Steady-state analysis of three-photon absorption spectra via density-matrix method in a three-coupled-quantum-well nanostructure*

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We numerically simulate three-photon absorption spectra in a three-coupled-quantum-well nanostructure interacting with a pump field, a coherent coupling field, and a probe field. We find that the three-photon absorption spectra can be dramatically influenced due to the intensities of the coupling field and pump field changing under the three-photon resonance condition. The effect of the frequency detuning of the pump field on the three-photon absorption spectra is also discussed. The study in our case is much more practical than the study in the case of its atomic counterpart in the sense of flexible design and the wide adjustable parameters. Thus it may open up some new possibilities for technological applications in optoelectronics and solid-state quantum information science.

Keywords: three-coupled-quantum-well, absorption spectra, three-photon resonance
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1. Introduction

In the past few years, a great many quantum optical phenomena based on coherence and quantum interference have attracted a lot of attention of many researchers in the quantum conherent media.[1−18] Meanwhile, as one of the fundamental components of the interaction of light with matter, the multiphoton processes have aroused the interest of many researchers since the two-photon absorption was first described in the 1930s.[19] The following respects have been studied: the two-photon fluorescence,[20] two-photon spectroscopy,[21] two-photon inhibition,[22−24] two-photon enhancement via the coupling field,[25] multi-photon absorption in four-wave mixing,[3] three-photon absorption in crystal,[26] and so on.

On the other hand, a great number of nonlinear quantum optical phenomena based on the quantum interference and coherence have also been extensively studied in the semiconductor quantum wells (SQWs), such as gain without inversion,[27] electromagnetically induced transparency,[28] optical bistability,[29] Kerr nonlinearity,[30] optical soliton,[31,32] four-wave mixing,[33] and so on. This is due mainly to the fact that the phenomena in SQWs have many potential important applications in optoelectronics and solid-state quantum information science. It is shown that devices based on intersubband transitions in the SQWs have many inherent advantages that the atomic systems do not have, such as the large electric dipole moments due to the small effective electron mass, the great flexibilities in device design by choosing the materials and structure dimensions, the high nonlinear optical coefficients, and the transition energies and the dipoles as well as the symmetries that can also be engineered as desired.

In the present work, we theoretically investigate the three-photon absorption spectra in a three-coupled-quantum-well nanostructure based on intersubband transitions. Our study and the system are based on the Refs. [29] and [33] but drastically different from theirs. First, what we are interested in is to show the three-photon absorption spectra under the different conditions. Second, we study mainly the effects of the frequency detuning and the intensity of the fields on the three-photon absorption spectra. Third, it is well known that the multi-photon processes play a crucial role in nonlinear quantum optics. Therefore, our work, which is feasible in practical situations, may open up new possibilities for technological applications. The rest of the present paper is organized as follows: in Section 2, we give a theoretical model and

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establish its corresponding equations. Our numerical results and physical analysis are shown in Section 3. Finally, some simple conclusions are given in Section 4.

2. Model and dynamic equations

We consider an asymmetric AlInAs/GaInAs three-coupled-quantum-well nanostructure with four levels as shown in Fig. 1 (for more details please see Ref. [33]). The optical transition $|3\rangle \leftrightarrow |2\rangle$ is induced by a coupling field $E_{\rm c}$ (Rabi frequency $\Omega_{\rm c}$), while the transitions $|4\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |1\rangle$ are derived by a relatively weak pump field $E_{\rm pump}$ (Rabi frequency $\Omega_{\rm pump}$) and a weak probe field $E_{\rm p}$ (Rabi frequency $\Omega_{\rm p}$) respectively. Let ω_{21}, ω_{32} and ω_{43} denote resonant frequencies which are associated with the corresponding optical transitions $|2\rangle \leftrightarrow |1\rangle$, $|3\rangle \leftrightarrow |2\rangle$ and $|4\rangle \leftrightarrow |3\rangle$ respectively.

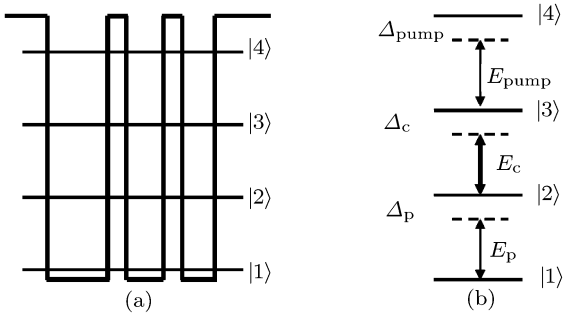


Fig. 1. Schematic diagrams of the three-coupled-quantum-well nanostructure (a), and energy level arrangement under study (b).

If we take the level $|1\rangle$ as the energy origin and choose the Hamiltonian as follows:

$$H_0 = \omega_{\rm p}\,|2\rangle\langle 2| + (\omega_{\rm p}+\omega_{\rm c})\,|3\rangle\langle 3| + (\omega_{\rm p}+\omega_{\rm pump}+\omega_{\rm c})\,|4\rangle\langle 4|\,, \tag{1}$$

where the $\omega_{\rm p}$, $\omega_{\rm c}$ and $\omega_{\rm pump}$ are the weak probe field frequency, coupling field frequency and relatively weak pump field frequency, respectively. In the interaction picture and under the rotating-wave approximation, the Hamiltonian of this system is given by (let $\hbar = 1$)[3]

$$\begin{aligned} H_{\rm int} = {} & \Delta_{\rm p}\,|2\rangle\langle 2| + (\Delta_{\rm c}+\Delta_{\rm p})\,|3\rangle\langle 3| \\ & + (\Delta_{\rm c}+\Delta_{\rm p}+\Delta_{\rm pump})\,|4\rangle\langle 4| \\ & - (\Omega_{\rm p}\,|2\rangle\langle 1| + \Omega_{\rm c}\,|3\rangle\langle 2| \\ & + \Omega_{\rm pump}\,|4\rangle\langle 3| + {\rm H.c}), \end{aligned} \tag{2}$$

where the H.c means Hermitian conjugation; $\Delta_{\rm p} = \omega_{21} - \omega_{\rm p}$, $\Delta_{\rm c} = \omega_{32} - \omega_{\rm c}$ and $\Delta_{\rm pump} = \omega_{43} - \omega_{\rm pump}$ are the detunings of the weak probe field, relatively weak pump field and coupling field respectively; $\Omega_{\rm p} = \mu_{21}E_{\rm p}/\hbar$, $\Omega_{\rm pump} = \mu_{43}E_{\rm pump}/\hbar$ and $\Omega_{\rm c} = \mu_{32}E_{\rm c}/\hbar$ are the Rabi frequencies for the respective intersubband optical transitions; μ_{21}, μ_{32} and μ_{43} are the relevant dipole moments.

By adopting the standard approach,[29] the density-matrix equations can easily be derived to be

$$\begin{aligned}
{\rm i}\,\dot\rho_{44} &= \frac{1}{2}\Omega^*_{\rm pump}\rho_{43} - \frac{1}{2}\Omega_{\rm pump}\rho_{34} - {\rm i}\,\gamma_4\rho_{44},\\
{\rm i}\,\dot\rho_{33} &= \frac{1}{2}\Omega_{\rm pump}\rho_{34} - \frac{1}{2}\Omega^*_{\rm pump}\rho_{43} + \frac{1}{2}\Omega^*_{\rm c}\rho_{32} + \tfrac{1}{2}\Omega_{\rm c}\rho_{23} + {\rm i}\,\gamma_{43}\rho_{44} - {\rm i}\,\gamma_3\rho_{33},\\
{\rm i}\,\dot\rho_{22} &= -\frac{1}{2}\Omega_{\rm p}\rho_{12} + \frac{1}{2}\Omega^*_{\rm p}\rho_{21} - \frac{1}{2}\Omega^*_{\rm c}\rho_{32} + \tfrac{1}{2}\Omega_{\rm c}\rho_{23} + {\rm i}\,\gamma_{42}\rho_{44} + {\rm i}\,\gamma_{32}\rho_{33} - {\rm i}\,\gamma_2\rho_{22},\\
{\rm i}\,\dot\rho_{21} &= \frac{1}{2}\Omega_{\rm p}(\rho_{22}-\rho_{11}) - \frac{1}{2}\Omega^*_{\rm c}\rho_{31} + \Delta_{\rm p}\rho_{21} - \frac{\rm i}{2}\Gamma_{21}\rho_{21},\\
{\rm i}\,\dot\rho_{31} &= \frac{1}{2}\Omega_{\rm p}\rho_{32} - \frac{1}{2}\Omega_{\rm c}\rho_{21} - \frac{1}{2}\Omega^*_{\rm pump}\rho_{41} + (\Delta_{\rm p}+\Delta_{\rm c})\rho_{31} - \frac{\rm i}{2}\Gamma_{31}\rho_{31},\\
{\rm i}\,\dot\rho_{41} &= \frac{1}{2}\Omega_{\rm p}\rho_{42} - \frac{1}{2}\Omega_{\rm pump}\rho_{31} + (\Delta_{\rm p}+\Delta_{\rm c}+\Delta_{\rm pump})\rho_{41} - \frac{\rm i}{2}\Gamma_{41}\rho_{41},\\
{\rm i}\,\dot\rho_{32} &= \frac{1}{2}\Omega^*_{\rm p}\rho_{31} - \frac{1}{2}\Omega^*_{\rm pump}\rho_{42} - \frac{1}{2}\Omega_{\rm c}(\rho_{33}-\rho_{22}) + \Delta_{\rm c}\rho_{32} - \frac{\rm i}{2}\Gamma_{32}\rho_{32},\\
{\rm i}\,\dot\rho_{42} &= \frac{1}{2}\Omega^*_{\rm p}\rho_{41} - \frac{1}{2}\Omega_{\rm pump}\rho_{32} + \frac{1}{2}\Omega_{\rm c}\rho_{43} + (\Delta_{\rm c}+\Delta_{\rm pump})\rho_{42} - \frac{\rm i}{2}\Gamma_{42}\rho_{42},\\
{\rm i}\,\dot\rho_{43} &= \frac{1}{2}\Omega^*_{\rm c}\rho_{42} + \frac{1}{2}\Omega_{\rm pump}(\rho_{44}-\rho_{33}) + \Delta_{\rm pump}\rho_{43} - \frac{\rm i}{2}\Gamma_{43}\rho_{43},
\end{aligned} \tag{3}$$

which are constrained by $\rho_{nm} = \rho^*_{mn}$ $(m, n = 1,\ 2,\ 3,\ 4)$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1$. The population decay rates and dephasing decay rates are added phenomenologically into the above density-matrix equations.[29,34,35]

The population decay rate for subband $|i\rangle$, denoted by γ_i, is due primarily to longitudinal optical (LO) phonon emission events at low temperature. The total decay rates $\Gamma_{ij}(i \neq j)$ are given by $\Gamma_{21} = \gamma_2 + \gamma_{21}^{\rm dph}$, $\Gamma_{31} = \gamma_3 + \gamma_{31}^{\rm dph}$ ($\gamma_3 = \gamma_{31} + \gamma_{32}$), $\Gamma_{41} = \gamma_4 + \gamma_{41}^{\rm dph}$ ($\gamma_4 = \gamma_{41} + \gamma_{42} + \gamma_{43}$), $\Gamma_{32} = \gamma_2 + \gamma_3 + \gamma_{32}^{\rm dph}$, $\Gamma_{42} = \gamma_2 + \gamma_4 + \gamma_{42}^{\rm dph}$ and $\Gamma_{43} = \gamma_4 + \gamma_3 + \gamma_{43}^{\rm dph}$, where $\gamma_{ij}^{\rm dph}$ determined by electron–electron, interface roughness, and phonon scattering processes, is the dephasing decay rate of the quantum coherence of the $|i\rangle \leftrightarrow |j\rangle$ transition.

3. Numerical results

It is well known that the population of the level $|4\rangle$ (ρ_{44}) can be used to display the three-photon absorption spectra in this three-coupled-quantum-well nanostructure. Now we conduct some numerical studies under the steady-state conditions, and the results are shown in Figs. 2–4. In the following numerical calculations, we keep $\Omega_n = \Omega_n^*$ ($n = p, c, pump$) and also choose the parameters to be dimensionless by scaling them with γ. In addition, we assume that all subbands have the same effective mass and the electron–electron effects have a very small influence on our results, and the choices of the parameters are based on some results from Refs. [29] and [33]. It should be noted that the choice of the dephasing and decay rates might be not very reasonable in our paper, however, it is believed that this problem can be solved via properly adjusting other physical variables such as temperature, interface roughness scattering, the ratio of the dipole moment and so on.

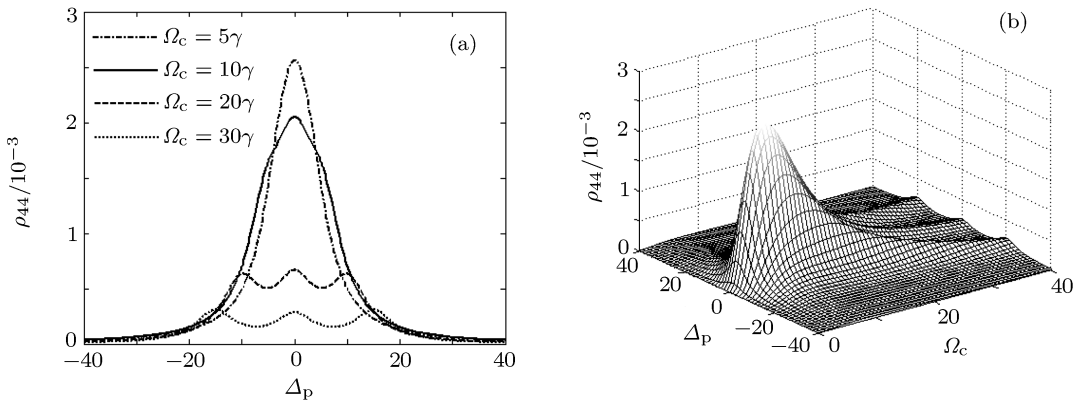


Fig. 2. Three-photon absorption spectra with coupling field Rabi frequency as a function of probe field detuning (a) and the three-photon absorption spectra as a function of probe field detuning and coupling field Rabi frequency (b), with parameters being $\Omega_{\rm pump} = 5\gamma$, $\Omega_{\rm p} = 0.5\gamma$, $\Gamma_{21} = 10\gamma$, $\Gamma_{43} = 7\gamma$, $\Gamma_{41} = \Gamma_{42} = 9\gamma$, $\Gamma_{31} = \Gamma_{32} = 8\gamma$, $\gamma_{41} = \gamma_{42} = \gamma_{43} = \gamma_{31} = \gamma_{32} = \gamma = \frac{1}{2}\gamma_2$, and $\Delta_{\rm pump} = \Delta_{\rm c} = 0$.

First of all, we analyse how the coupling-field intensity modifies the three-photon absorption spectra, with all other parameters kept fixed. Figure 2 demonstrates the dependence of the three-photon absorption spectra on coupling-field intensity. It can be easily seen from Fig. 2(a) that increasing the intensity of the coupling-field leads to a significant decrease in the line centre of the three-photon absorption spectrum. The reason for the above result can be qualitatively explained as follows. By applying an increasingly strong coupling-field to the intersubband transition $|3\rangle \leftrightarrow |2\rangle$, the absorption for the probe field on the intersubband transition $|2\rangle \leftrightarrow |1\rangle$ of the electronic medium can be reduced dramatically,[29] thereby influencing the three-photon absorption spectra. In order to further show the influence of the intensity of the coupling-field on the three-photon absorption spectra as a function of the probe field frequency detuning and the intensity of the coupling-field in Fig. 2(b), we can also easily see that the increasingly strong coupling-field reduces the three-photon absorption obviously.

In the following, we show the dependence of the three-photon absorption spectra on the intensity of the pump field in Fig. 3. In Fig. 3(a), we can see that as the intensity of the pump field changes, the three-photon absorption becomes more and more large in the line centre of the three-photon absorption spectrum. But when the pump and coupling fields are both very strong compared with the probe field, the three-photon absorption is suppressed. A reasonable explanation for this is that due to the presence of the relatively weak pump field, the destructive interference induced by the pump field is present in this three-coupled-quantum-well nanostructure,

which is bad for the three-photon absorption spectrum. However, when the pump and coupling fields are very strong, the constructive quantum interference induced by the pump field is present and plays an important role in this system. So, we can see that the three-photon absorption is suppressed. The three-dimensional plots of the dependence of the three-photon absorption spectrum on the intensity of the pump field and the probe field frequency detuning are given in Fig. 3(b), which also supports the above comments.

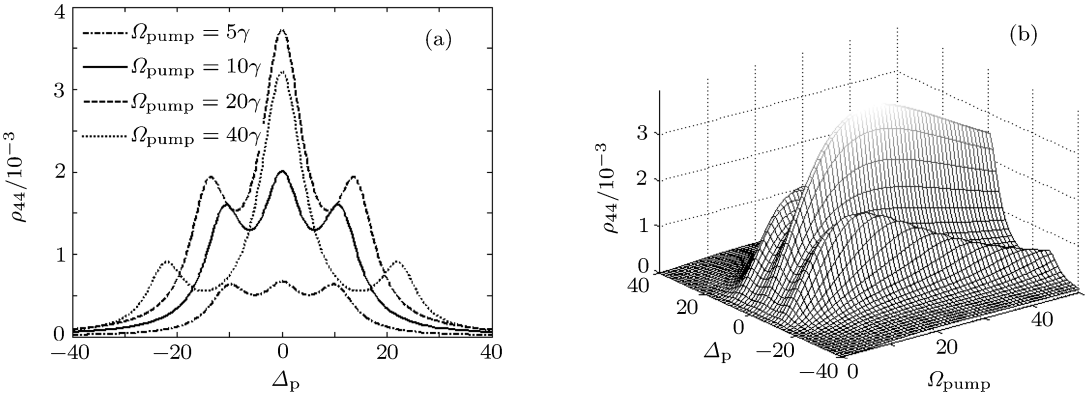


Fig. 3. Variations in three-photon absorption spectrum with pump field Rabi frequency as a function of probe field detuning(a), and three-dimensional plots of three-photon absorption spectrum as a function of probe field detuning and pump field Rabi frequency (b), with parameters being $\Omega_{\rm c} = 20\gamma$, $\Omega_{\rm p} = 0.5\gamma$, $\Gamma_{21} = 10\gamma$, $\Gamma_{43} = 7\gamma$, $\Gamma_{41} = \Gamma_{42} = 9\gamma$, $\Gamma_{31} = \Gamma_{32} = 8\gamma$, $\gamma_{41} = \gamma_{42} = \gamma_{43} = \gamma_{31} = \gamma_{32} = \gamma = \frac{1}{2}\gamma_2$, $\Delta_{\rm pump} = \Delta_{\rm c} = 0$.

Finally, the effect of the detuning of pump field on the three-photon absorption spectrum is given in Fig. 4. Clearly, the pump field detuning reduces the three-photon absorption in the line centre of the three-photon absorption spectrum spectra in Fig. 4(a), which can be easily explained as follows: when the pump field is relatively weak compared with the coupling-field, the effect of the destructive interference induced by the pump field on the three-photon absorption is small under the three-photon nonresonant condition. We can also easily see the corresponding results from the three-dimensional plots of the detuning of the pump field and the probe field frequency detuning as shown in Fig. 4(b).

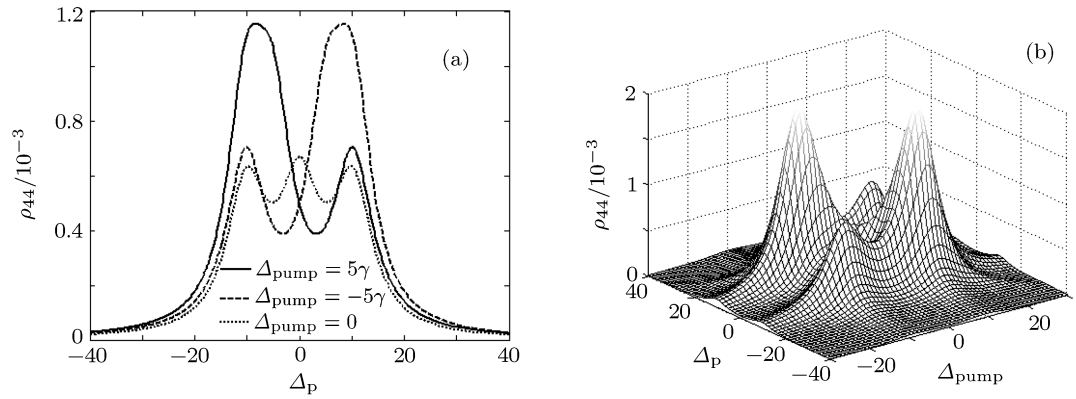


Fig. 4. Variations in three-photon absorption spectrum with pump field detuning as a function of probe field detuning (a), and three-dimensional plot of three-photon absorption spectra as a function of probe field detuning and pump field detuning (b), with parameters being $\Omega_{\rm c} = 20\gamma$, $\Omega_{\rm p} = 0.5\gamma$, $\Gamma_{21} = 10\gamma$, $\Gamma_{43} = 7\gamma$, $\Gamma_{41} = \Gamma_{42} = 9\gamma$, $\Gamma_{31} = \Gamma_{32} = 8\gamma$, $\gamma_{41} = \gamma_{42} = \gamma_{43} = \gamma_{31} = \gamma_{32} = \gamma = \frac{1}{2}\gamma_2$, $\Delta_{\rm c} = 0$, and $\Omega_{\rm pump} = 5\gamma$.

4. Conclusions

We numerically simulate the three-photon absorption spectra via a density-matrix method in a three-coupled-quantum-well nanostructure. We find that the three-photon absorption spectra can be dramatically influenced due to the fact that the intensities of the coupling field and pump field change under the three-photon resonance condition. And the effect of the frequency detuning of the pump field on the three-photon absorption

spectrum is also discussed. The study in our case is much more practical than that in the case of its atomic counterpart in the sense of flexible design and the wide adjustable parameters. Thus our study may open up some new possibilities for technological applications in optoelectronics and solid-state quantum information science.

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