Negative refraction in ferromagnetic materials under external magnetic field: a theoretical analysis*

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We present a detailed theoretical analysis on the possibilities and conditions for negative permeability and negative refraction occuring in the magnetic materials with both pronounced magnetic and dielectric responses to electromagnetic waves. The results indicate that the permeability is always positive for $\delta = (2q + 0.5)\pi$ (δ is the initial phase difference of magnetic components h_x and h_y of incident electromagnetic wave, q is integer), which means that it is difficult to realize negative refraction. However, for $\delta = 2q\pi, \delta = (2q + 1)\pi$, or $\delta = (2q - 0.5)\pi$, the negative permeability occurs at some range of free procession frequency, which means that the refraction can become negative under certain conditions. Further analysis reveals that for general positive permittivity there are various opportunities for realizing the negative refraction provided that some requirements are met. One concludes also that the refractive index for $\delta = 2q\pi$ case is similar to $\delta = (2q + 1)\pi$. The only difference between two cases of $\delta = 2q\pi$ and $\delta = (2q + 1)\pi$ is that the x-direction for $\delta = 2q\pi$ corresponds to the y-direction for $\delta = (2q + 1)\pi$, and the y-direction for $\delta = 2q\pi$ corresponds to the x-direction for $\delta = (2q + 1)\pi$. The results are valuable for designing and analysing the complex negative refraction of magnetic materials.

Keywords: negative refraction, ferromagnetic materials

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1. Introduction

Negative refraction materials (NRMs) tend to reverse all our understanding of optics.[1] When light enters the NRM at an angle, it is refracted to an opposite side of the normal to standard positive index materials,[2−6] which means that lenses can be constructed from flat NRM blocks with resolution beyond the diffraction limit.[7] The super-resolution ability of NRMs could lead to applications in microscopy and lithography, which attracts lots of attentions in this field.[8−10] Specially in recent years, different ways to design and fabricate NRMs are proposed and realized,[11,12] such as metamaterials,[13] photonic crystals,[14,15] multilayer stacks,[16,17] and nonmagnetic materials etc.[18,19] However, these materials are mainly limited to artificial structures created in the laboratory. In order to explore some simple and low-cost fabrication methods for NRMs, one gradually transfers the attentions from artificial structures to natural materials. In natural materials, the magnetic materials with dielectric or metallic characteristics are potential candidates. For example, the ferromagnetic materials (such as ferrite materials) present both the ferromagnetic properties and the dielectric characteristics.[20−24] Some magnetized metal magnetic films can present both the metallic properties that the permittivity is negative below the plasma resonance frequency and the strong ferromagnetic resonance properties that the negative permeability can also be achieved close to the ferromagnetic resonance.[25−28] Pimenov *et al.* have experimentally shown that such properties could be found in ferromagnetic natural materials. Using 150 GHz electromagnetic wave irradiation, a negative refraction index for $La_{2/3}Ca_{1/3}MnO_3$ ferromagnetic material was verified under external magnetic fields close to the ferromagnetic resonance, which is a valuable contribution to the field.[29,30] Xu and Bai *et al.* reported that the negative refraction properties of composite material consisting of ferrite and copper wires were observed at the microwave frequencies under an external magnetic field.[31,32] Zhao *et al.* found the magnetotunable material consisting of yttrium iron garnet slab and metallic wires. The negative refraction passband through the NRMs can be dynamically and continu-

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ously tuned in a wide frequency region by an applied external magnetic field.[33,34]

In the present work, we give a detailed theoretical analysis on the negative refraction in ferromagnetic materials under external magnetic fields and some possible requirements for realizing the negative refraction, which can be very useful for designing and fabricating NRMs from natural materials.

The rest of the paper is organized as follows. In Section 2, the theoretical basis of the magnetic susceptibility for homogeneous isotropic infinite medium under an external magnetic field and a response to an electromagnetic wave are presented, and the formula for calculating the magnetic susceptibility tensor is derived. In Section 3, the dependence of magnetic susceptibility on the polarization characteristics of electromagnetic waves is discussed. In Section 4, the permittivity for general ferromagnetic dielectric materials is analyzed. In Section 5, the criterion and calculation formula for complex refractive index are derived. The possible complex refractive index of ferromagnetic materials is described in Section 6. The work is finally concluded in Section 7.

2. Theoretical basis on the magnetic susceptibility tensor for homogeneous isotropic infinite sample under an external magnetic field[35]

A homogeneous isotropic infinite sample is magnetized under both an external direct current magnetic field $\boldsymbol{H}_{\rm e}$ and an electromagnetic wave magnetic field $\boldsymbol{h}$, and the magnetization intensity is $\boldsymbol{M}$. The total effective field is the sum of the two fields $\boldsymbol{H}_{\rm eff} = \boldsymbol{h} + \boldsymbol{H}_{\rm e}$. Let us assume that $\boldsymbol{M}$ deviates by an angle θ from $\boldsymbol{H}_{\rm e}$ direction (shown in Fig. 1) and $\boldsymbol{M}$ moves due to precession around $\boldsymbol{H}_{\rm e}$. The motion equation of $\boldsymbol{M}$ with damping taken into account can be expressed as

$$\frac{{\rm d}\boldsymbol{M}}{{\rm d}t} = -\gamma.\boldsymbol{M} \times \boldsymbol{H}_{\rm eff} + \frac{a}{M}\boldsymbol{M} \times \frac{{\rm d}\boldsymbol{M}}{{\rm d}t}, \qquad (1)$$

where γ is the gyromagnetic ratio, and a is damping parameter. One further writes down $\boldsymbol{H}_{\rm eff}$ and $\boldsymbol{M}$ as follows

$$H_{\rm eff} = h_x\boldsymbol{i} + h_y\boldsymbol{j} + (H_{\rm e} + h_z)\boldsymbol{k}, \qquad (2)$$

$$\boldsymbol{M} = m_x\boldsymbol{i} + m_y\boldsymbol{j} + (M_0 + m_z)\boldsymbol{k}. \qquad (3)$$

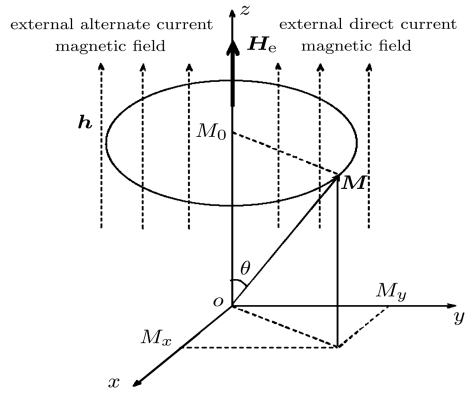


Fig. 1. The movement schematic of magnetization under external magnetic field.

Assuming $\boldsymbol{h} = \boldsymbol{h}_0\,{\rm e}^{{\rm i}\omega_0 t}$ (where ω_0 is the electromagnetic wave frequency) and $\boldsymbol{m} = \boldsymbol{m}_0\,{\rm e}^{{\rm i}\omega_0 t}$, $\boldsymbol{M}_0$ is the constant component of $\boldsymbol{M}$ at the z-axis. The m_x, m_y, and m_z are the alternate components of $\boldsymbol{M}$ at x-direction, y-direction, and z-direction, respectively. Substituting Eqs. (2) and (3) into Eq. (1) one gets the formula as follows:

$$\begin{aligned}
&{\rm i}\omega_0 \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \\
&= -\gamma \begin{pmatrix} m_y(H_{\rm e} + h_z) - (M_0 + m_z)h_y \\ (M_0 + m_z)h_x - m_x(H_{\rm e} + h_z) \\ m_x h_y - m_y h_x \end{pmatrix} \\
&\quad + \frac{a}{M} \begin{pmatrix} m_y{\rm i}\omega_0 m_z - (M_0 + m_z){\rm i}\omega_0 m_y \\ (M_0 + m_z){\rm i}\omega_0 m_x - m_x{\rm i}\omega_0 m_z \\ m_x{\rm i}\omega_0 m_y - m_y{\rm i}\omega_0 m_x \end{pmatrix}. \qquad (4)
\end{aligned}$$

Generally speaking, the magnetic field intensity for electromagnetic waves is much smaller than the external direct current magnetic field $\boldsymbol{H}_{\rm e}$, so we assume that $|\boldsymbol{h}| \ll |\boldsymbol{H}_{\rm e}|$, and $|\boldsymbol{m}| \ll |\boldsymbol{M}|$, here the second-order terms in Eq. (4) are ignored, one can get

$$\begin{aligned}
&{\rm i}\omega_0 m_x = -\gamma(m_y H_{\rm e} - M_0 h_y) - \frac{{\rm i}a\omega_0}{M} M_0 m_y, \\
&{\rm i}\omega_0 m_y = -\gamma(M_0 h_x - m_x H_{\rm e}) + \frac{{\rm i}a\omega_0}{M} M_0 m_x, \\
&m_z = 0. \qquad (5)
\end{aligned}$$

Due to $M_0 \approx M \approx M_{\rm s}$ ($M_{\rm s}$ is the saturation magnetization intensity), Eq. (5) can be simplified as

$$\begin{aligned}
&{\rm i}\omega_0 m_x + (\omega + {\rm i}a\omega_0)m_y = \omega_m h_y, \\
&-(\omega + {\rm i}a\omega_0)m_x + {\rm i}\omega_0 m_y = -\omega_m h_x, \\
&m_z = 0, \qquad (6)
\end{aligned}$$

where $\omega(=\gamma H_{\rm e})$ is called free precession frequency of the magnetization and can be tuned by the external direct current magnetic field $H_{\rm e}$, and $\omega_m = \gamma M_{\rm s} \approx \gamma M_0$. One can get from Eq. (6)

$$\begin{aligned} m_x &= \frac{\omega_m(\omega + {\rm i}a\omega_0)}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2} h_x + {\rm i}\frac{\omega_m\omega_0}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2} h_y, \\ m_y &= -{\rm i}\frac{\omega_m\omega_0}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2} h_x + \frac{\omega_m\omega_0}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2} h_y, \\ m_z &= 0. \end{aligned} \tag{7}$$

Equation (7) can be rewritten as

$$\boldsymbol{m} = \overleftrightarrow{\boldsymbol{\chi}} \cdot \boldsymbol{h} = (\overleftrightarrow{\boldsymbol{\mu}} - 1) \cdot \boldsymbol{h}, \tag{8}$$

where $\overleftrightarrow{\boldsymbol{\mu}}(= \overleftrightarrow{\boldsymbol{\chi}} + 1)$ is the permeability tensor. $\overleftrightarrow{\boldsymbol{\chi}}$ is called magnetic susceptibility tensor, and

$$\overleftrightarrow{\boldsymbol{\chi}} = \overleftrightarrow{\boldsymbol{\mu}} - 1 = \begin{pmatrix} \chi & -{\rm i}\chi_a & 0 \\ {\rm i}\chi_a & \chi & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \mu - 1 & -{\rm i}\mu_a & 0 \\ {\rm i}\mu_a & \mu - 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

$$\begin{aligned} \chi = \mu - 1 &= \frac{\omega_m(\omega + {\rm i}a\omega_0)}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2}, \\ \chi_a = \mu_a &= \frac{-\omega_m\omega_0}{(\omega + {\rm i}a\omega_0)^2 - \omega_0^2}. \end{aligned} \tag{10}$$

The χ and χ_a can be written as $\chi = \chi' - {\rm i}\chi''$ and $\chi_a = \chi'_a - {\rm i}\chi''_a$, respectively, and

$$\chi' = \mu' - 1 = \frac{\omega_m\omega(\omega^2 - \omega_0^2) + \omega_m\omega a^2\omega_0^2}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}, \tag{11}$$

$$\chi'' = \mu'' = \frac{\omega_m a\omega_0[\omega^2 + \omega_0^2(1+a^2)]}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}, \tag{12}$$

$$\chi'_a = \mu'_a = \frac{-\omega_m\omega_0[\omega^2 - \omega_0^2(1+a^2)]}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}, \tag{13}$$

$$\chi''_a = \mu''_a = \frac{-2\omega_m\omega a\omega_0^2}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}. \tag{14}$$

Equations (11)–(14) indicate that the components of magnetic susceptibility tensor depend on the free precession frequency ω, which is in turn determined by the external direct current magnetic field intensity due to $\omega = \gamma H_{\rm e}$.

In order to have a better understanding, Fig. 2 gives the dependence of susceptibility on the free precession frequency, where we assume that $\omega_m/\omega_0 = 3$ and $a = 0.1$ (It is noted that $\omega_m/\omega_0 = 3$ and $a = 0.1$ will be still chosen in following calculations and analysis in order to keep the consistency). One can find that when the frequency ω_0 is fixed, the χ'' (shown in Fig. 2(b)) and χ''_a (shown in Fig. 2(d)) reach up to positive and negative maximum values, respectively if $\omega(=\gamma H_{\rm e}) = \omega_0$, which can be realized by tuning external direct current magnetic field intensity $H_{\rm e}$. The $H_{\rm e}$ is called ferromagnetic resonance magnetic field. In addition, the χ' (shown in Fig. 2(a)) and χ'_a (shown in Fig. 2(c)) are less than 0 at left and right side of the ferromagnetic resonance magnetic field, respectively.

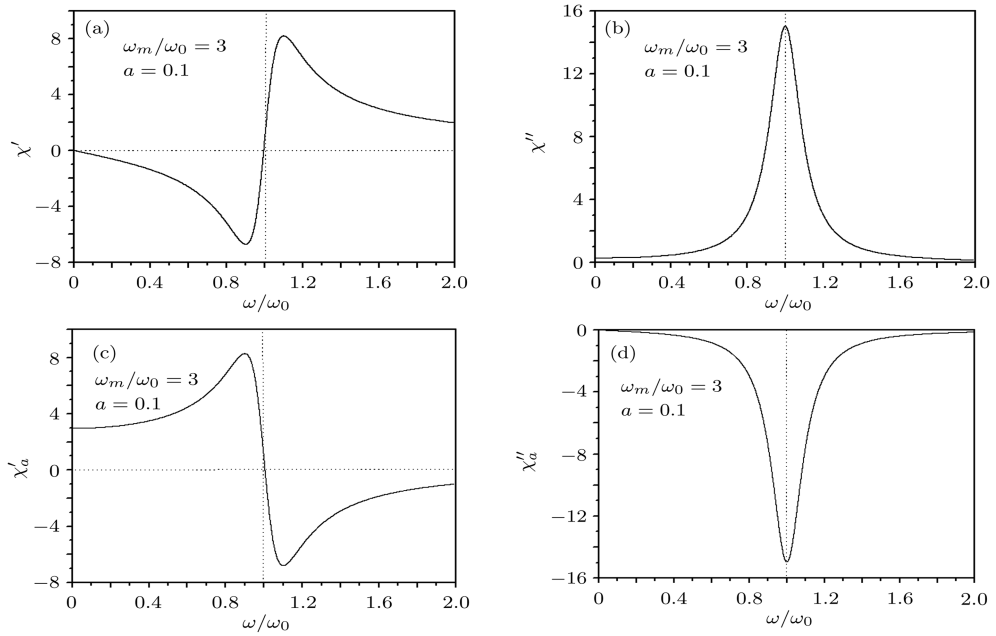


Fig. 2. The dependence of magnetic susceptibility on the free precession frequency. (a) the relation of χ' with ω; (b) the relation of χ'' with ω; (c) the relation of χ'_a with ω; (d) the relation of χ''_a with ω.

3. Magnetic susceptibility for homogeneous isotropic infinite sample under polarization light and direct current magnetic field

As is shown in Fig. 1, we choose z-axis which is the external direct current magnetic field direction to be the propagation direction of electromagnetic wave. To simplify the analysis, the electromagnetic wave magnetic field in this coordinate system can be expressed as

$$h_x = -A\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_2)}, \quad h_y = A\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_1)}, \\ h_z = 0, \tag{15}$$

where A is the amplitude, δ_1 and δ_2 are the initial phases of h_x and h_y, respectively. Substituting χ and χ_a into Eq. (7), one can get

$$\begin{aligned} m_x &= \chi h_x - \mathrm{i}\chi_a h_y \\ &= \chi' h_x - \chi_a'' h_y - \mathrm{i}(\chi'' h_x + \chi_a' h_y), \\ m_y &= \mathrm{i}\chi_a h_x + \chi h_y \\ &= \chi_a'' h_x + \chi' h_y - \mathrm{i}(\chi'' h_y - \chi_a' h_x), \\ m_z &= 0. \end{aligned} \tag{16}$$

Substituting Eq. (15) into Eq. (16), one can get

$$\begin{aligned} m_x &= -A([\chi'\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_2)} + \chi_a''\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_1)}] \\ &\quad + \mathrm{i}[\chi_a' A\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_1)} - \chi''\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_2)}]), \\ m_y &= A([\chi'\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_1)} - \chi_a''\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_2)}] \\ &\quad -\mathrm{i}[\chi''\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_1)} + \chi_a'\,\mathrm{e}^{\mathrm{i}(\omega_0 t+\delta_2)}]), \\ m_z &= 0. \end{aligned} \tag{17}$$

In the following four subsections, as examples, the dependence of complex permeability on the initial phases is analysed and discussed in detail.

Case 1 $\delta = \delta_2 - \delta_1 = 2q\pi$, ($q$ is integer);
Case 2 $\delta = \delta_2 - \delta_1 = (2q+1)\pi$;
Case 3 $\delta = \delta_2 - \delta_1 = (2q+1/2)\pi$;
Case 4 $\delta = \delta_2 - \delta_1 = (2q-1/2)\pi$.

3.1. $\delta = \delta_2 - \delta_1 = 2q\pi, \delta_2 = \delta_1 + 2q\pi$

According to Eq. (15) and $\delta_2 = \delta_1 + 2q\pi$, one can get

$$h_x = -h_y. \tag{18}$$

Substituting Eqs. (18) and (15), and $\delta_2 = \delta_1 + 2q\pi$ into Eq. (17) one can get

$$\begin{aligned} m_x &= [(\chi' + \chi_a'') - \mathrm{i}(\chi'' - \chi_a')]\,h_x, \\ m_y &= [(\chi' - \chi_a'') - \mathrm{i}(\chi'' + \chi_a')]\,h_y, \\ m_z &= 0. \end{aligned} \tag{19}$$

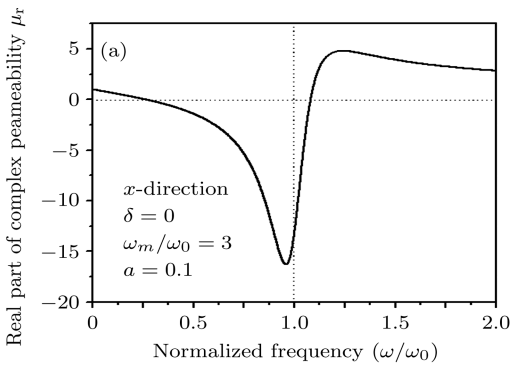
One can see from Eq. (19) the complex permeability $\mu(= \mu_\mathrm{r} + \mathrm{i}\mu_\mathrm{i})$ at x-direction can be written as

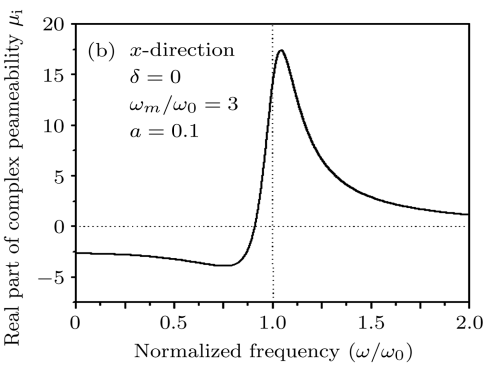
$$\begin{aligned} \mu_\mathrm{r} &= \chi' + \chi_a'' + 1 \\ &= \frac{\omega_m\omega\left(\omega^2 + \omega_0^2[(a-1)^2 - 2]\right)}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2} + 1, \\ \mu_\mathrm{i} &= \chi'' - \chi_a' \\ &= \frac{\omega_m\omega_0\left(\omega_0^2[a(a^2 - a + 1) - 1] + \omega^2(a+1)\right)}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}, \end{aligned} \tag{20}$$

and at y-direction the complex permeability $\mu(= \mu_\mathrm{r} + \mathrm{i}\mu_\mathrm{i})$ can be expressed as

$$\begin{aligned} \mu_\mathrm{r} &= \chi' - \chi_a'' + 1 \\ &= \frac{\omega_m\omega\left\{\omega^2 + \omega_0^2[(a+1)^2 - 2]\right\}}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2} + 1, \\ \mu_\mathrm{i} &= \chi'' + \chi_a' \\ &= \frac{\omega_m\omega_0[\omega_0^2(a^3 + a^2 + a + 1) + \omega^2(a-1)]}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2\omega_0^2}. \end{aligned} \tag{21}$$

Equations (20) and (21) are plotted in Fig. 3. Figures 3(a) and 3(b) are the real part μ_r and imaginary part





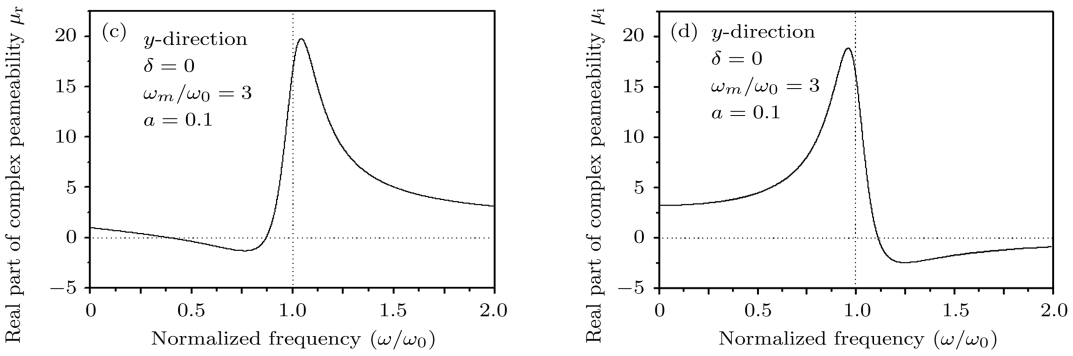


Fig. 3. Complex permeability under $\delta = 2q\pi$ (a) real part (b) imaginary part at x-direction, (c) real part (d) imaginary part at y-direction.

$\mu_{\rm i}$ at x-direction, respectively. Figures 3(c) and 3(d) are the real part $\mu_{\rm r}$ and imaginary part $\mu_{\rm i}$ at y-direction, respectively. One can see from Fig. 3 that the negative values for the complex permeability occur at some range of free procession frequency, which is also a condition required for realizing the negative refraction.

3.2. $\delta = \delta_2 - \delta_1 = (2q+1)\pi, \delta_2 = \delta_1 + (2q+1)\pi$

According to Eq. (15) and $\delta_2 = \delta_1 + (2q+1)\pi$, one can get

$$h_x = h_y. \tag{22}$$

Substituting Eqs. (22) and (15), and $\delta_2 = \delta_1 + (2q+1)\pi$ into Eq. (17) one can get

$$\begin{aligned} m_x &= [(\chi' - \chi''_a) - {\rm i}(\chi'' + \chi'_a)]\, h_x, \\ m_y &= [(\chi' + \chi''_a) - {\rm i}(\chi'' - \chi'_a)]\, h_y, \\ m_z &= 0. \end{aligned} \tag{23}$$

One can see from Eq. (23) the complex permeability $\mu(= \mu_{\rm r} + {\rm i}\mu_{\rm i})$ at x-direction can be written as

$$\begin{aligned} \mu_{\rm r} &= \chi' - \chi''_a + 1 \\ &= \frac{\omega_m \omega \left\{\omega^2 + \omega_0^2[(a+1)^2 - 2]\right\}}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2 \omega_0^2} + 1, \\ \mu_{\rm i} &= \chi'' + \chi'_a \\ &= \frac{\omega_m \omega_0 [\omega_0^2(a^3 + a^2 + a + 1) + \omega^2(a-1)]}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2 \omega_0^2}, \end{aligned} \tag{24}$$

and at y-direction the complex permeability $\mu(= \mu_{\rm r} + {\rm i}\mu_{\rm i})$ can be expressed as

$$\begin{aligned} \mu_{\rm r} &= \chi' + \chi''_a + 1 \\ &= \frac{\omega_m \omega \left\{\omega^2 + \omega_0^2[(a-1)^2 - 2]\right\}}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2 \omega_0^2} + 1, \\ \mu_{\rm i} &= \chi'' - \chi'_a \\ &= \frac{\omega_m \omega_0 \left\{\omega_0^2[a(a^2 - a + 1) - 1] + \omega^2(a+1)\right\}}{[\omega^2 - (1+a^2)\omega_0^2]^2 + 4\omega^2 a^2 \omega_0^2}. \end{aligned} \tag{25}$$

Comparing Eq. (24) with Eq. (21) and Eq. (25) with Eq. (20) it is easy to find that the complex permeability at x-direction for $\delta = (2q+1)\pi$ is the same as that at y-direction for $\delta = 2q\pi$, and the complex permeability at y-direction for $\delta = (2q+1)\pi$ is the same as that at x-direction for $\delta = 2q\pi$. This indicates the complex permeability for $\delta = (2q+1)\pi$ can be also described in Fig. 3. The differences are that for $\delta = (2q+1)\pi$ Figs. 3(a) and 3(b) are at the y-direction, while Figs. 3(c) and 3(d) are at x-direction.

3.3. $\delta = \delta_2 - \delta_1 = (2q+0.5)\pi, \delta_2 = \delta_1 + (2q+0.5)\pi$

According to Eq. (15) and $\delta_2 = \delta_1 + (2q+0.5)\pi$, one can get

$$\begin{aligned} h_x &= -{\rm i} h_y, \\ h_y &= {\rm i} h_x. \end{aligned} \tag{26}$$

Substituting Eqs. (26) and (15), and $\delta_2 = \delta_1 + (2q+0.5)\pi$ into Eq. (17), one can get

$$\begin{aligned} m_x &= [(\chi' + \chi'_a) - {\rm i}(\chi'' + \chi''_a)]\, h_x, \\ m_y &= [(\chi' + \chi'_a) - {\rm i}(\chi'' + \chi''_a)]\, h_y, \\ m_z &= 0. \end{aligned} \tag{27}$$

Equation (27) shows that the complex permeability $\mu(= \mu_{\rm r} + {\rm i}\mu_{\rm i})$ at x-direction is the same as that at y-direction, and can be written as

$$\begin{aligned} \mu_{\rm r} &= \chi' + \chi'_a + 1 = \frac{\omega_m(\omega + \omega_0)}{(\omega + \omega_0)^2 + a^2\omega_0^2} + 1, \\ \mu_{\rm i} &= \chi'' + \chi''_a = \frac{\omega_m \alpha \omega_0}{(\omega + \omega_0)^2 + a^2\omega_0^2}. \end{aligned} \tag{28}$$

Equation (28) is plotted in Fig. 4. Figures 4(a) and 4(b) are the real part $\mu_{\rm r}$ and imaginary part $\mu_{\rm i}$ respectively. Figure 4 is showing that whether the real part or the imaginary part is always more than 0 at any free procession frequency. This means that it is difficult to realize negative refraction for $\delta = (2q + 0.5)\pi$.

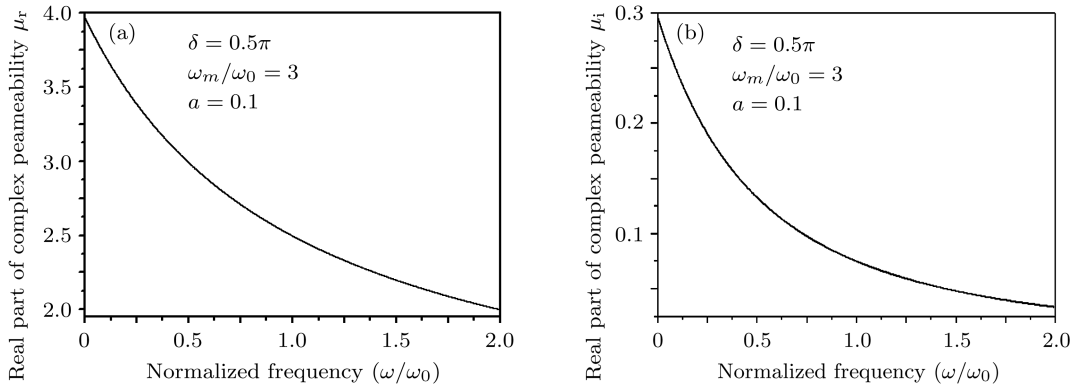


Fig. 4. Complex permeability under $\delta = (2q + 0.5)\pi$ (a) real part (b) imaginary part.

3.4. $\delta = \delta_2 - \delta_1 = (2q - 0.5)\pi, \delta_2 = \delta_1 + (2q - 0.5)\pi$

According to Eq. (15) and $\delta_2 = \delta_1 + (2q - 0.5)\pi$, one can get

$$\begin{aligned} h_x &= {\rm i}h_y, \\ h_y &= -{\rm i}h_x. \end{aligned} \tag{29}$$

Substituting Eqs. (29) and (15), and $\delta_2 = \delta_1 + (2q - 0.5)\pi$ into Eq. (17), one can get

$$\begin{aligned} m_x &= [(\chi' - \chi'_a) - {\rm i}(\chi'' - \chi''_a)]\, h_x, \\ m_y &= [(\chi' - \chi'_a) - {\rm i}(\chi'' - \chi''_a)]\, h_y, \\ m_z &= 0. \end{aligned} \tag{30}$$

Similar to the $\delta = (2q + 0.5)\pi$ case, one can see from Eq. (30) that the complex permeability $\mu(= \mu_{\rm r} + {\rm i}\mu_{\rm i})$ at x-direction is the same as that at y-direction, and can be written as

$$\begin{aligned} \mu_{\rm r} &= \chi' - \chi'_a + 1 = \frac{\omega_m(\omega - \omega_0)}{(\omega - \omega_0)^2 + a^2\omega_0^2} + 1, \\ \mu_{\rm i} &= \chi'' - \chi''_a = \frac{\alpha\omega_m\omega_0}{(\omega - \omega_0)^2 + a^2\omega_0^2}. \end{aligned} \tag{31}$$

Figures 5(a) and 5(b) are the real part $\mu_{\rm r}$ and imaginary part $\mu_{\rm i}$ in Eq. (31), respectively. One can see from Fig. 5 that the real part could be less than 0 at a certain range of free procession frequency, and the imagery part is always more than 0. This indicates that the negative refraction may be realized at some special requirements, which will be discussed in Section 6.

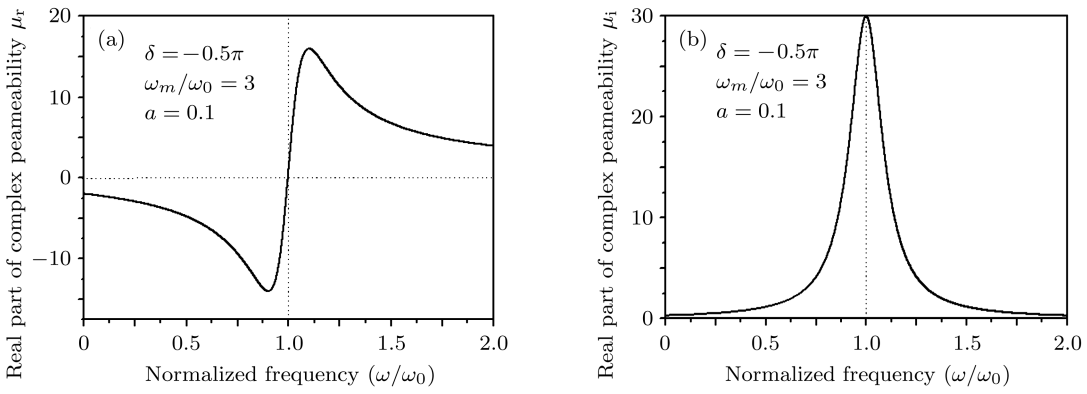


Fig. 5. Complex permeability under $\delta = (2q - 0.5)\pi$ (a) real part (b) imaginary part.

4. Permittivity analysis for homogeneous isotropic infinite sample[35]

The magnetic response and susceptibility have been derived in above sections. For the dielectric characteristics, generally speaking, the ferromagnetic medium is isotropic. In the following, we analyse the permittivity properties and give the corresponding calculation formula. For the convenience of analysis, we write electric field of the electromagnetic wave as

$$\boldsymbol{E} = \boldsymbol{E}_m \, \mathrm{e}^{\mathrm{i}\omega_0 t}. \tag{32}$$

The electric dipole moments are produced within the ferromagnetic materials under the electric field of the electromagnetic wave, and the electric displacement vector is marked as $\boldsymbol{D}$. The $\boldsymbol{D}$ may lag for phase angle δ_{e} due to the damping factors, and is expressed as

$$\boldsymbol{D} = \boldsymbol{D}_m \, \mathrm{e}^{\mathrm{i}(\omega_0 t - \delta_{\mathrm{e}})}. \tag{33}$$

Here the complex permittivity $\varepsilon(= \varepsilon_{\mathrm{r}} + \mathrm{i}\varepsilon_{\mathrm{i}})$ is defined as

$$\varepsilon = \frac{1}{\varepsilon_0}\frac{\boldsymbol{D}}{\boldsymbol{E}} = \frac{1}{\varepsilon_0}\frac{D_{\mathrm{m}}}{E_{\mathrm{m}}} \, \mathrm{e}^{-\mathrm{i}\delta_{\mathrm{e}}}. \tag{34}$$

For homogeneous ferromagnetic dielectric materials, the electric displacement is mainly from the electric polarization $\boldsymbol{P}$, and can be expressed as $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$. The electric polarization includes electronic polarization, ion polarization, and orientation polarization. The corresponding dielectric constant can be expressed as

$$\varepsilon = \varepsilon_\infty + \frac{\varepsilon_{\mathrm{s}} - \varepsilon_\infty}{1 + \mathrm{i}\omega_0 \tau}, \tag{35}$$

where τ is a finite relaxation time for orientation polarization of electric dipole, ε_∞ is the dielectric constant for $\omega_0 \to \infty$ and ε_{s} is the dielectric constant for $\omega_0 \to 0$. According to Eqs. (34) and (35), one can get

$$\varepsilon_{\mathrm{r}} = \varepsilon_\infty + (\varepsilon_{\mathrm{s}} - \varepsilon_\infty)\frac{1}{1 + (\omega_0 \tau)^2}, \tag{36}$$

$$\varepsilon_{\mathrm{i}} = \varepsilon_\infty + (\varepsilon_{\mathrm{s}} - \varepsilon_\infty)\frac{\omega_0 \tau}{1 + (\omega_0 \tau)^2}. \tag{37}$$

For the ferromagnetic materials, from Eqs. (36) and (37) one can find that both the real part and imaginary part are possible to be less than 0 under certain special requirements. In this work, in order to be general true for most natural materials, we assume that $\varepsilon_{\mathrm{r}} > 0$ and $\varepsilon_{\mathrm{i}} > 0$.

5. The criterion and calculation for complex refractive index

The complex refractive index n is determined simultaneously by both the complex permittivity $\varepsilon(= \varepsilon_{\mathrm{r}} + \mathrm{i}\varepsilon_{\mathrm{i}})$ and complex permeability $\mu(= \mu_{\mathrm{r}} + \mathrm{i}\mu_{\mathrm{i}})$. The n can be expressed as

$$n = \sqrt{\varepsilon\mu} = \sqrt{(\varepsilon_{\mathrm{r}} + \mathrm{i}\varepsilon_{\mathrm{i}})(\mu_{\mathrm{r}} + \mathrm{i}\mu_{\mathrm{i}})}. \tag{38}$$

For convenience, the ε and μ are written in the form of complex number

$$\varepsilon = |\varepsilon| \, \mathrm{e}^{\mathrm{i}(\alpha + 2s\pi)} (0 \le \alpha < 2\pi, s = 0, 1, 2, 3 \ldots), \tag{39}$$

$$\mu = |\mu| \, \mathrm{e}^{\mathrm{i}(\beta + 2t\pi)} (0 \le \beta < 2\pi, t = 0, 1, 2, 3 \ldots) \tag{40}$$

with

$$|\varepsilon| = \sqrt{\varepsilon_{\mathrm{r}}^2 + \varepsilon_{\mathrm{i}}^2},$$
$$|\mu| = \sqrt{\mu_{\mathrm{r}}^2 + \mu_{\mathrm{i}}^2}. \tag{41}$$

Based on Einstein's causality and Ref. [36], the sign of n ought to be determined by

$$n = \sqrt{|\varepsilon| \, |\mu|} \, \mathrm{e}^{\mathrm{i}[(\alpha+\beta)/2 + (s+t)\pi]}. \tag{42}$$

The complex refractive index can be expressed as

$$n = n_{\mathrm{r}} + \mathrm{i}n_{\mathrm{i}}, \tag{43}$$

where n_{r} is the refractive index, and n_{i} is the extinction coefficient, and

$$n_{\mathrm{r}} = \sqrt{|\varepsilon| \, |\mu|} \cos\left[\frac{\alpha + \beta}{2} + (s + t)\pi\right], \tag{44}$$

$$n_{\mathrm{i}} = \sqrt{|\varepsilon| \, |\mu|} \sin\left[\frac{\alpha + \beta}{2} + (s + t)\pi\right]. \tag{45}$$

The above solutions can be divided into two groups:

1) even solutions $s + t = 2p$ for $p = 0, \pm 1, \pm 2 \ldots$,

2) odd solutions $s + t = 2p + 1$ for $p = 0, \pm 1, \pm 2 \ldots$.

The even solution corresponds to $s + t = 0$ and the odd solution corresponds to $s + t = \pm 1$. A causal solution has to be required so that the waves could travel from the source points to observation points, no matter how the sign of refractive index changes in the materials. Violating the causality condition would result in a negative energy density which cannot be accepted in physics. The principle applies also to the optically gaining materials where external pumping sources sustain an amplifying function but the energy

flow remains in the positive direction. That is to say, the wave impedance Z has to be positive to maintain the Poynting vector in the causal direction, i.e.

$$\mathrm{Re}\{Z\} \approx \mathrm{Re}\left\{\sqrt{\frac{\mu}{\varepsilon}}\right\} > 0. \tag{46}$$

Equation (46) is the criterion we should use to select the causal solution for the square root of the refractive index in Eqs. (44) and (45).

6. The possible complex refractive index of ferromagnetic materials

For the ferromagnetic materials, generally one can find from Eqs. (36) and (37) $\varepsilon_\mathrm{r} > 0$ and $\varepsilon_\mathrm{i} > 0$, which is also general true for most natural materials. According to analysis and discussion in Section 3, here we can give the possible sign of the complex refractive index. The results are presented in Table 1. One sees from Table 1 that there are various opportunities for the real part of the refractive index to be negative provided that some requirements are satisfied. In the light of analysis one concludes on the general conditions for negative refraction as follows. There must be positive refractive index for $\delta = (2q + 1/2)\pi$. The refractive index for $\delta = 2q\pi$ case is similar to $\delta = (2q + 1)\pi$, only the x-direction for $\delta = 2q\pi$ corresponds to the y-direction for $\delta = (2q + 1)\pi$, and the y-direction for $\delta = 2q\pi$ corresponds to the x-direction for $\delta = (2q + 1)\pi$.

Let us consider two typical cases, one has a relatively small ε_i and the other has a relatively large ε_i. We note that the selected data for the dielectric function is in consistence with typical data in the literature for magnetic materials in high frequencies.[37−39] For these two cases, both the real and imaginary parts of the refractive index at x-direction for $\delta = 2q\pi$ are plotted in Fig. 6. In Fig. 6(a), the proportion $|\varepsilon_\mathrm{i}|/|\varepsilon_\mathrm{r}|$ is low, and the spectral range for negative refraction is narrow at about $\omega = 0.89\,\omega_0 \sim 0.93\,\omega_0$. In Fig. 6(b)

Table 1. The sign of complex refractive index.

<table>
<tr><td rowspan="2">$\delta = (2q - 0.5)\pi$</td><td colspan="4" rowspan="2">$\mu_\mathrm{r} < 0 \;\; \mu_\mathrm{i} > 0$</td><td colspan="4" rowspan="2">$n_\mathrm{i} > 0$</td><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} > \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} > 0$</td></tr>
<tr><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} < \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} < 0$</td></tr>
<tr><td></td><td colspan="4">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} > 0$</td><td colspan="4">$n_\mathrm{r} > 0 \;\; n_\mathrm{i} > 0$</td></tr>
<tr><td>$\delta = (2q + 0.5)\pi$</td><td colspan="4">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} > 0$</td><td colspan="4">$n_\mathrm{r} > 0 \;\; n_\mathrm{i} > 0$</td></tr>
<tr><td rowspan="8">$\delta = 2q\pi$</td><td rowspan="8">x-direction</td><td rowspan="2">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} < 0$</td><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} > \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} > 0 \;\; n_\mathrm{i} < 0$</td><td rowspan="8">$y$-direction</td><td rowspan="2">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} < 0$</td><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} > \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} > 0 \;\; n_\mathrm{i} < 0$</td></tr>
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<tr><td rowspan="2">$\mu_\mathrm{r} < 0 \;\; \mu_\mathrm{i} < 0$</td><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} > \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} > 0 \;\; n_\mathrm{i} < 0$</td><td colspan="3" rowspan="2">/</td></tr>
<tr><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} < \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} < 0 \;\; n_\mathrm{i} > 0$</td></tr>
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<tr><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} < \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} < 0 \;\; n_\mathrm{i} > 0$</td><td>$\frac{\mu_\mathrm{i}}{\mu_\mathrm{r}} < \frac{\varepsilon_\mathrm{i}}{\varepsilon_\mathrm{r}}$</td><td>$n_\mathrm{r} < 0 \;\; n_\mathrm{i} > 0$</td></tr>
<tr><td colspan="2">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} > 0$</td><td colspan="2">$n_\mathrm{r} > 0 \;\; n_\mathrm{i} > 0$</td><td colspan="2">$\mu_\mathrm{r} > 0 \;\; \mu_\mathrm{i} > 0$</td><td colspan="2">$n_\mathrm{r} > 0 \;\; n_\mathrm{i} > 0$</td></tr>
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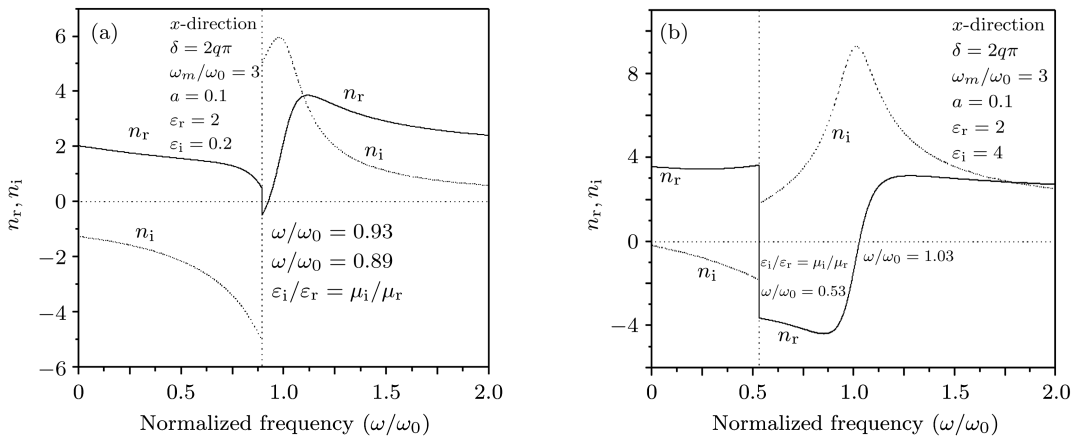


Fig. 6. The dependence of complex refractive index on free precession frequency at x-direction (a) $\varepsilon_\mathrm{i} = 0.2$ and $\varepsilon_\mathrm{r} = 2$, (b) $\varepsilon_\mathrm{i} = 4$, and $\varepsilon_\mathrm{r} = 2$.

the proportion $|\varepsilon_{\rm i}|/|\varepsilon_{\rm r}|$ is high and the spectral range for negative refraction is broadened at about $\omega = 0.53\,\omega_0 \sim 1.03\,\omega_0$. It is interesting to note from Fig. 6(b) that in the most part of the spectrum within the range of negative refraction, there is $n_{\rm r} > n_{\rm i}$, i.e. the real part is larger than the imaginary part of the refractive index. In other words, the refraction effect is dominant in comparison with the absorption, which would help us to avoid confusion between the negative refraction and inhomogeneity.

7. Conclusions

We have presented a theoretical analysis and discussions for negative refraction occuring in the magnetic materials. Under an external magnetic field, the material has a resonance in the spectrum of the magnetic permeability. On one side of the resonance, under certain conditions, there is a broadened spectral range where the refraction becomes negative when linearly or rotationally polarized electromagnetic waves are incident upon the material. In some special directions, these conditions are always satisfied and, in other words, there is certainly a spectral range of negative refraction. In addition, in most cases the real part of the refractive index is larger than the imaginary part of the refractive index, and meanwhile the electric permittivity keeps in general positive with or without plasma resonance. Finally, since the magnetic resonance is induced by the external field, for a given incident electromagnetic wave, one is always able to tune the external direct current magnetic field to reach negative refraction.

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