The initial decoherence of the mesoscopic Josephson junction flux qubit*

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For a mesoscopic radio frequency superconducting quantum interference device (rfSQUID), at a degeneracy point, the system reduces to a quantum two-state system which can be used as a flux qubit. When the noise environment is equivalent to a harmonic oscillators bath, by virtue of an operator-norm measure for the short time decoherence, this paper investigates the initial decoherence of the flux qubit operating in the ohmic noise environment and illustrates its property by means of the numerical evaluation.

Keywords: mesoscopic rfSQUID, flux qubit, short-time decoherence
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1. Introduction

It is well known that the superconducting circuits based on Josephson junction are considered to be a promising candidate for large-scale quantum computing. Many kinds of the superconducting qubits, such as Josephson charge, flux and phase qubits,[1] are proposed by the scientists and the engineers to imagine building the novel quantum information processors in the past two decades. According to DiVincenzo's proposal,[2] however, a qualified qubit should have a low decoherence degree to make sure that there is sufficient coherence time to implement quantum manipulation and measurement. So, the mechanisms and the reasons of decoherence for the Josephson junction qubit had been studied both theoretically and experimentally.[3−6] Generally, the conventional measurements for decoherence are based on the Markov approximation. It has been pointed out that, however, the Markov approximation cannot be adopted in low temperature and short time scales.[7,8] For the superconducting Josephson qubit, whose working temperature should be as low as few 10 mK and the single-qubit controlled times are 10^{-11}–10^{-7} second correspondingly, the conventional methods are not suitable any more. So, the new more appropriate measures based on the short-time approximation has been introduced by Privman *et al.*[9] for short-time decoherence. One can employ these measures to investigate whether the physical qubit model satisfies the decoherence criterion and then decide a more suitable one for quantum computation. The flux qubit, differing from the Josephson charge qubit which has been extensively studied,[1,8,10−13] has a longer decoherence time demonstrated by experiments.[14] Thus, research of the decoherence properties of the flux qubit seems to have a more practical significance.

In this paper, we present a detailed study of the initial decoherence for the simplest flux qubit on mesoscopic radio frequency superconducting quantum interference device (rfSQUID) by virtue of the Privman's measures. It is arranged as follows. In Section 2, we give the Hamiltonian of the flux qubit interacted with the noise environment. In Section 3, we present the dynamics of short-time based on the short-time approximation. In Section 4, we investigate the initial decoherence for the flux qubit operations in the operator-norm measure and use the numerical evaluation analysing the initial decoherence time.

2. Hamiltonian of the mesoscopic rfSQUID and ohmic noise

For a single rfSQUID, which is formed by a loop with one junction. Using L, we let $E_{\rm J}$ denote the self-inductance of the loop and the Josephson tunneling energy. If there is no dissipation, when $E_{\rm J} \gg E_{\rm c}$

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($E_{\rm c} = 2e^2/C_{\rm J}$, $C_{\rm J}$ is the junction capacitance), the circuit acts as a simplest flux qubit. The phase difference φ across the junction is related to the flux Φ in the loop (in units of the flux quantum $\Phi_0 = h/2e$, h is the Plank constant) by $\varphi/2\pi = \Phi/\Phi_0 + \text{integer}$. If there is an external flux $\Phi_{\rm ex}$ applied in the system, the Hamiltonian is[1]

$$\mathcal{H}_{\rm rf} = \frac{Q^2}{2C_{\rm J}} - E_{\rm J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi-\Phi_{\rm ex})^2}{2L}, \quad (1)$$

where the charge $Q = -{\rm i}\hbar\partial/\partial\Phi$ is canonically conjugate to the flux Φ, i.e. $[\Phi, Q] = {\rm i}\hbar$. If the externally applied flux $\Phi_{\rm ex}$ is close to $\Phi_0/2$, the last two terms in Eq. (1) form a double-well potential near $\Phi = \Phi_0/2$. When the temperature T is low enough, the certain two states in the two wells contribute. Thus, the system can be reduced to an effective two-state system[1]

$$\mathcal{H}_{\rm s} = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x, \quad (2)$$

where σ_z and σ_x are the Pauli matrices, $B_z = I_{\rm p}(\Phi_{\rm ex} - \Phi_0/2)$ is the energy bias between the two states, $I_{\rm p}$ is the persistent current along the loop. The off-diagonal term B_x describes the tunnel amplitude between the two states with opposite directions of persistent current along the loop, which is in proportion to the capacitance $C_{\rm J}$ for a fixed $\beta_L = 4\pi^2 L E_{\rm J}/\Phi_0^2$.

From Eq. (2), one can perform a required one-bit operation by changing the parameter B_z through controlling the external flux $\Phi_{\rm ex}$. If we choose the degeneracy point $\Phi_{\rm ex} = \Phi_0/2$, then by considering the noise environment, the Hamiltonian becomes[13,15]

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_{\rm s} + \mathcal{H}_{\rm I} + \mathcal{H}_{\rm b} \\ &= -\frac{1}{2}B_x\sigma_x + \sigma_z\sum_k c_k(a_k^\dagger + a_k) \\ &\quad + \sum_k \hbar\omega_k a_k^\dagger a_k, \end{aligned} \quad (3)$$

where the noise environment is modeled by a bath with a large set of harmonic oscillators. In thermal equilibrium, the spectral density of the fluctuating field which is related to the power spectrum is[13]

$$S(\omega) \equiv 2\hbar J(\omega)\coth\frac{\hbar\omega}{2k_{\rm B}T}. \quad (4)$$

Here ω is the continuous frequency of the bath mode, $k_{\rm B}$ and T are the Boltzmann constant and temperature respectively. The bath spectral density $J(\omega)$ is

$$\begin{aligned} J(\omega) &\equiv \frac{\pi}{\hbar}\sum_k c_k^2\delta(\omega-\omega_k) \\ &= \frac{\pi}{\hbar}\int_{-\infty}^{\infty} N(\omega)c^2(\omega_k)\delta(\omega-\omega_k)\,{\rm d}\omega \\ &= \frac{\pi}{\hbar}N(\omega)c^2(\omega), \end{aligned} \quad (5)$$

where the $N(\omega)$ is the density of the states of environment modes. For the standard ohmic dissipation, taking some high-frequency cutoff $\omega_{\rm c}$ into account, we obtain the behaviour of spectral density $J(\omega)$ at low frequencies as

$$J(\omega) = \frac{\pi}{2}\alpha\hbar\omega\,{\rm e}^{-\frac{\omega}{\omega_{\rm c}}}. \quad (6)$$

The coefficient α is the dimensionless strength of this kind of dissipation. In a physical system it depends on the amplitude of the noise and the coupling strength. Comparing Eq. (5) with Eq. (6), we have

$$N(\omega)c^2(\omega) = \frac{1}{2}\alpha\hbar^2\omega\,{\rm e}^{-\frac{\omega}{\omega_{\rm c}}}, \quad (7)$$

which describes how the power of the bath is distributed with the frequency.

3. Short-time dynamics

According to Privman's proposal,[15] the density matrix $\boldsymbol{R}$ of the system plus bath at time $t = 0$ is assumed to be the direct product

$$R(0) = \rho(0)\otimes_k \theta_k, \quad (8)$$

where $\rho(0)$ is the initial state of the flux qubit and θ_k is density matrices of the thermalized bath modes k,

$$\theta_k = \frac{{\rm e}^{-\beta\hbar\omega_k a_k^\dagger a_k}}{{\rm Tr}_k({\rm e}^{-\beta\hbar\omega_k a_k^\dagger a_k})}, \quad \left(\beta = \frac{1}{k_{\rm B}T}\right). \quad (9)$$

Thus, the time dependence of the density martix for the system and bath is

$$R(t) = {\rm e}^{-{\rm i}(\mathcal{H}_{\rm s}+\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar}R(0)\,{\rm e}^{{\rm i}(\mathcal{H}_{\rm s}+\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar}. \quad (10)$$

Utilizing the short-time approximation[15]

$$\begin{aligned} &{\rm e}^{{\rm i}(\mathcal{H}_{\rm s}+\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar+O(t^3)} \\ =\ &{\rm e}^{{\rm i}\mathcal{H}_{\rm s}t/2\hbar}\,{\rm e}^{{\rm i}(\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar}\,{\rm e}^{{\rm i}\mathcal{H}_{\rm s}t/2\hbar}, \end{aligned} \quad (11)$$

we can express the elements of the reduced density martix $\rho(t)$ as

$$\begin{aligned} &\rho_{mn}(t) \\ =\ &{\rm Tr}_{\rm b}\langle m|\,{\rm e}^{-{\rm i}\mathcal{H}_{\rm s}t/2\hbar}\,{\rm e}^{-{\rm i}(\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar} \\ &\times\,{\rm e}^{-{\rm i}\mathcal{H}_{\rm s}t/2\hbar}R(0) \\ &\times\,{\rm e}^{{\rm i}\mathcal{H}_{\rm s}t/2\hbar}\,{\rm e}^{{\rm i}(\mathcal{H}_{\rm I}+\mathcal{H}_{\rm b})t/\hbar}\,{\rm e}^{{\rm i}\mathcal{H}_{\rm s}t/2\hbar}|n\rangle\,. \end{aligned} \quad (12)$$

Inserting several completeness relations $\sum_{j=0}^{N} |j\rangle \langle j| = 1$ into Eq. (12), after some calculations one can obtain[16]

$$\rho_{10}(t) = \frac{1}{2} \mathrm{e}^{-\mathrm{i}B_x t}[1 + \mathrm{e}^{-B^2(t)}]\rho_{10}(0) + \frac{1}{2}[1 - \mathrm{e}^{-B^2(t)}]\rho_{01}(0), \quad (13)$$

$$\rho_{11}(t) = \frac{1}{2}[1 + \mathrm{e}^{-B^2(t)}]\rho_{11}(0) + \frac{1}{2}[1 - \mathrm{e}^{-B^2(t)}]\rho_{00}(0), \quad (14)$$

where

$$B^2(t) \equiv 8 \sum_k \frac{|c_k|^2}{(\hbar\omega_k)^2} \sin^2 \frac{\omega_k t}{2} \coth \frac{\beta\hbar\omega_k}{2}. \quad (15)$$

In the limit of infinite number of the bath modes, Eq. (15) can be replaced by the integration form,

$$B^2(t) = 8 \int \mathrm{d}\omega N(\omega) |c(\omega)|^2 (\hbar\omega)^{-2} \times \sin^2 \frac{\omega t}{2} \coth \frac{\beta\hbar\omega}{2}. \quad (16)$$

Substituting Eq. (7) into Eq. (16) yields

$$B^2(t) = 8 \int \mathrm{d}\omega \frac{\alpha}{2\omega} \mathrm{e}^{-\frac{\omega}{\omega_\mathrm{c}}} \sin^2 \frac{\omega t}{2} \coth \frac{\beta\hbar\omega}{2}. \quad (17)$$

Note that the spectral function $B^2(t)$ is the mean-squared value of the magnitude of the phase noise,[4] which is decided by the choice of bath model and its coupling forms with the system. As the time increases, $B^2(t)$ grows monotonically from zero. Specifically, for the standard ohmic dissipation, $B^2(t)$ is quadratic growth for short times, logarithmic growth for the intermediate region and the linear behaviour in large time scales.[15,16]

4. The initial decoherence of the flux qubit

Fedichkin *et al.*[16] introduced an operator-norm measure, which can be used for quantifying the degree of decoherence. If the deviation operator is expressed as

$$\sigma(t) \equiv \rho(t) - \rho^{\mathrm{ideal}}(t), \quad (18)$$

where $\rho(t)$, $\rho^{\mathrm{ideal}}(t)$ are the density matrices of the real evolution and ideal evolution (no interaction) respectively, then operator norms $||\sigma||_\lambda$ is defined as

$$||\sigma||_\lambda = \sup_{\varphi \neq 0} \left(\frac{\langle\varphi| \sigma^\dagger \sigma |\varphi\rangle}{\langle\varphi| |\varphi\rangle} \right)^{1/2}. \quad (19)$$

Here λ denotes the eigenvalue norm. For the single qubit discussed in Section 1, the norms can be simplified as[16]

$$||\sigma||_\lambda = \sqrt{|\sigma_{10}|^2 + |\sigma_{11}|^2}. \quad (20)$$

For Eqs. (13) and (14), the evolutions of the ideal condition are $\rho_{10}^{\mathrm{ideal}}(t) = \rho_{10}(0) \mathrm{e}^{-\mathrm{i}B_x t}$ and $\rho_{11}^{\mathrm{ideal}}(t) = \rho_{11}(0)$. Thus, from Eq. (18) we have

$$\rho_{10}(t) = \frac{1}{2}\rho_{10}(0)[1 - \mathrm{e}^{-B^2(t)}][1 - \mathrm{e}^{-\mathrm{i}B_x t}],$$
$$\rho_{11}(t) = \frac{1}{2}[1 - \mathrm{e}^{-B^2(t)}][\rho_{00}(0) - \rho_{11}(0)]. \quad (21)$$

Substituting Eq. (21) into Eq. (20) yields

$$||\sigma(t)||_\lambda = \frac{1}{2}[1 - \mathrm{e}^{-B^2(t)}] \times \left\{ [\rho_{00}(0) - \rho_{11}(0)]^2 + 4 |\rho_{10}(0)|^2 \sin^2 \frac{B_x t}{2} \right\}^{1/2}, \quad (22)$$

where the value of the norm is equal to 0 at $t = 0$, and then it increases to positive as time goes by. This namely is the degree of the initial decoherence based on the short-time approximation. If the initial pure state is known, the evolution of the norms will depend on the system parameter B_x to some extent. However, the initial state is not always given, we must turn to the maximal norm,[16] $D = \sup_{\rho(0)}(||\sigma(t, \rho(0))||_\lambda)$, for characterizing the decoherence for an arbitrary initial state, pure or mixed. For the above discussion, the expression of the maximal norm is

$$D(t) = \frac{1}{2}[1 - \mathrm{e}^{-B^2(t)}]. \quad (23)$$

Though $D(t)$ can not give the decoherence process in detail, we still obtain an upper-bound estimate of decoherence for an arbitrary initial state. It is suitable for the most practical cases. Consequently, we can use the results in Eqs. (22) and (23) for evaluating the appreciate flux qubit in the quantum computing designs.

In the following, we can analyse the initial decoherence time by means of the numerical evaluation. For the different pure initial states $\psi^{(i)} = (\cos\varphi_i, \sin\varphi_i)^{\mathrm{T}}$ (where $0 \leq \varphi_i \leq 2\pi$), we can plot the decoherence varying with the time t. Here, we confine ω in 1 Hz–50 GHz, the cutoff frequency $\omega_\mathrm{c} \sim 10^{-12}$,[8] the temperature $T = 50$ mK,[17] the tunneling energy $B_x = 16.4$ μeV,[17] and the coefficient α is an estimated value about 10^{-6}–10^{-8}.[14] By using these parameters, we can plot respectively the norms $||\sigma(t)||_\lambda$

versus time t when we choose $\varphi_0 = 0$, $\varphi_1 = \pi/4$, $\varphi_2 = \pi/6$.

In Fig. 1, the upmost line corresponds to $\varphi_0 = 0$, the middle dash line and the lowest dot line represent $\varphi_2 = \pi/6$, $\varphi_1 = \pi/4$ respectively. The upmost line is the maximal, while the others are under it and oscillate with time t periodically. So, it shows that the different initial states has different decoherent degree accordingly, but in the mean time they have the same maximum. Comparing the inset of Figs. 1(a), 1(b) and 1(c), we can see that the different value of α can produce a marked amplitude variation of the norms from the scale of vertical axis. So, we know that the strength of decoherence is closely associated with the strength of the dissipation. The larger value of dissipation coefficient α, the stronger of the decohernce accordingly. The norm curves, however, keep the similar shape and have nothing to do with the value of α. Specially, when $\varphi_0 = 0$, which corresponds to the initial state $\psi^{(0)} = (1, 0)^{\mathrm{T}}$, the norm $||\sigma(t)||_\lambda$ is the maximum and it equals to the maximal norm $D(t)$. If we denote the low decoherence ($D(t)_{\mathrm{a}} < 5 \times 10^{-6}$, $D(t)_{\mathrm{b}} < 5 \times 10^{-7}$ and $D(t)_{\mathrm{c}} < 5 \times 10^{-8}$ which are depicted in the insets in Fig. 1) time by τ, then it can be obtained about 1.5×10^{-10} s = 0.15 ns or more, while the characteristic time of the gate operation $\tau_0 = \hbar/B_x = 12.7$ ps.[8] Obviously $\tau \gg \tau_0$, which shows that there is enough decoherence time for completing the quantum manipulation. This also is why the flux qubit is proposed as a candidate for quantum memory. At the same time, it proves that the flux qubit investigated above can be taken as the qubit for quantum computing because it satisfies the DiVincenzo's low decoherence criterion.[2]

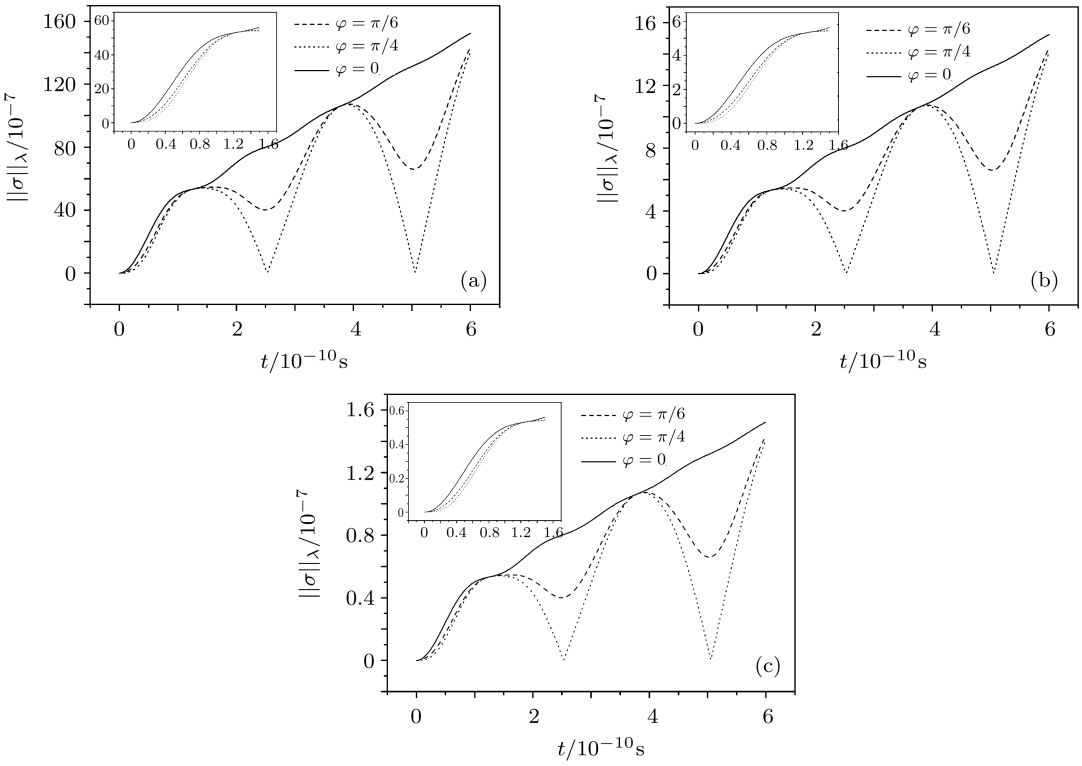


Fig. 1. The norms $\|\sigma\|_\lambda$ versus time t for (a) $\alpha = 10^{-6}$, (b) $\alpha = 10^{-7}$, (c) $\alpha = 10^{-8}$. The insets are the traces in a shorter time for the different α respectively.

5. Conclusion

In summary, we use the short-time approximation and norm measurement method to demonstrate the evolution of the initial decoherence for the flux qubit in the ohmic noise environment. We know that the initial decoherence decided by not only the parameters of tunnel energy B_x and the working temperature T, but also the dimensionless strength coefficient α and the kind of the pure initial states. In practice, for a rfSQUID circuit, B_x and T are always determined, whether the device is qualified for a qubit or not mainly depends on the coupling strength and the pure initial state. By means of the numerical evaluation, it is shown that the

decoherence increases with the increase of dimensionless strength of the dissipation. Furthermore, no matter what the initial state is, the low decoherence time can be estimated by Eq. (23). Because we employ the short-time approximation, it is more accurate to acquire the initial decoherence process. So, by virtue of this measure we further confirm that the flux qubit may be a good candidate of qubit for quantum computation.

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