Separate holographic screening soliton pairs in a biased series photorefractive crystal circuit*

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Holographic dark (bright) screening solitons are predicted in one dimension for a series circuit consisting of two photorefractive crystals connected electronically by electrode leads in a chain with a voltage source. Each crystal can support a holographic screening soliton. The two solitons are known collectively as a separate holographic screening soliton pair with three types: bright–bright, bright–dark and dark–dark. The numerical results show that the two solitons in a soliton pair can affect each other through a light-induced current and their coupling can affect their spatial profiles under the limit in which the optical wave has a spatial extent much less than the width of the crystal.

Keywords: photorefractive effect, holographic soliton, soliton pairs
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1. Introduction

Holographic solitons which are supported by the holographic focusing mechanism in photorefractive (PR) crystals are widely investigated theoretically and experimentally.[1−5] The holographic focusing mechanism arises from a grating in the refractive index induced by interference between the two beams (i.e. pump and signal beams) via two-wave mixing (TWM). It was predicted that, based on such a holographic focusing mechanism, both beams in the TWM process could form holographic solitons simultaneously under a special condition that the asymmetric energy exchange between the beams does not take place.[1] It was proposed that the signal beam alone could form PR solitons based on the holographic focusing mechanism if the asymmetric energy exchange from the pump beam to the signal beam took place, and such solitons could be known as dissipative holographic solitons because they could exist in a dissipative PR system.[5] Combining the holographic focusing mechanism with the self-phase-modulation self-focusing mechanism that originates from the spatially nonuniform screening of the applied field to form screening solitons,[6,7] or from the photovoltaic effects to form photovoltaic solitons,[8,9] or from both the screening and photovoltaic effects simultaneously to form screening-photovoltaic solitons,[10,11] holographic screening solitons,[12] holographic photovoltaic solitons[13] and holographic screening-photovoltaic solitons[14] were proposed to occur in dissipative PR systems in which the asymmetric energy exchange takes place. Soliton solutions in dissipative systems are the result of a double balance, i.e. diffraction is balanced by nonlinearity and loss is balanced by gain. At present, all investigations on the holographic solitons are concerned with only one piece of PR crystal. Can a holographic screening soliton form individually within each crystal in a biased series PR crystal circuit in which two PR crystals, denoted by P and $\hat{P}$, and a voltage source are connected in a chain by electrode leads (Fig. 1)? That is, when two sets of signal beam and pump beam are, appropriately and respectively, incident on the two crystals, can a holographic screening soliton form individually within each crystal? If so, do the two solitons, formed separately in the two crystals, interact or affect each other?

In the present paper, we investigate holographic screening solitons formed in a series PR crystal circuit. By use of the well-known transport model of photorefractive effect, which was successfully used to develop the theories of a holographic screening soliton,[12] we predicate that each crystal can support a holographic screening soliton in the case that the spatial extent of the optical wave is much less than the width of the

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crystal. We name the two solitons, formed separately in the two crystals, a separate holographic screening soliton pair. Both bright and dark solitons can form in the crystals. As a result, there are three types of the separate holographic screening soliton pairs: bright–bright, bright–dark, dark–dark.

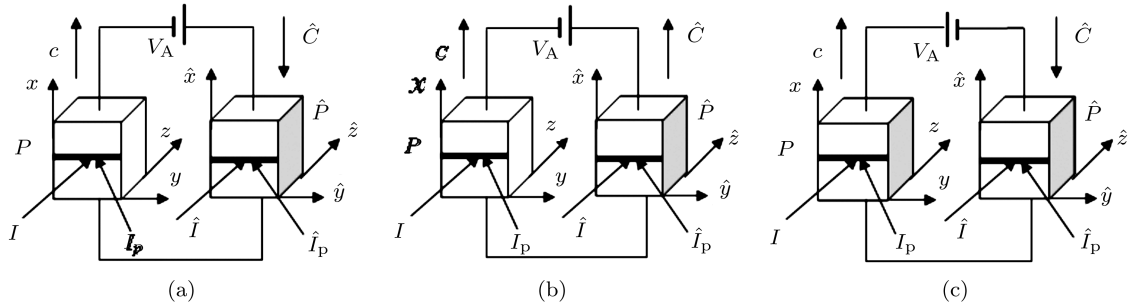


Fig. 1. Illustration of a biased series PR crystal circuit consisting of two PR crystals to support a bright–bright soliton pair (a), a bright–dark soliton pair (b), and a dark–dark soliton pair (c) separately, where P and $\hat{P}$ denote the two crystals; C and $\hat{C}$ represent the two c axes; I and $\hat{I}$ refer to the incident bright or dark solitonlike one-dimensional laser beams respectively; I_p and $\hat{I}_p$ indicate the incident pump beams; V_A denotes the voltage of the source.

2. Theoretical model

Considering first crystal P, its optical c axis orients along the x axis and a bias field is applied in the same direction. A one-dimensional bright or dark soliton-like optical beam I as a signal beam is made to propagate in the crystal along the z axis, and it is permitted to diffract only along the x direction. An optical beam I_p with a uniform spatial distribution as a pump beam is also made to propagate in the crystal along the z axis and makes a small angle θ in the crystal with respect to the signal beam. The two coherent beams couple by PR codirectional degenerate two-beam coupling, and then the pump beam can amplify the signal beam.[15,16] Moreover, we assume that the signal beam is linearly polarised along the x direction. The pump beam is also linearly polarised, and its polarised direction makes an angle φ with respect to the x axis.

When the signal beam is much weaker than the pump beam, which makes a very weak index modulation, the perturbed extraordinary refractive index $\bar{n}_e$ (along the c axis) of the signal beam can be written as $\bar{n}_e = n_e + \Delta n_1 + \Delta n_2$, where n_e is the unperturbed extraordinary refractive index, Δn_1 results from the spatial nonuniform screening of the applied field,[6,7] and Δn_2 results from the two-beam coupling process.[15] Δn_1 can be expressed as[6,7] $\Delta n_1 = -n_e^3 r_{33} E_{sc}/2$, where r_{33} is the electro-optic coefficient, and E_{sc} is the space-charged field induced by the signal beam. Δn_2 can be expressed as[15] $\Delta n_2 = n_1[\mathrm{e}^{\mathrm{i}\alpha}\phi_p\phi^*\,\mathrm{e}^{\mathrm{i}\Delta} + \text{c.c.}]/[2(|\phi_p|^2 + |\phi|^2)]$, where φ and φ_p are the slowly varying envelopes of the electric-field components of the signal and pump beams, respectively, $\Delta = k_p z\cos\theta + k_p x\sin\theta - kz$, $k = n_e k_0$, $k_0 = 2\pi/\lambda_0$, λ_0 is the free-space wavelength of the light-wave employed, k and k_p are the wave vectors of the signal and the pump beams respectively. The term $n_1\,\mathrm{e}^{\mathrm{i}\alpha}$ can be expressed as $n_1\,\mathrm{e}^{\mathrm{i}\alpha} = \lambda_0(\Gamma_0 + \mathrm{i}\Gamma/2)/\pi$, where Γ and Γ_0 are the intensity and the phase coupling coefficients, respectively.

On the other hand, $n_1\,\mathrm{e}^{\mathrm{i}\alpha} = \mathrm{i}n_e^3 r_{\mathrm{eff}}\tilde{E}E_s/(\tilde{E} + E_s)$ which results in $\Gamma_0 + \mathrm{i}\Gamma/2 = \mathrm{i}\delta\tilde{E}E_s/(\tilde{E} + E_s)$, where $\delta = n_e^3 r_{\mathrm{eff}} k_0/2$, $\tilde{E} = E_d - \mathrm{i}E_0$, E_0 is the bias field, $E_d = 4\pi k_B T\sin\theta/(e\lambda_0)$ is the diffusion field, k_B is the Boltzmann constant, e is the electron charge, T is the absolute temperature, $E_s = eN_A\lambda_0/(4\pi\varepsilon_r\varepsilon_0\sin\theta)$ is the saturation field, N_A is the acceptor density, ε_r is the effective dielectric constant, and ε_0 is the permittivity of vacuum.

We use a method similar to that in Ref. [15], in which the couple-wave equation for codirectional degenerate two-beam coupling was derived. But we only consider the variation of φ in the x direction, and ignore the quadratic terms of Δn_1 and Δn_2 as well as the product of Δn_1 and Δn_2, in the slowly varying approximation (i.e. we ignore the term of $\partial^2\varphi/\partial z^2$), and we find that φ satisfies the following paraxial wave equation:

$$\frac{1}{2k}\frac{\partial^2\phi}{\partial x^2} + \mathrm{i}\frac{\partial\phi}{\partial z} - \frac{k_0}{2}n_e^3 r_{33}E_{sc}\phi$$

$$+\left(\Gamma_0-\frac{\mathrm{i}}{2}\Gamma\right)\frac{I_\mathrm{p}}{I_\mathrm{p}+I}\phi+\mathrm{i}\frac{\vartheta_0}{2}\phi=0, \tag{1}$$

where ϑ_0 is the absorption coefficient of the crystal, $I=I(x,z)=(n_\mathrm{e}/2\eta_0)\left|\phi(x,z)\right|^2$, $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$ and $I_\mathrm{p}=(n_\mathrm{e}/2\eta_0)\left|\varphi_\mathrm{p}\right|^2$. The induced space-charge field E_sc can be obtained from the standard set of rate and continuity equations and Gauss's law, which describe the photorefractive effect in a medium. In the steady state, the one-dimensional equations are[7]

$$\gamma_\mathrm{R}nN_\mathrm{D}^+=s_\mathrm{i}(I+I_\mathrm{b})(N_\mathrm{D}-N_\mathrm{D}^+), \tag{2}$$

$$\frac{\partial E_\mathrm{sc}}{\partial x}=\frac{e}{\varepsilon_0\varepsilon_\mathrm{r}}(N_\mathrm{D}^+-N_\mathrm{A}-n), \tag{3}$$

$$J=e\mu nE_\mathrm{sc}+k_\mathrm{B}T\mu\frac{\partial n}{\partial x}, \tag{4}$$

$$\frac{\partial J}{\partial x}=0\quad\text{or}\quad J=\text{const}, \tag{5}$$

where N_D^+ and N_D are the ionised donor density and the donor density, respectively, n is the electron density, J is the current density, s_i is the photoexcitation cross section, γ_R is the carrier recombination rate, μ is the electron mobility, and I_b is the intensity of the background light. For the system considered in this paper, we have $I_\mathrm{b}=I_\mathrm{p}+I_\mathrm{d}$, where I_d is the dark irradiance. We assume that $I_\mathrm{d}\ll I_\mathrm{p}$, thus we have $I_\mathrm{b}\approx I_\mathrm{p}$. Moreover, in Eqs. (2)–(5) we have ignored any z spatial dependence by assuming that the variables involved vary much more rapidly in the x direction.

We can take a similar way to that in Ref. [7] to greatly simplify Eqs. (2)–(5) by keeping in mind that the following inequalities hold true in typical PR media: $N_\mathrm{D}^+\gg n$, $N_\mathrm{D}\gg n$, $N_\mathrm{A}\gg n$ and $\left|(\varepsilon_0\varepsilon_\mathrm{r}/eN_\mathrm{A})\left(\partial E_\mathrm{sc}/\partial x\right)\right|\ll 1$. In this case, Eqs. (2) and (3) yield the following results: $N_\mathrm{D}^+\approx N_\mathrm{A}$ and $n=s_\mathrm{i}(N_\mathrm{D}-N_\mathrm{A})(I+I_\mathrm{p})/(\gamma_\mathrm{R}N_\mathrm{A})$. At this point, we also assume that the power density $I=I(x,z)$ attains asymptotically to a constant value at $x\to\pm\infty$, that is $I(x\to\pm\infty,z)=I_\infty$. In the region of constant illumination, Eqs. (2)–(5) require that the space-charge field E_sc is independent of x, i.e., $E_\mathrm{sc}(x\to\pm\infty,z)=E_0$. From Eq. (4), the current density J in the regions $(x\to\pm\infty)$, denoted by $J_\infty=J(x\to\pm\infty,z)$, can be subsequently determined and given by $J_\infty=e\mu n_\infty E_0$, where $n_\infty=s_\mathrm{i}(N_\mathrm{D}-N_\mathrm{A})(I_\infty+I_\mathrm{p})/(\gamma_\mathrm{R}N_\mathrm{A})$. For the crystal $\hat{P}$, we similarly have $\hat{J}_\infty=e\hat{\mu}\hat{n}_\infty\hat{E}_0$ and $\hat{n}_\infty=\hat{s}_\mathrm{i}(\hat{N}_\mathrm{D}-\hat{N}_\mathrm{A})(\hat{I}_\infty+\hat{I}_\mathrm{p})/(\hat{\gamma}_\mathrm{R}\hat{N}_\mathrm{A})$.

Let V_A denote the applied voltage source, and V and $\hat{V}$ represent the potentials measured between the electrodes of P and $\hat{P}$ crystals, with separations between the electrodes being W and $\hat{W}$, respectively. Let S and $\hat{S}$ denote the surfaces of the electrodes of the crystals P and $\hat{P}$ respectively. In the series circuit (Fig. 1), we have $V_\mathrm{A}=V+\hat{V}$ and $SJ=\hat{S}\hat{J}$. If the spatial extent of the optical wave is much less than the x width W, E_0 is approximately expressed as $E_0=V/W$. For the crystal $\hat{P}$, we have $\hat{E}_0=\hat{V}/\hat{W}$. Obviously, we find $V_\mathrm{A}=V+\hat{V}=WE_0+\hat{W}\hat{E}_0$. Keeping in mind that Eq. (5) implies that the current density J is constant everywhere in the crystal, that is, $J\left(x,z\right)=J_\infty$. For the crystal $\hat{P}$, we also have $\hat{J}\left(x,z\right)=\hat{J}_\infty$. As a result, we have $SJ_\infty=\hat{S}\hat{J}_\infty$. Substituting J_∞ and $\hat{J}_\infty$ into $SJ_\infty=\hat{S}\hat{J}_\infty$ and using $V_\mathrm{A}=V+\hat{V}=WE_0+\hat{W}\hat{E}_0$, we find that

$$E_0=\hat{\Omega}E_\mathrm{A}, \tag{6}$$

$$\hat{E}_0=\Omega\hat{E}_\mathrm{A}, \tag{7}$$

$$\hat{\Omega}=\hat{\delta}\left(\hat{I}_\infty+\hat{I}_\mathrm{p}\right)/\left[\delta\left(I_\infty+I_\mathrm{p}\right)+\hat{\delta}\left(\hat{I}_\infty+\hat{I}_\mathrm{p}\right)\right], \tag{8}$$

$$\Omega=\delta\left(I_\infty+I_\mathrm{p}\right)/\left[\delta\left(I_\infty+I_\mathrm{p}\right)+\hat{\delta}\left(\hat{I}_\infty+\hat{I}_\mathrm{p}\right)\right], \tag{9}$$

where $\delta=S\mu s_\mathrm{i}(N_\mathrm{D}-N_\mathrm{A})/(\gamma_\mathrm{R}N_\mathrm{A}W)$, $E_\mathrm{A}=V_\mathrm{A}/W$, $\hat{\delta}=\hat{S}\hat{\mu}\hat{s}_\mathrm{i}(\hat{N}_\mathrm{D}-\hat{N}_\mathrm{A})/(\hat{\gamma}_\mathrm{R}\hat{N}_\mathrm{A}\hat{W})$ and $\hat{E}_\mathrm{A}=V_\mathrm{A}/\hat{W}$. The parameters Ω and $\hat{\Omega}$ are known as coupling coefficients between the two solitons and $\Omega+\hat{\Omega}=1$.

In the region of $I\left(x,z\right)$ varying with x, the current density J is determined by Eqs. (2) and (4), and then from $J\left(x,z\right)=J_\infty$, the induced space-charge field E_sc in the crystal P can be determined by entirely ignoring the diffusion effect to be

$$E_\mathrm{sc}=E_0\frac{I_\infty+I_\mathrm{p}}{I+I_\mathrm{p}}. \tag{10}$$

For the crystal $\hat{P}$, we also have

$$\hat{E}_\mathrm{sc}=\hat{E}_0\frac{\hat{I}_\infty+\hat{I}_\mathrm{p}}{\hat{I}+\hat{I}_\mathrm{p}}. \tag{11}$$

Although the expression E_sc (as well as $\hat{E}_\mathrm{sc}$) has a similar form to that for the space-charge field in a single biased photorefractive crystal,[12] the value of E_sc (as well as $\hat{E}_\mathrm{sc}$) depends on the parameters of the two crystals, including I_∞ and $\hat{I}_\infty$. On the other hand, E_sc and $\hat{E}_\mathrm{sc}$ are not independent. They couple each other by the coupling coefficients Ω and $\hat{\Omega}$.

Then the evolution equations of the soliton formed in the crystal P can be established by substituting Eq. (10) into Eq. (1). It proves more convenient to study this equation in a normalised fashion. To do so, we adopt the following dimensionless coordinates and variables, i.e. let $\xi=z/\left(kx_0^2\right)$, $s=x/x_0$ and $\phi=\left(2\eta_0I_\mathrm{p}/n_\mathrm{e}\right)^{1/2}U$. Here x_0 is an arbitrary spatial width. The dynamical evolution equations of the

normalised envelope U can be determined as follows:

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} - [\beta(1+\rho) - g_0 + \mathrm{i}g]\frac{U}{1+|U|^2} + \mathrm{i}\vartheta U = 0, \tag{12}$$

where $\rho = I_\infty/I_p$, $g_0 = kx_0^2\Gamma_0$, $g = kx_0^2\Gamma/2$, $\beta = \chi r_{33}E_0$, $\chi = (k_0x_0)^2(n_e^4/2)$ and $\vartheta = kx_0^2\vartheta_0/2$. From $\Gamma_0 + \mathrm{i}\Gamma/2 = \mathrm{i}\delta\tilde{E}E_s/\left(\tilde{E}+E_s\right)$, we can obtain the expressions for g and g_0 to be $g = \left[\chi r_{\mathrm{eff}}E_s\left(E_0^2 + E_dE_{ds}\right)\right]/\left(E_{ds}^2 + E_0^2\right)$ and $g_0 = \left(\chi r_{\mathrm{eff}}E_0E_s^2\right)/\left(E_{ds}^2 + E_0^2\right)$, where $E_{ds} = E_d + E_s$. It is difficult to obtain an exact analytical solution of Eq. (12) in general conditions. In what follows, we solve this equation in the condition of $|U|^2 = I/I_p \ll 1$, and we can take $\left(1+|U|^2\right)^{-1} \approx 1 - |U|^2$; thus Eq. (12) becomes

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} - [P_d + \mathrm{i}G]U + [P_d + \mathrm{i}g]U|U|^2 = 0, \tag{13}$$

where $G = g - \vartheta$ and $P_d = (1+\rho)\beta - g_0$.

Similarly, we can obtain results for $\hat{P}$ as follows:

$$\mathrm{i}\hat{U}_{\hat{\xi}} + \frac{1}{2}\hat{U}_{\hat{s}\hat{s}} - \left[\hat{P}_d + \mathrm{i}\hat{G}\right]\hat{U} + [\hat{P}_d + \mathrm{i}\hat{g}]\hat{U}|\hat{U}|^2 = 0, \tag{14}$$

where $\hat{G} = \hat{g} - \hat{\vartheta}$, $\hat{P}_d = (1+\hat{\rho})\hat{\beta} - \hat{g}_0$, $\hat{\rho} = \hat{I}_\infty/\hat{I}_p$, $\hat{g} = [\hat{\chi}\hat{r}_{\mathrm{eff}}\hat{E}_s(\hat{E}_0^2 + \hat{E}_d\hat{E}_{ds})]/(\hat{E}_{ds}^2 + \hat{E}_0^2)$, $\hat{g}_0 = (\hat{\chi}\hat{r}_{\mathrm{eff}}\hat{E}_0\hat{E}_s^2)/(\hat{E}_{ds}^2 + \hat{E}_0^2)$, $\hat{E}_d = 4\pi k_B\hat{T}\sin\hat{\theta}/(e\hat{\lambda}_0)$, $\hat{E}_s = e\hat{N}_A\hat{\lambda}_0/(4\pi\hat{\varepsilon}_r\varepsilon_0\sin\hat{\theta})$, $\hat{E}_{ds} = \hat{E}_d + \hat{E}_s$, $\hat{\beta} = \hat{\chi}\hat{r}_{33}\hat{E}_0$, $\hat{\chi} = (\hat{k}_0^2\hat{x}_0^2)(\hat{n}_e^4/2)$, $\hat{\vartheta} = \hat{k}\hat{x}_0^2\hat{\vartheta}_0/2$, $\hat{\xi} = \hat{z}/(\hat{k}\hat{x}_0^2)$ and $\hat{s} = \hat{x}/\hat{x}_0$. Although the two dynamical evolution equations have a similar form to that for a single biased photorefractive crystal,[12] they couple each other by the coupling coefficients Ω and $\hat{\Omega}$.

3. Spatial profiles of three types of separate holographic screening soliton pairs and coupling effects

We consider the effects of the interaction between the two solitons in a separate holographic screening soliton pair on the intensity profilers of the two solitons. In a biased series PR crystal circuit, two crystals are connected electronically. When optical beams illuminate the crystals, the light-induced current will flow from one crystal to another, and as a result, the bias voltage source applied to each crystal will vary with the intensities of the incident laser beams. However, for a single biased PR crystal the bias voltage depends only on the voltage source and does not vary with the intensity of the input laser beam. Therefore, for a biased series PR crystal circuit, changing the intensity of the laser beam incident on one crystal leads not only the soliton formed in that crystal but also the soliton formed in the other crystal to change. That is to say, the two solitons in a separate holographic screening soliton pair can interact or affect each other by the light-induced current. The interaction is collisionless. Because Ω and $\hat{\Omega}$ depend on the parameters of the two crystals, the character of the soliton in one crystal depends on not only the parameters of that crystal but also the parameters of the other crystal. In other words, the character of any soliton in a soliton pair depends on the parameters of the two crystals.

In order to provide some relevant examples, two strontium barium niobate (SBN) crystals are separately taken as P and $\hat{P}$. The two crystals have the following parameters: $\gamma_{33} = \hat{\gamma}_{33} = 220$ pm/V, $n_e = \hat{n}_e = 2.33$, $n_0 = \hat{n}_0 = 2.36$, $\varepsilon_r = \hat{\varepsilon}_r = 880$, $\vartheta_0 = \hat{\vartheta}_0 = 0.27$ cm^{-1}, $N_A = \hat{N}_A = 1.2 \times 10^{17}$ cm^{-3}, $\theta = \hat{\theta} = 2°$, $\varphi = \hat{\varphi} = 87°$, $T = \hat{T} = 300$ K, $W = \hat{W} = 1$ cm, $S = \hat{S}$, $\delta = \hat{\delta}$, $I_p = m\hat{I}_p$, $r_{\mathrm{eff}} = (n_e/n_0)^3 r_{33}\cos\theta\cos(\theta/2)\cos\phi$, and $\hat{r}_{\mathrm{eff}} = (\hat{n}_e/\hat{n}_0)^3\hat{r}_{33}\cos\hat{\theta}\cos\left(\hat{\theta}/2\right)\cos\hat{\varphi}$. The arbitrary scales are $x_0 = \hat{x}_0 = 25$ μm and the wavelengths are $\lambda_0 = \hat{\lambda}_0 = 0.5$ μm. The voltage of the source is $V_A = 100$ V.

3.1. Bright–dark holographic screening soliton pair

We begin our analysis by considering a bright–dark holographic screening soliton pair. We assume that the bright soliton forms in the crystal P and the dark soliton forms in the crystal $\hat{P}$. First, the bright soliton solution of Eq. (13) in the crystal P can be obtained by taking an approach similar to that in Refs. [17] and [18] as follows:

$$U = F\,\mathrm{sech}(Bs)\exp\{\mathrm{i}b\ln[\mathrm{sech}(Bs)]\} \times \exp(-\mathrm{i}\nu\xi), \tag{15}$$

where $F = [3G/(2g)]^{1/2}$, $B = (G/b)^{1/2}$, $b = [-3P_b + (9P_b^2 + 8g^2)^{1/2}]/(2g)$, $\nu = (b^2-1)G/(2b) + P_b$ and $P_b = \beta - g_0$.

The dark soliton solution of Eq. (14) can be obtained by taking an approach similar to that in Refs. [17] and [18] as follows:

$$\hat{U} = \hat{D}\tanh(\hat{H}\hat{s})\exp\left\{\mathrm{i}\hat{d}\ln\left[\cosh\left(\hat{H}\hat{s}\right)\right]\right\}$$

$$\times \exp\left(-\mathrm{i}\hat{\sigma}\hat{\xi}\right), \tag{16}$$

where $\hat{D} = (\hat{G}/\hat{g})^{1/2}$, $\hat{H} = [2\hat{G}/(3\hat{d})]^{1/2}$, $\hat{d} = [3\hat{P}_\mathrm{d} + (9\hat{P}_\mathrm{d}^2 + 8\hat{g}^2)^{1/2}]/(2\hat{g})$ and $\hat{\sigma} = 2\hat{G}/(3\hat{d}) + \hat{P}_\mathrm{d}$.

Equations (15) and (16) predict that a bright–dark soliton pair can exist in a biased series PR crystal circuit. The conditions necessary for holographic screening bright (dark) solitons are $g > \vartheta$ ($\hat{g} > \hat{\vartheta}$) and $b > 0$ ($\hat{d} > 0$). From the definitions of b, $\hat{d}$, P_b and $\hat{P}_\mathrm{d}$ it follows that b is always positive, no matter whether $P_\mathrm{b} > 0$ or $P_\mathrm{b} < 0$, and $\hat{d}$ is always positive, no matter whether $\hat{P}_\mathrm{d} > 0$ or $\hat{P}_\mathrm{d} < 0$. That is to say, holographic screening bright (dark) solitons can exist, no matter whether $P_\mathrm{b} > 0$ ($\hat{P}_\mathrm{d} > 0$) or $P_\mathrm{b} < 0$ ($\hat{P}_\mathrm{d} < 0$), which leads both holographic screening bright and dark solitons to exist, no matter whether $E_0 > 0$ ($\hat{E}_0 > 0$) or $E_0 < 0$ ($\hat{E}_0 < 0$). Note that P_b, $\hat{P}_\mathrm{d} > 0$ or P_b, $\hat{P}_\mathrm{d} < 0$ means that the crystal possesses a focusing or defocusing nonlinearity. Therefore, both holographic screening bright and dark solitons can exist, no matter whether the medium possesses a focusing or a defocusing nonlinearity.

We consider the case where a bright–dark holographic screening soliton pair is formed in the configuration shown in Fig. 1(b). In this configuration, we have $E_\mathrm{A} = 10^4$ V/m and $\hat{E}_\mathrm{A} = -10^4$ V/m. Assume that the crystal P supports a bright soliton, then we will have $\rho = I_\infty/I_\mathrm{p} = 0$. In the first place, we take $\hat{\rho} = \hat{I}_\infty/\hat{I}_\mathrm{p} = 0.03$ and $m = I_\mathrm{p}/\hat{I}_\mathrm{p} = 1$, and then we have two sets of parameters, i.e. a) $\Omega = 0.4926$, $F = 0.6909$, $b = 0.1547$, and $B = 0.8635$ and b) $\hat{\Omega} = 0.5074$, $\hat{D} = 0.5641$, $\hat{d} = 0.1545$, and $\hat{H} = 0.7055$. With these values, the normalised intensity profiles of the bright soliton in the crystal P and the dark soliton in the crystal $\hat{P}$ are obtained by solving Eqs. (15) and (16) as shown in Fig. 2(a) (solid curve) and Fig. 2(b) (solid curve), respectively. When the input intensity of pump beam of the crystal P increases, for example, m increases from 1 to 5, but the other parameters remain unchanged, not only the bright soliton in crystal P changes (see dash line in Fig. 2(a)), but also the dark soliton in the crystal $\hat{P}$ varies (see dash line in Fig. 2(b)). The two curves are calculated at $\Omega = 0.9798$, $F = 0.6906$, $b = 1.1827$, $B = 0.3121$ and $\hat{\Omega} = 0.0202$, $\hat{D} = 0.5648$, $\hat{d} = 0.0784$, $\hat{H} = 0.9918$. By the same token, when the input intensity of pump beam of the crystal $\hat{P}$ increases, for example, m increases from 1 to 0.2, but the other parameters remain unchanged, not only the dark soliton in the crystal $\hat{P}$ changes (see dot line in Fig. 2(b)), but also the bright soliton in the crystal P varies (see dot line in Fig. 2(a)). The two curves are calculated at $\Omega = 0.1626$, $F = 0.6914$, $b = 0.0945$, $B = 1.1059$ and $\hat{\Omega} = 0.8374$, $\hat{D} = 0.5639$, $\hat{d} = 0.4297$, $\hat{H} = 0.4228$. The above results imply that for a bright–dark soliton pair, one soliton can affect the profile of the other soliton by the light-induced current. In other words, the two solitons in a bright–dark holographic screening soliton pair can affect each other.

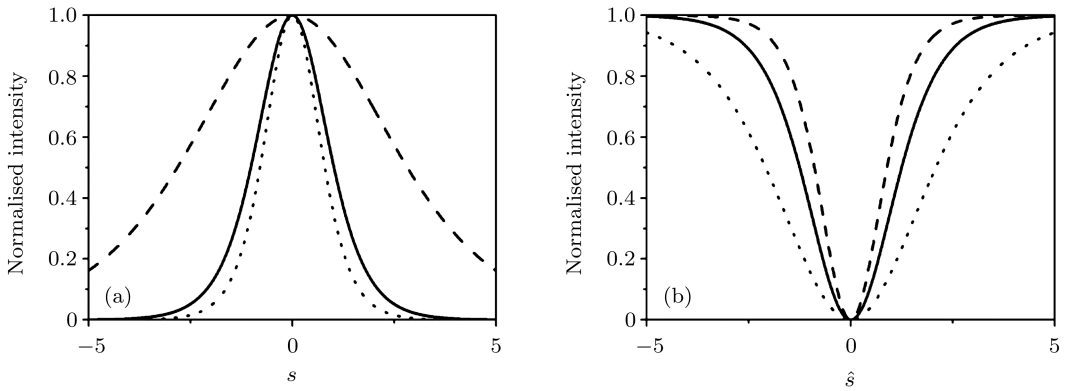


Fig. 2. Bight–dark separate holographic screening soliton pairs in a biased series SBN crystal circuit showing (a) bright soliton profiles in the crystal P and (b) darkt soliton profiles in the crystal $\hat{P}$.

3.2. Bright–bright holographic screening soliton pair

Then we consider the case of bright–bright holographic screening soliton pair, i.e., where both two crystals support holographic screening bright soliton. The bright soliton profiles in the bright–bright soliton pair can be

obtained by use of a similar way to the above and determined by the following equation:

$$U = F\,\mathrm{sech}\,(Bs)\exp\{\mathrm{i}b\ln[\mathrm{sech}\,(Bs)]\} \times \exp(-\mathrm{i}\nu\xi)\,. \quad (17)$$

Similarly, for the crystal $\hat{P}$, we have

$$\hat{U} = \hat{F}\,\mathrm{sech}\left(\hat{B}\hat{s}\right)\exp\left\{\mathrm{i}\hat{b}\ln\left[\mathrm{sech}\left(\hat{B}\hat{s}\right)\right]\right\} \times \exp\left(-\mathrm{i}\hat{\nu}\hat{\xi}\right), \quad (18)$$

where $\hat{F} = [3\hat{G}/(2\hat{g})]^{1/2}$, $\hat{B} = (\hat{G}/\hat{b})^{1/2}$, $\hat{b} = [-3\hat{P}_\mathrm{b} + (9\hat{P}_\mathrm{b}^2 + 8\hat{g}^2)^{1/2}]/(2\hat{g})$, $\hat{\nu} = (\hat{b}^2 - 1)\hat{G}/(2\hat{b}) + \hat{P}_\mathrm{b}$ and $\hat{P}_\mathrm{b} = \hat{\beta} - \hat{g}_0$.

Equations (17) and (18) predict that a bright–bright soliton pair can exist in a biased series PR crystal circuit. We consider the case of a bright–bright holographic screening soliton pair formed in the configuration shown in Fig. 1(a). In this configuration, we have $E_\mathrm{A} = \hat{E}_\mathrm{A} = 10^4$ V/m For the two bright solitons, we have $\rho = I_\infty/I_\mathrm{p} = 0$ and $\hat{\rho} = \hat{I}_\infty/\hat{I}_\mathrm{p} = 0$. Let $m = I_\mathrm{p}/\hat{I}_\mathrm{p} = 1$, then we will have $\Omega = \hat{\Omega} = 0.5$, $F = \hat{F} = 0.6909$, $B = \hat{B} = 0.8573$, and $b = \hat{b} = 0.1569$. With these values, the normalised intensity profiles of the two bright solitons in the crystals P and $\hat{P}$ are obtained by solving Eqs. (17) and (18) which are shown in Fig. 3(a) (solid curve) and Fig. 3(b) (solid curve), respectively. When one of the input intensities of pump beams changes, for example, m increases from 1 to 50, but the other parameters remain unchanged, we have $\Omega = 0.9804$, $F = 0.6906$, $B = 0.3113$, $b = 1.1887$ and $\hat{\Omega} = 0.0196$, $\hat{F} = 0.6917$, $\hat{B} = 1.1964$, $\hat{b} = 0.0808$. It can be found that not only the bright soliton in that crystal changes as shown by dash line in Fig. 3(a), but also the bright soliton in the other crystal varies as shown by dash line in Fig. 3(b) if one of the incident beams of the two crystals changes. In other words, the two bright solitons in a bright–bright holographic screening soliton pair can affect each other.

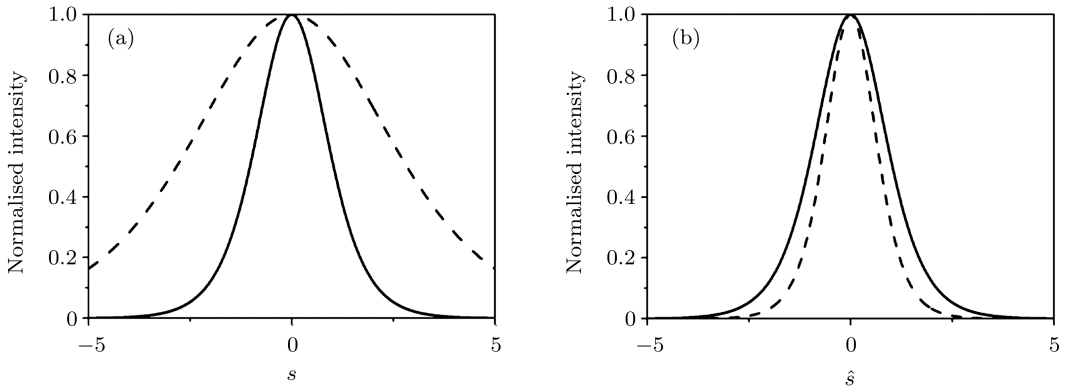


Fig. 3. Bright–bright separate holographic screening soliton pairs in a biased series SBN crystal circuit, showing (a) bright soliton profiles in the crystal P and (b) bright soliton profiles in the crystal $\hat{P}$.

3.3. Dark–dark holographic screening soliton pair

Finally, we consider a dark–dark holographic screening soliton pair. The two dark soliton profiles in a dark–dark soliton pair can be obtained in a similar way to the above and determined by the following expressions:

$$U = D\tanh(Hs)\exp\{\mathrm{i}d\ln[\cosh(Hs)]\} \times \exp(-\mathrm{i}\sigma\xi)\,, \quad (19)$$

$$\hat{U} = \hat{D}\tanh(\hat{H}\hat{s})\exp\left\{\mathrm{i}\hat{d}\ln\left[\cosh\left(\hat{H}\hat{s}\right)\right]\right\} \times \exp\left(-\mathrm{i}\hat{\sigma}\hat{\xi}\right), \quad (20)$$

where $D = (G/g)^{1/2}$, $H = [2G/(3d)]^{1/2}$, $d = [3P_\mathrm{d} + (9P_\mathrm{d}^2 + 8g^2)^{1/2}]/(2g)$ and $\sigma = 2G/(3d) + P_\mathrm{d}$.

Equations (19) and (20) predict that there can exist a dark–dark soliton pair in a biased series PR crystal circuit. We consider the case of a dark–dark holographic screening soliton pair formed in the configuration shown in Fig. 1(c). In this configuration, we have $E_\mathrm{A} = \hat{E}_\mathrm{A} = -10^4$ V/m. Let $\rho = \hat{\rho} = 0.03$ and $m = I_\mathrm{p}/\hat{I}_\mathrm{p} = 1$, then we will have $\Omega = \hat{\Omega} = 0.5$, $D = \hat{D} = 0.5641$, $d = \hat{d} = 0.1522$ and $H = \hat{H} = 0.7107$. With these values, the normalised intensity profiles of the two dark solitons in the crystals P and $\hat{P}$ are obtained by solving Eqs. (19) and (20) (see solid

line in Fig. 4(a) and solid line in Fig. 4(b) respectively). When one of the input intensities of pump beams changes, for example, m increases from 1 to 50, but the other parameters remain unchanged, we have $\Omega = 0.9804$, $D = 0.5639$, $d = 1.1822$, $H = 0.2549$ and $\hat{\Omega} = 0.0196$, $\hat{D} = 0.5648$, $\hat{d} = 0.0784$, $\hat{H} = 0.9921$. It can be found that not only the dark soliton in that crystal changes as shown by dash line in Fig. 4(a), but also the dark soliton in the other crystal varies as indicated by dash line in Fig. 4(b) if one of the incident beams of the two crystals changes. In other words, the two dark solitons in a dark–dark holographic screening soliton pair can affect each other.

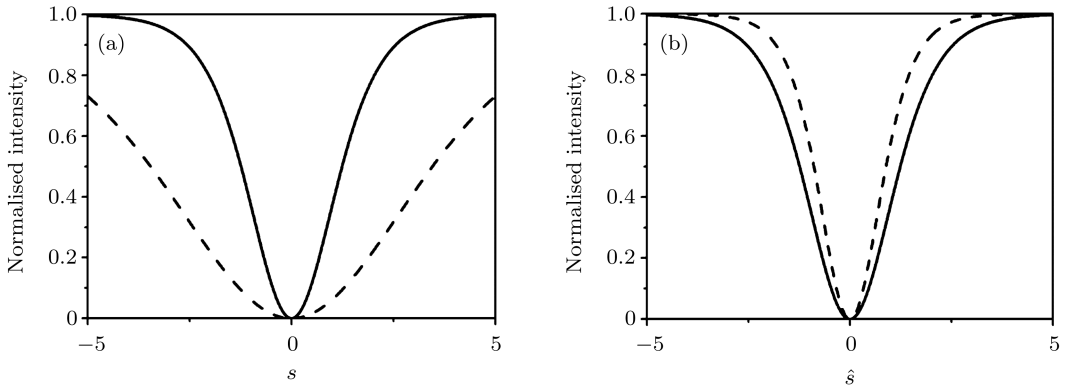


Fig. 4. Dark–dark separate holographic screening soliton pairs in a biased series SBN crystal circuit, showing (a) dark soliton profiles in the crystal P and (b) dark soliton profiles in the crystal $\hat{P}$.

4. Conclusions

In this paper, we show theoretically that a separate holographic screening soliton pair can be formed in a series PR crystal circuit consisting of two PR crystals and a biased voltage resource connected in a chain. Because both bright and dark holographic screening solitons are possible to exist in each crystal, there are three types of separate holographic screening soliton pairs: dark–dark, bright–dark and bright–bright. As a separate holographic screening soliton pair forms in a biased series PR crystal circuit, because the two PR crystals are connected electronically, changing the input intensity of pump beam of a crystal leads not only the characters of the soliton formed in that crystal but also the characters of the other soliton formed in the other crystal to change. That is to say, the two solitons in a separate holographic screening soliton pair can interact with or affect each other by the light-induced current.

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