1} - t_n)^2} \\ &= \frac{\mu T_0}{\rho \left(1 + \dfrac{u_{n+1} - u_n}{x_{n+1} - x_n}\right)} \frac{u_{n+2} - 2u_{n+1} + u_n}{(x_{n+1} - x_n)^2} \\ &\quad + \frac{\mu T_0}{\rho} \log\left(1 + \frac{u_{n+1} - u_n}{x_{n+1} - x_n}\right). \end{aligned} \quad (16)$$

3.2. A scaling-invariant solution

A scaling-invariant solution will be written as: $u(x,t) = u(z), z = x/t$. Then u only depends on one variable z. Furthermore, equation (16) will be put into another form,

$$\begin{aligned} &\frac{u(z_{n+2}) - 2u(z_{n+1}) + u(z_n)}{(t_{n+1} - t_n)^2} \\ &= \frac{\mu T_0 [u(z_{n+2}) - 2u(z_{n+1}) + u(z_n)]}{\rho \left(1 + \dfrac{u(z_{n+1}) - u(z_n)}{x_{n+1} - x_n}\right) (x_{n+1} - x_n)^2} \\ &\quad + \frac{\mu T_0}{\rho} \log\left(1 + \frac{u(z_{n+1}) - u(z_n)}{x_{n+1} - x_n}\right). \end{aligned} \quad (17)$$

From Eq. (9), we conclude

$$\begin{aligned} \frac{x_{n+1}}{t_{n+1}} &= \frac{v_n t_n}{-x_n + \lambda t_n - \omega_n t_n} \\ &= \frac{v_n}{-x_n/t_n + \lambda - \omega_n} \ (\lambda = \omega_n) \\ &= z_{n+1} \\ &= -\frac{v_n}{z_n}. \end{aligned} \quad (18)$$

Let $z_n = z$. Combined with Eq. (18), equation (17) can be reduced to

$$\begin{aligned} &\frac{u\left(z\dfrac{v_{n+1}}{v_n}\right) - 2u\left(-\dfrac{v_n}{z}\right) + u(z)}{(t_{n+1} - t_n)^2} \\ &= \frac{\mu T_0 \left[u\left(z\dfrac{v_{n+1}}{v_n}\right) - 2u\left(-\dfrac{v_n}{z}\right) + u(z)\right]}{\rho \left(1 + \dfrac{u\left(-\dfrac{v_n}{z}\right) - u(z)}{x_{n+1} - x_n}\right) (x_{n+1} - x_n)^2} \\ &\quad + \frac{\mu T_0}{\rho} \log\left(1 + \frac{u\left(-\dfrac{v_n}{z}\right) - u(z)}{x_{n+1} - x_n}\right). \end{aligned} \quad (19)$$

With the condition

$$u\left(z\frac{v_{n+1}}{v_n}\right)-2u\left(-\frac{v_n}{z}\right)+u(z)=0,$$
$$u\left(-\frac{v_n}{z}\right)-u(z)=0, \tag{20}$$

the general solution can be written as

$$u=az+b/z+cz^2+d/z^2+f, \tag{21}$$

where a,b,c,d and f must satisfy the following condition

$$\begin{aligned}&az\frac{v_{n+1}}{v_n}+b\frac{v_v}{zv_{n+1}}+c\frac{z^2v_{n+1}^2}{v_n^2}+d\frac{v_n^2}{z^2v_{n+1}^2}+f\\ =&-a\frac{v_n}{z}-b\frac{z}{v_n}+c\frac{v_n^2}{z^2}+d\frac{z^2}{v_n^2}+f\\ =&az+b\frac{1}{z}+cz^2+\frac{1}{z^2}+f,\end{aligned}$$

where $a=a(v_n),b=b(v_n),c=c(v_n)$.

3.3. A translational invariant solution

From Eq. (9), we obtain

$$\begin{aligned}t_{n+1}-t_n&=-x_n+\lambda t_n-\omega_n t_n\\&\quad-(-x_{n-1}+\lambda t_{n-1}-\omega_{n-1}t_{n-1})\\&=-[x_n-(\lambda-\omega_n)t_n]+x_{n-1}\\&\quad-(\lambda-\omega_n)t_{n-1}.\end{aligned} \tag{22}$$

With the translational invariant $z=x+at$, assuming $\lambda-\omega_n=-a$, we get the variable steps

$$\begin{aligned}\sigma_t&=t_{n+1}-t_n\\&=-(x_n+at_n)+x_{n-1}+at_{n-1}\\&=-z_n+z_{n-1},\end{aligned} \tag{23}$$

$$\begin{aligned}\sigma_x&=x_{n+1}-x_n\\&=v_nt_n-v_{n-1}t_{n-1}\\&=v_n(t_n-t_{n-1})+t_{n-1}(v_n-v_{n-1}).\end{aligned} \tag{24}$$

If $v_n=v_{n-1}$ $(n=1,2\ldots)$, i.e. $v_n=c$, equation (24) reads

$$\begin{aligned}\sigma_x&=x_{n+1}-x_n\\&=v_n(t_n-t_{n-1})\\&=c(-z_{n-1}+z_{n-2}).\end{aligned} \tag{25}$$

Substituting Eqs. (23) and (24) into Eq. (19), we have

$$\begin{aligned}&\frac{u(z_{n+2})-2u(z_{n+1})+u(z_n)}{(z_n-z_{n-1})^2}\\=&\frac{\mu T_0\left(u(z_{n+2})-2u(z_{n+1})+u(z_n)\right)}{c\rho\left[c(z_{n-1}-z_{n-2})^2-(z_{n-1}-z_{n-2})(u(z_{n+1})-u(z_n))\right]}+\frac{\mu T_0}{\rho}\log\left(1+\frac{u(z_{n+1})-u(z_n)}{c(z_{n-1}-z_{n-2})}\right).\end{aligned} \tag{26}$$

3.4. The r-parameter Lie groups of transformations

For an r-parameter Lie group of transformations

$$x^*=X(x;\varepsilon), \tag{27}$$

assuming that $x=(x_1,x_2\ldots,x_n)$, and the parameters are denoted by $\varepsilon=(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n)$, we make that

$$\begin{aligned}&\xi_{\alpha j}=\frac{\partial x_j^*}{\partial\varepsilon_\alpha}\Big|_{\varepsilon=0}=\frac{\partial X_j(x;\varepsilon)}{\partial\varepsilon_\alpha}\Big|_{\varepsilon=0},\\&\alpha=1,2,\ldots,r;\quad j=1,2,\ldots,n.\end{aligned} \tag{28}$$

The infinitesimal generator X_α, corresponding to the parameter ε_α of the r-parameter Lie group of transformations, is

$$X_\alpha=\sum_{j=1}^{n}\xi_{\alpha j}(x)\frac{\partial}{\partial x_j},\quad \alpha=1,2,\ldots,r. \tag{29}$$

Equation (26) is related to five points. Assuming that $z_{n-2},z_{n-1},z_n,z_{n+1}$ and z_{n+2} are independent variables, we have

$$\begin{aligned}z_{n-2}^*&=\frac{(1+\varepsilon_3)z_{n-2}+\varepsilon_4z_{n-1}+\varepsilon_5}{\varepsilon_1z_{n-2}+\varepsilon_2z_{n-1}+1}+z_n\varepsilon_3^2+z_{n-2},\\z_{n-1}^*&=\frac{\varepsilon_6z_{n-2}+(1+\varepsilon_7)z_{n-1}+\varepsilon_8}{\varepsilon_1z_{n-2}+\varepsilon_2z_{n-1}+1}+z_{n+1}\varepsilon_1^4,\\z_n^*&=\frac{z_{n+2}\varepsilon_1^2}{\varepsilon_1^2z_{n-2}+\varepsilon_2^3z_{n-1}+1},\\z_{n+1}^*&=z_{n+1},\\z_{n+2}^*&=z_{n+2}.\end{aligned} \tag{30}$$

Then substituting Eq. (30) into Eq. (28), the coefficients of infinitesimal generator X_α are determined. Then, from Eq. (29), the eight infinitesimal generators are obtained as follows:

$$X_1=\sum_{i=-2}^{2}\frac{\partial z_{n+i}^*}{\partial\varepsilon_1}\Big|_{\varepsilon=0}\frac{\partial}{\partial z_{n+i}}$$

$$= -z_{n-2}^2 \frac{\partial}{\partial z_{n-2}} - z_{n-2} z_{n-1} \frac{\partial}{\partial z_{n-1}},$$

$$X_2 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_2}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}}$$

$$= -z_{n-2} z_{n-1} \frac{\partial}{\partial z_{n-2}} - z_{n-1}^2 \frac{\partial}{\partial z_{n-1}},$$

$$X_3 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_3}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = z_{n-2} \frac{\partial}{\partial z_{n-2}},$$

$$X_4 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_4}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = z_{n-1} \frac{\partial}{\partial z_{n-2}},$$

$$X_5 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_5}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = \frac{\partial}{\partial z_{n-2}},$$

$$X_6 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_6}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = z_{n-2} \frac{\partial}{\partial z_{n-1}},$$

$$X_7 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_7}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = z_{n-1} \frac{\partial}{\partial z_{n-1}},$$

$$X_8 = \sum_{i=-2}^{2} \frac{\partial z_{n+i}^*}{\partial \varepsilon_8}\Big|_{\varepsilon=0} \frac{\partial}{\partial z_{n+i}} = \frac{\partial}{\partial z_{n-1}}. \tag{31}$$

In order to find out the invariants, we give the characteristic equations

$$\frac{\mathrm{d}x_1}{\xi_1(x)} = \frac{\mathrm{d}x_2}{\xi_2(x)} = \cdots = \frac{\mathrm{d}x_n}{\xi_n(x)}, \tag{32}$$

where $x_1, x_2, \ldots, x_n$ are independent variables, and ξ_i are the coefficients of infinitesimal generator X_α, $i = 1, 2, \ldots, 5$, $\alpha = 1, 2, \ldots, 8$, then we show the characteristic equation with respect to X_1

$$\frac{\mathrm{d}z_{n-2}}{z_{n-2}^2} = \frac{\mathrm{d}z_{n-1}}{z_{n-2} z_{n-1}}. \tag{33}$$

From Eq. (33), we calculate the invariant

$$I_1 = \frac{z_{n-1}}{z_{n-2}} = \frac{x_{n-1} + a t_{n-1}}{x_{n-2} + a t_{n-2}}, \tag{34}$$

the characteristic equation of X_2 is

$$\frac{\mathrm{d}z_{n-2}}{z_{n-2} z_{n-1}} = \frac{\mathrm{d}z_{n-1}}{z_{n-1}^2},$$

then the invariant with respect to X_2 will be written as

$$I_2 = \frac{z_{n-2}}{z_{n-1}} = \frac{x_{n-2} + a t_{n-2}}{x_{n-1} + a t_{n-1}}. \tag{35}$$

In the same way, we will calculate the other six invariants of X_i for $i = 3, 4, \ldots, 8$.

$$I_3 = I\boldsymbol{n} z_{n-2} = In\left(x_{n-2} + a t_{n-2}\right),$$
$$I_4 = \frac{z_{n-2}}{z_{n-1}} = \frac{x_{n-2} + a t_{n-2}}{x_{n-1} + a t_{n-1}},$$
$$I_5 = z_{n-2} = x_{n-2} + a t_{n-2},$$
$$I_6 = \frac{z_{n-1}}{z_{n-2}} = \frac{x_{n-1} + a t_{n-1}}{x_{n-2} + a t_{n-2}},$$
$$I_7 = I\boldsymbol{n} z_{n-2} = In\left(x_{n-2} + a t_{n-2}\right),$$
$$I_8 = z_{n-1} = x_{n-1} + a t_{n-1}. \tag{36}$$

3.5. Ratio invariant

Make new variables that can be written as

$$I_1 = u_{n+1} - u_n, \quad I_2 = u_n - u_{n-1}, \quad \xi_1 = \frac{I_1}{I_2}. \tag{37}$$

The operator X, keeping only the necessary terms, is

$$X = I_1 \partial_{I_1} + I_2 \partial_{I_2}, \tag{38}$$

where I_1 and I_2 are not invariants of X but ξ_1 is. It is vital for us to consider a nonuniform net

$$\sigma_t^{n+1} = \phi(x, t). \tag{39}$$

We extend the operator by σ_t^{n+1} and σ_t^n, i.e.,

$$X = x^2 \frac{\partial}{\partial x} + \sigma_t^{n+1}\left(2x + 2t + \sigma_t^{n+1}\right) \frac{\partial}{\partial \sigma_t^{n+1}} + \sigma_t^n\left(2x + 2t - \sigma_t^n\right) \frac{\partial}{\partial \sigma_t^n}. \tag{40}$$

It is important to construct an invariant net starting from the invariants of the group G:

$$y_1 = \frac{\mathrm{e}^{2t/x} \sigma_t^{n+1}}{x^2\left(\sigma_t^{n+1} + 2x + 2t\right)},$$
$$y_2 = \frac{x^2\left(2x + 2t - \sigma_t^n\right)}{\sigma_t^n\, \mathrm{e}^{2t/x}}. \tag{41}$$

We construct, for example, an invariant nonuniform net on the interval $(0, L_0)$. Taking $L_0 = y_1$, we get the equality

$$\sigma_t^{n+1} = \frac{L_0 x^2\left(2x + 2t\right)}{\mathrm{e}^{2t/x} - L_0 x^2}. \tag{42}$$

Owing to both y_1 and y_2 being arbitrary constants, we make

$$y_2 = \mathrm{e}^{f(x,t)} y_1 = L_0\, \mathrm{e}^{f(x,t)}, \tag{43}$$

then we get

$$\sigma_t^n = \frac{x^2\left(2x + 2t\right)}{L_0\, \mathrm{e}^{f(x,t) + 2t/x} + x^2}. \tag{44}$$

Theorem For time steps σ_t^n and σ_t^{n+1} if there exists a function $f = f(x, t)$ that satisfies the following equation

$$L_0x^2\frac{\left[\mathrm{e}^{f(x,t)+2t/x}\left(\dfrac{\partial f}{\partial x}-2t/x^2\right)+2x\right]\left(\mathrm{e}^{2t/x}-L_0x^2\right)+\left(\mathrm{e}^{f(x,t)+2t/x}+x^2\right)\left(\dfrac{2t}{x^2}\mathrm{e}^{2t/x}+2xL_0\right)}{\left(\mathrm{e}^{2t/x}-L_0x^2\right)^2}$$

$$+\frac{\left(\sigma_t^{n+1}\right)^2}{\sigma_t^n}+\sigma_t^{n+1}=0,$$

then the ratio[22] can be written as

$$\frac{\sigma_t^{n+1}}{\sigma_t^n}=\frac{L_0^2\left(\mathrm{e}^{f(x,t)+2t/x}+x^2\right)}{\mathrm{e}^{2t/x}-L_0x^2}, \tag{45}$$

which also gives an invariant manifold.

3.6. Lie algebras

The commutator of any two infinitesimal generators of an r-parameter Lie group of transformations[23−25] is also an infinitesimal generator, in particular

$$[X_\alpha,X_\beta]=C_{\alpha\beta}^{\gamma}X_\gamma,$$

where $C_{\alpha\beta}$ are constants called structure constants, $\alpha,\beta,\gamma=1,2,\ldots,r$.

Consider an r-parameter Lie group of transformations with infinitesimal generator $\{X_\alpha\}$, $\alpha=1,2,\ldots,8$; the commutation relation of X_α and X_β is another first order operator

$$\begin{aligned}&[X_\alpha,X_\beta]\\&=X_\alpha X_\beta-X_\beta X_\alpha\\&=\sum_{i,j=1}^{n}\left[\left(\xi_{\alpha i}(x)\frac{\partial}{\partial x_i}\right)\left(\xi_{\beta j}(x)\frac{\partial}{\partial x_j}\right)-\left(\xi_{\beta i}(x)\frac{\partial}{\partial x_i}\right)\left(\xi_{\alpha j}(x)\frac{\partial}{\partial x_j}\right)\right]\\&=\sum_{j=1}^{n}\eta_j(x)\frac{\partial}{\partial x_j},\end{aligned} \tag{46}$$

where

$$\eta_j(x)=\sum_{i=1}^{n}\left[\xi_{\alpha i}(x)\frac{\partial\xi_{\alpha j}(x)}{\partial x_i}-\xi_{\beta i}(x)\frac{\partial\xi_{\alpha j}(x)}{\partial x_i}\right], \tag{47}$$

then

$$[X_\alpha,X_\beta]=-[X_\beta,X_\alpha].$$

It is convenient to display the commutators of a Lie algebra through its commutator table whose (i,j)-th entry is $[X_i,X_j]$. It follows that the table is antisymmetric with its diagonal elements all zero. The structure constants are easily read off from the commutator table.

For the infinitesimal generators we give the following commutator table.

Table 1. Commutators of a Lie algebra.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
X_1	0	0	$-X_1$	$-X_2$	$-2X_3-X_7$	0	0	$-X_6$
X_2	0	0	0	0	$-X_4$	$-X_1$	$-X_2$	$-X_3-2X_7$
X_3	X_1	0	0	$-X_4$	$-X_5$	X_6	0	0
X_4	X_2	0	X_4	0	0	X_7-X_3	$-X_4$	$-X_5$
X_5	$2X_3+X_7$	X_4	X_5	0	0	X_8	0	0
X_6	0	X_1	$-X_6$	X_3-X_7	$-X_8$	0	X_6	0
X_7	0	X_2	0	X_4	0	$-X_6$	0	$-X_8$
X_8	X_6	X_3+2X_7	0	X_5	0	0	X_8	0

4. Conclusions

In this paper, a new lattice equation is applied to reduce the discrete wave equation of the lattice equation. The result reveals how the discrete wave equation keeps the symmetry on the new lattice. The Lie algebras show the relationships between infinitesimal generators. It is practicable to extend this result to the heat conduction equation and other wave equations.

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