The propagation of dipole solitons in highly nonlocal medium*

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This paper studies the propagation of dipole solitons in highly nonlocal medium by using the variational method. It finds that the dipole solitons will be stable when the input power obeys a restrict value. When the incident power does not satisfy the stable conditions, the nonlocal accessible dipole solitons will undergo linear harmonic oscillation. It shows such evolution behaviours in detail.

Keywords: nonlinear, highly nonlocal media, dipole solitons
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1. Introduction

Nonlocality has been studied in different branches of physics and the formation of solitons in a nonlocal nonlinear medium is one of the most interesting phenomena.[1−12] It has been proved that the nonlocal nonlinearity tends to suppress the modulation instability of the plane wave,[1−3] provides a simple physical mechanism for stabilization of multihump solitons[6] and causes the attraction between dark solitons.[7] Nonlocal nonlinear media also supports vector solitons,[8,9] which have robust properties. Nonlocality has also potential application in beam steering and optical devices.[10,11]

In highly nonlocal medium, the propagation of solitons can be treated as linear harmonic oscillation(accessible solitons).[13] Experimental observation of complex soliton states in lead glass has been reported.[14,15] There is also a kind of rotating soliton clustersd in nonlocal nonlinear medium,[16] this is a kind of exotic soliton. Due to the high degree of nonlocality, the observation of accessible solitons in nematic liquid crystal turns to reality.[17] Nonlocal incoherent solitons have also been discussed theoretically in Kerr media with extremely large nonlocality.[18,19] In nonlocal nonlinear media there also exists dipole-like solitons,[20] corresponding properties have been studied in nonlocal nematic liquid crystal[21] and in thermal nonlinear medium.[22]

In this paper we investigate the properties of dipole solitons in highly nonlocal medium. Using the variational approach, we obtain the critical incident power for the accessible dipole-solitons. When the input power does not satisfy the critical value, the nonlocal accessible dipole solitons will suffer linear harmonic oscillation.

2. Model and approximately variational theory

We consider a dipole beam propagating along the z axis within a nonlocal Kerr-type nonlinear medium. The propagation equations of the system[2] are given by

$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{1}{2k}\frac{\partial^2 u}{\partial x^2}+u\frac{k}{n_0}\int R(x-x')|u(x',z)|^2\mathrm{d}x'=0, \tag{1}$$

$$R(x-x')=\frac{1}{\sqrt{\pi}\sigma}\exp\left[-\frac{(x-x')^2}{\sigma^2}\right], \tag{2}$$

where u is the component of the beam, k is wavenumber, $R(x-x')$ is the normalized nonlocal response function with a characteristic width denoted by σ. Here we assume that the medium has a Gaussian nonlocal response.[5,6] We investigate the solitons by using the variational approach, and the Lagrange density of

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Eq. (1) is

$$\mathscr{L} = \frac{\mathrm{i}}{2}\left(u^*\frac{\partial u}{\partial z} - u\frac{\partial u^*}{\partial z}\right) - \frac{1}{2k}\left|\frac{\partial u}{\partial x}\right|^2 + \frac{k}{2n_0}|u|^2\int R(x-x')|u(x',z)|^2\mathrm{d}x'. \quad (3)$$

The variational method is represented as $\delta\int\int\mathscr{L}\mathrm{d}x\mathrm{d}z = 0$, $\delta\int\mathscr{L}_r\mathrm{d}z = 0$, where $\mathscr{L}_r = \int_{-\infty}^{\infty}\mathscr{L}\mathrm{d}x$.

We look for the solution of Eqs. (1) in dipole Gaussian form

$$u(x,z) = A(z)x\exp\left[-\frac{x^2}{2W^2(z)}\right]\exp[\mathrm{i}\,\theta(z)+\mathrm{i}\,c(z)x^2], \quad (4)$$

where $A(z)$ and $\theta(z)$ are the amplitude and the phase respectively, $W(z)$ is the beam width, and $c(z)$ represents the phase-front curvature of the beam. Then we can obtain the presentation of the Lagrange density

$$\mathscr{L}_r = -\frac{3\sqrt{\pi}}{4}A^2W^5\frac{\mathrm{d}c}{\mathrm{d}z} - \frac{\sqrt{\pi}}{2}A^2W^3\frac{\mathrm{d}\theta}{\mathrm{d}z} - \frac{3\sqrt{\pi}A^2}{8k}(W+4c^2W^5) + \frac{\sqrt{\pi}kA^4W^6(\sigma^4+2\sigma^2W^2+3W^4)}{8n_0(\sigma^2+2W^2)^{5/2}}. \quad (5)$$

Following the standard procedures of the variational approach, we obtain

$$\frac{\delta\mathscr{L}_r}{\delta\theta} = 0 \Rightarrow \frac{\mathrm{d}(A^2W^3)}{\mathrm{d}z} = 0, \quad (6)$$

$$\frac{\delta\mathscr{L}_r}{\delta c} = 0 \Rightarrow 2W\frac{\mathrm{d}A}{\mathrm{d}z} + 5A\frac{\mathrm{d}W}{\mathrm{d}z} - \frac{4}{k}AWc = 0, \quad (7)$$

$$\frac{\delta\mathscr{L}_r}{\delta A} = 0 \Rightarrow \frac{3}{2}W^5\frac{\mathrm{d}c}{\mathrm{d}z} + W^3\frac{\mathrm{d}\theta}{\mathrm{d}z} = -\frac{3W}{4k} - \frac{3c^2W^5}{k} + \frac{kA^2W^6(\sigma^4+2\sigma^2W^2+3W^4)}{2n_0(\sigma^2+2W^2)^{5/2}}, \quad (8)$$

$$\frac{\delta\mathscr{L}_r}{\delta W} = 0 \Rightarrow W^3\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{5W^5}{2}\frac{\mathrm{d}c}{\mathrm{d}z} = -\frac{W}{4k} - \frac{5c^2W^5}{k} + \frac{kA^2W^6(\sigma^2+W^2)^3}{2n_0(\sigma^2+2W^2)^{7/2}}. \quad (9)$$

Equation (6) represents the fact that the beam power is conserved, from which we can obtain

$$\int_{-\infty}^{\infty}|u(x,z)|^2\mathrm{d}x = \frac{1}{2}\sqrt{\pi}A^2W^3 = P_0. \quad (10)$$

Inserting Eq. (6) into Eq. (7) yields the relation

$$\frac{\mathrm{d}W}{\mathrm{d}z} = \frac{2}{k}Wc. \quad (11)$$

Combining Eqs. (8) and (9) gives the ordinary differential equations for the parameter c:

$$\frac{\mathrm{d}c}{\mathrm{d}z} = \frac{1}{2kW^4} - \frac{2c^2}{k} - \frac{kA^2W^3(\sigma^4+4\sigma^2W^2+5W^4)}{2n_0(\sigma^2+2W^2)^{7/2}}. \quad (12)$$

Equations (11) and (12) give the evolution equation for the beam width

$$\frac{\mathrm{d}^2W}{\mathrm{d}z^2} = \frac{1}{k^2W^3} - \frac{2P_0W(\sigma^4+4\sigma^2W^2+5W^4)}{\sqrt{\pi}n_0(\sigma^2+2W^2)^{7/2}}, \quad (13)$$

which describes the evolution dynamics of the width of the dipole solitons in nonlocal medium and shows a competition among diffraction, nonlinearity as well as nonlocality.

Let $W(z) = W(0) = W_0$, which represents that the dipole beam keeps its incident width, in such case, a stationary nonlocal dipole solitons can be formed. Then we can obtain the critical incident power for the nonlocal dipole solitons:

$$P_0 = \frac{\sqrt{\pi}n_0(\sigma^2+2W_0^2)^{7/2}}{2k^2W_0^4(\sigma^4+4\sigma^2W^2+5W_0^4)}. \quad (14)$$

For a single dipole beam is not stable in weakly nonlocal medium, the weak nonlocality can not balance the repulsion of its two out-of-phase lobes. This beam becomes stable only for high degree of nonlocality.[23]

3. Accessible harmonic oscillation of nonlocal dipole solitons

In the following, we concentrate on the propagation of dipole solitons in highly nonlocal media. In such case, σ is much larger than W, it means that W/σ is very small, so Eq. (13) can be simplified as follows:

$$\frac{\mathrm{d}^2W}{\mathrm{d}z^2} - \frac{1}{k^2W^3} + \frac{2P_0W}{\sqrt{\pi}n_0\sigma^3} = 0. \quad (15)$$

Assume that the beam at $z = 0$ has $\mathrm{d}W/\mathrm{d}z|_{z=0} = 0$, integrating Eq. (15) once, we can obtain

$$\left[\frac{\mathrm{d}W}{\mathrm{d}z}\right]^2 + \frac{1}{k^2W_0^2}\left[\frac{W_0^2}{W^2} - 1\right] + \frac{2P_0}{\sqrt{\pi}n_0\sigma^3}\left[W^2 - W_0^2\right] = 0. \quad (16)$$

If let

$$V(W) = \frac{1}{k^2W_0^2}\left[\frac{W_0^2}{W^2} - 1\right] + \frac{\gamma P_0}{n_0}\left[W^2 - W_0^2\right]$$

denote the potential of the soliton, then Eq.(16) depicts a classical Newton's equation describing an effective particle with kinetic energy $(\mathrm{d}W/\mathrm{d}z)^2$ moving in the potential $V(W)$. The asymmetric potential of the

accessible dipole solitons is illustrated in Fig. 1. From Fig. 1 we can see that as incident power increases, the minimum and the width of the potential will decrease synchronously.

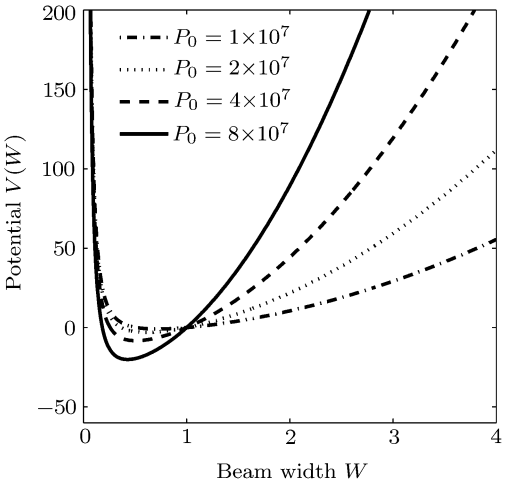


Fig. 1. Potential $V(W)$ in strongly nonlocal nonlinear medium for different values of incident power P_0. The initial parameters are $n_0 = 3$, $\sigma = 100$, $k = 1$ and $W_0 = 1$.

Accessible nonlocal dipole solitons[19] can be obtained by

$$\left[\frac{\partial V(W)}{\partial W}\right]\Big|_{W(z)=W_0} = 0,$$

$$W_0^4 = \frac{\sqrt{\pi}n_0\sigma^3}{2k^2P_0}, \tag{17}$$

so the critical input power for the of propagation accessible solitons is

$$P_{\rm c} = \frac{\sqrt{\pi}n_0\sigma^3}{2k^2W_0^4}. \tag{18}$$

It indicates that the solitons will be stable when the input power has a restricted value which is related to the beam width and the response function of the medium.

When the incident power do not satisfy the expression of Eq. (18), the nonlocal accessible dipole solitons will undergo linear harmonic oscillations.

Let $W(z)/W_0 = Y(z)$ and rewrite Eq. (16) as

$$\left(\frac{{\rm d}Y}{{\rm d}z}\right)^2 = \frac{m(Y^2-1)(\eta-Y^2)}{Y^2W_0^2}, \tag{19}$$

where $m = 2pW_0^2/\sqrt{\pi}n_0\sigma^3$, $\eta = P_{\rm c}/P_0 > 0$, $P_c = \sqrt{\pi}n_0\sigma^3/2k^2W_0^4$. Integrating Eq. (19) we obtain the simple form of dipole solitons in strongly nonlocal medium[13]

$$W^2(z) = W_0^2\left[\cos^2(\beta z) + \eta\sin^2(\beta z)\right], \tag{20}$$

where $\beta = \sqrt{m}/W_0 = \sqrt{2P_0/\sqrt{\pi}n_0\sigma^3}$. When $\eta = 1$, $W(z) = W_0$, the accessible nonlocal dipole solitons maintain its initial width during the propagation and the solitons is stable. When $\eta \neq 1$, the solitons will undergo linear harmonic oscillation.[13]

In Fig. 2 we show the evolution behaviour of the beamwidths of the dipole solitons in strongly nonlocal medium. As shown in Fig. 2(a), when the incident power of the dipole solitons in the highly nonlocal nonlinear medium is equal to the critical power ($\eta = 1$), the normalized intensity will maintain its initial form during propagation. Otherwise, the behaviour of the solitons mainly depends on whether the total incident power P_0 is above the critical value or not.

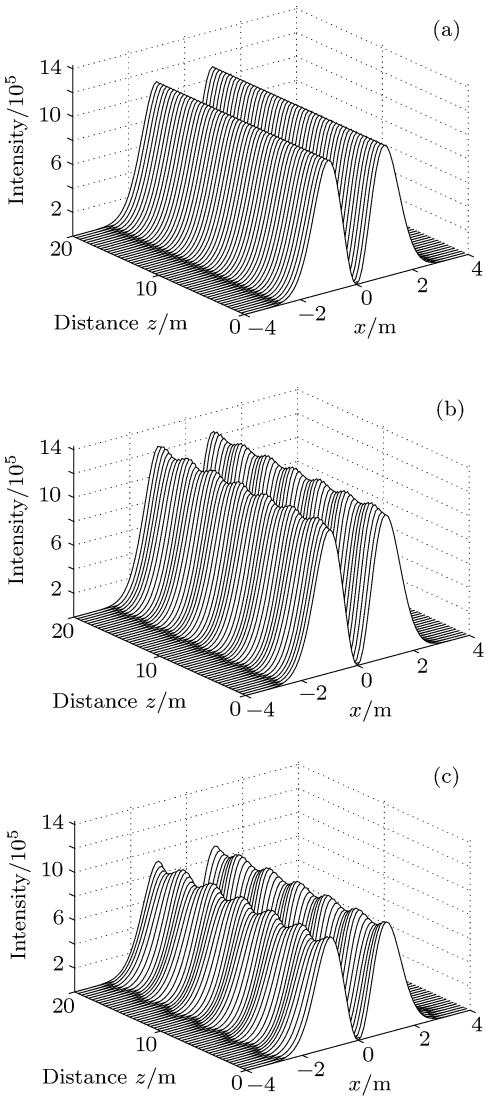


Fig. 2. Evolution of a beam with the beam width $W_0 = 1$, $n_0 = 3$, $\sigma = 100$, $k = 1$: (a) the power $P_0 = P_c = 4.71\times10^6(\eta = 1)$; (b) the power $P_0 = 5.11\times10^6(\eta < 1)$; (c) the power is $P_0 = 3.96\times10^6(\eta > 1)$.

In Fig. 2(b), when the total incident power $P_0 >$

$P_c(\eta < 1)$, the normalized intensity will firstly increase and then decrease, after that it will oscillate between the initial value and a bigger value. When the total incident power $P_0 < P_c(\eta > 1)$, the normalized intensity will firstly decrease and then increase, after that it will also oscillate between the initial value and a smaller value, as shown in Fig. 2(c).

4. Numerical result

In the following, accessible dipole solitons will be compared with directly numerical integration to Eq. (13) by using the standard fourth order Runge–Kutta scheme. As shown in Fig. 3(a), when the incident power is below the critical one, the beam width oscillates between the initial value and a bigger value. On the other hand, when the incident power is above the critical one, the beam width oscillates between the initial value and a smaller value. This is mainly because the bigger (smaller) the incident power is, the stronger (weaker) the nonlocal nonlinear effect is. As shown in Fig. 3(b), the beam width will also suffer from such regular linear harmonic oscillations when the degree of the nonlocality increases. This is because the nonlocality will effectively decrease the strength of the underlined nonlinearity. The larger the nonlocality (σ),the weaker the nonlinearity will be. Accordingly, underline that the diffraction of the beams will be stronger than the self-trapping effect, and thus the beam width will increase firstly, and then decrease when it reaches the maximum.

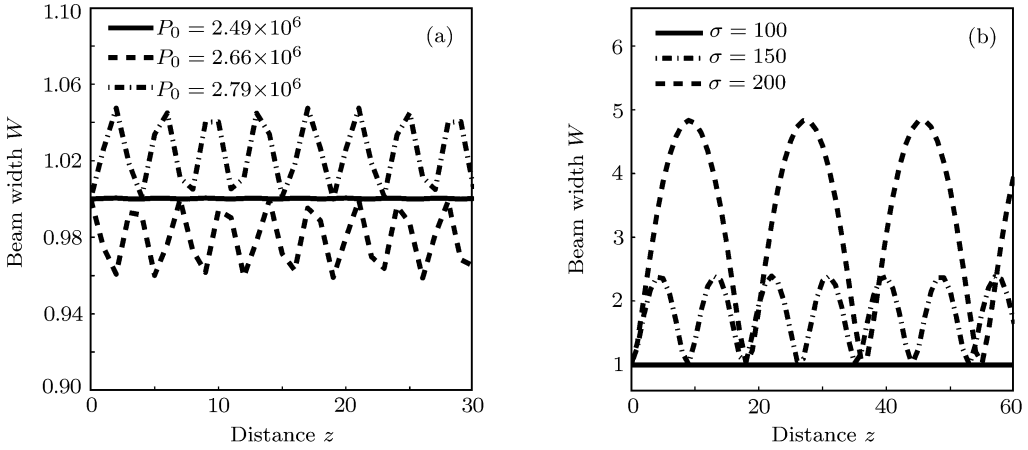


Fig. 3. Nonstationary propagation of the dipole beam in highly nonlocal nonlinear medium with $n_0 = 3.0$, $k = 1$ and $W_0 = 1$. The initial parameters are $\sigma = 100$, $P = 4.71 \times 10^6$ (solid line), $P = 4.97 \times 10^6$ (dashed line), and $P = 4.39 \times 10^6$ (dash-dotted line) for Fig. 3(a), and $P = 4.71 \times 10^6$, $\sigma = 100$ (solid line), $\sigma = 150$ (dash-dotted line), and $\sigma = 200$ (dashed line) for Fig. 3(b), respectively.

5. Conclusion

In conclusion, we have studied the propagation of dipole solitons in strongly nonlinear medium by employing the variational approach. The solitons will be stable when the incident power has a restricted value which is related to the beam and the response function of the medium. Otherwise, the nonlocal accessible dipole solitons will suffer linear harmonic oscillations.

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