Phase properties of odd and even circular states*

Wang Yue-Yuan(王月媛)[a)b)], Liu Zheng-Jun(刘正君)[c)],
Liao Qing-Hong(廖庆洪)[a)], and Liu Shu-Tian(刘树田)[a)†]

[a)] *Department of Physics, Harbin Institute of Technology, Harbin* 150001, *China*
[b)] *Department of Physics, Harbin Normal university, Harbin* 150001, *China*
[c)] *Department of Automation Measurement and Control Engineering, Harbin Institute of Technology, Harbin* 150001, *China*

(Received 13 July 2009; revised manuscript received 3 November 2009)

Phase properties of the even and odd circular states are studied within the Hermitian phase formalism of Pegg and Barnett. Exact analytical formulas for the distribution function and the variance of the phase operator are obtained and used to examine whether or not the even and odd circular states exhibit photon-number squeezing and phase squeezing.

Keywords: even and odd circular states, Pegg–Barnett phase formalism, phase probability distribution, phase squeezing and photon-number squeezing

PACC: 4250

1. Introduction

Coherent states have features as classical as possible and have proved to be the most fruitful idea since the states were introduced to quantum optics.[1,2] The superposition of coherent states, referred to as Schrödinger-cat states when the constituent coherent states are macroscopically distinguishable, has attracted lots of interest due to various nonclassical properties arising from the quantum interference between the components of such superposition states in phase space.[3−6] Circular states, which are concerned with a sum of N coherent states having the same amplitude and are distributed with equal probability on a circle, are very instructive examples in which many aspects of the nonclassical properties have been studied.[7−11]

Certain quantum states, such as squeezed displaced number states, squeezed coherent states, amplitude-squeezed states, binomial states, and Hermite-polynomial states were well approximated by discrete coherent-state superpositions.[9,12] These discrete coherent-state superpositions provide experimental possibilities for generating an arbitrary quantum state of electromagnetic field. The significance of applying a coherent-state expansion instead of the number-state one is to open new prospects in 'quantum-state engineering'. Various schemes to generate circular states have been suggested. Nonlinear interaction of the field, which is initially in a coherent state, with a Kerr-like medium[3] or in a degenerate parametric oscillator[13] lead to superpositions of a finite number of coherent states. Back-action evading and quantum nondemolition measurements can also yield such superposition states.[14,15] The atomic interference method and the centre-of-mass motion of a trapped ion have been developed, which can result in arbitrary superposition of coherent states on a circle in phase space.[16,17]

Studies of phase properties of the fields generated in nonlinear optical processes show that the phase distribution is new representations of the quantum state of the field, alternative to the Q function or the Wigner function, and they carry quite a bit of essential information characterising the quantum state of the field. Also the phase distribution has been realised experimentally by means of the optical homodyne tomography, which allows quantum phase mean values to be calculated via the measured field density matrix.[18] Hence the phase properties of the circular states have been a subject of interest.[19] But it is well known that there are different techniques for phase description of the optical field. Three approaches have found great acceptance. Dirac (1927)[20] proposed the existence of a quantum phase operator as a quantity conjugate to the number operator that was to perform a polar decomposition of the annihilation operator. It turned out later that such decomposition suffers from serious drawbacks, and the phase operator introduced in this way cannot be considered as a

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10674038 and 10604042) and the National Basic Research Program of China (Grant No. 2006CB302901)
†Corresponding author. E-mail: stliu@hit.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

properly defined Hermitian phase operator. Susskind and Glogower (1964)[21] exposed the contradictions inherent in Dirac's polar decomposition and introduced, instead of the phase operator that appeared to be non-Hermitian, the operators corresponding to the cosine and sine of the phase. Unfortunately, these operators, although being Hermitian, do not commute, so that they cannot represent a single phase angle. Pegg and Barnett (1988)[22−24] have proposed a new phase operator formalism which can avoid the above-mentioned difficulty. The theory distinguishes from the many others such as used an angle operator, rather than used somewhat cumbersome exponential or trigonometric function operator. In particular, the expectation values of $\cos^2\phi$ and $\sin^2\phi$ follow the trigonometric unity rule. Using the Pegg–Barnett formalism which ensures a Hermitian phase operator, one can obtain the phase probability distribution. Therefore this technique has been applied to various problems in quantum optics because it does not include such difficulties as the others.[25−30]

In this paper we employ the Pegg–Barnett phase formalism to study phase properties of the even and odd circular states. The phase probability distribution (PPD) and phase variance of the phase operator are calculated. The exact analytical formulas describing phase properties of the circular states are obtained, and are illustrated graphically for different constants. Subsequently we use the PPD to study photon-number and phase squeezing of these states. The results indicate that phase becomes more and more random with the increase of the number of coherent states. Multi-peak phase structure of the superposition state appears when the number of coherent states is greater than two. The paper is organized as follows. In Section 2, we present our model. Section 3 analyses in detail phase properties of the circular states. We compute the phase distribution, phase variance, and phase squeezing and compare the even circular states PPD with odd circular states and statistic mixture states PPD. Finally we conclude in Section 4.

2. Odd and even circular states

The circular states generalise the cat states: they constitute a superposition of the harmonic oscillator coherent states,

$$|\psi_N(\alpha_0)\rangle = \lambda_N \sum_{k=1}^{N} C_k|\alpha_k\rangle, \tag{1}$$

where N, an integer is the number of coherent states in the superposition. The constants C_k are arbitrary and, as we will show later, their values determine whether the quantum interference among the coherent states is constructive or destructive, which results in observable effects. The parameters of the coherent states are expressed as follows:

$$\alpha_k = \alpha_0 \exp\left(\frac{\mathrm{i}2k\pi}{N}\right) = r\exp(\mathrm{i}\phi_k), \tag{2}$$

$$\alpha_0 = r\exp(\mathrm{i}\phi_0), \tag{3}$$

$$\phi_k = \phi_0 + \frac{2k\pi}{N}. \tag{4}$$

The coherent states $|\alpha_k\rangle$ have the same modulus $|\alpha_k| = |\alpha_0| = r$ and the normalisation constant λ_N can be written as

$$|\lambda_N|^{-2} = \exp(-r^2) \sum_{j,k=1}^{N} C_j^* C_k \exp(\alpha_j^* \alpha_k). \tag{5}$$

The density operator $\hat{\rho}$ of the circular states in the coherent-state basis consists of N parts

$$\hat{\rho} = |\psi_N(\alpha_0)\rangle\langle\psi_N(\alpha_0)| = \hat{\rho}_{\mathrm{M}} + \hat{\rho}_{\mathrm{I}}. \tag{6}$$

The part of the density matrix corresponding to the statistical mixture of two coherent states (diagonal terms in the coherent-state basis) is

$$\hat{\rho}_{\mathrm{M}} = |\lambda_N|^2 \sum_{k=1}^{N} |C_k|^2 |\alpha_k\rangle\langle\alpha_k|, \tag{7}$$

while the quantum interference part has the form

$$\hat{\rho}_{\mathrm{I}} = |\lambda_N|^2 \sum_{\substack{j,k=1 \\ j\neq k}}^{N} C_k C_j^* |\alpha_k\rangle\langle\alpha_j|. \tag{8}$$

This off-diagonal part of the density matrix contains information about the quantum interference between component states $|\alpha_j\rangle$ and $|\alpha_k\rangle$, and is responsible for quadrature squeezing, fourth-order squeezing and oscillations in the photon number distribution.

Since N is even and $C_{k+N/2} = \pm C_k$, the parity of the state may be even or odd, i.e.

$$|\psi_N(-\alpha_0)\rangle = \pm|\psi_N(\alpha_0)\rangle. \tag{9}$$

The circular state is also an eigenstate of the N-th power of the annihilation operator,

$$\hat{a}^N|\psi_N(\alpha_0)\rangle = \alpha_0^N|\psi_N(\alpha_0)\rangle. \tag{10}$$

Here we shall consider two particular cases as in Ref. [17]. The case $C_k = 1$, that is the even circular states (ECS); and the case $C_k = \exp(\mathrm{i}2k\pi/N)$ when N is even, that is the odd circular states (OCS). And we shall analyse their phase properties, respectively.

3. Phase properties

The Pegg–Barnett Hermitian phase formalism, as the Hermitian phase operator is defined, allows the direct calculations of the quantum phase properties for a given state of optical fields. Now we turn to obtaining the phase probability distributions for the state given by Eq. (1). In terms of the Pegg–Barnett phase operator formalism in a finite-dimensional $s+1$ Hilbert space, a set of complete orthonormal phase states $|\theta_m\rangle$ can be defined as

$$|\theta_m\rangle = (s+1)^{-1/2}\sum_{n=0}^{s}\exp\left(\mathrm{i}n\theta_m\right)|n\rangle. \tag{11}$$

Given a reference phase θ_0, the values of θ_m can be expressed as

$$\theta_m = \theta_0 + \frac{2\pi m}{s+1}, \quad (m = 0, 1, 2, \ldots, s). \tag{12}$$

In this space the Hermitian phase operator $\hat{\Phi}_\theta$ is defined as

$$\hat{\Phi}_\theta = \sum_{m=0}^{s}\theta_m|\theta_m\rangle\langle\theta_m|. \tag{13}$$

The expectation value of the phase operator $\hat{\Phi}_\theta$ in a pure state $|\psi\rangle$ is given by

$$\langle\psi|\hat{\Phi}_\theta|\psi\rangle = \sum_{m=0}^{s}\theta_m|\langle\theta_m|\psi\rangle|^2, \tag{14}$$

where $|\langle\theta_m|\psi\rangle|^2$ denotes the probability $P(\theta)$ of $|\psi\rangle$ in the phase states $|\theta_m\rangle$. The density of phase states is $(s+1)/2\pi$, so in the continuum limit as s tends to infinity, we can write Eq. (14) as

$$\langle\psi|\hat{\Phi}_\theta|\psi\rangle = \int_{\theta_0}^{\theta_0+2\pi}\theta P(\theta)\,\mathrm{d}\theta. \tag{15}$$

Using the above definitions, one can study phase properties of any physical state of light.[22,24]

3.1. Phase probability distribution

For the circular states $|\psi_N\rangle$, we can find the phase amplitude

$$\begin{aligned}&\langle\theta_m|\psi_N\rangle\\ &= (s+1)^{-1/2}\lambda_N\sum_{k=1}^{N}C_k\langle\theta_m|\alpha_k\rangle\\ &= (s+1)^{-1/2}\lambda_N\sum_{k=1}^{N}\sum_{n=0}^{\infty}C_k\\ &\quad\times\exp\left(-\frac{1}{2}r^2\right)\frac{r^n}{\sqrt{n!}}\exp\left[-\mathrm{i}n(\theta_m-\phi_k)\right].\end{aligned} \tag{16}$$

From these amplitudes we can construct the phase probability distribution and calculate the expectation value and variance of phase operator. As $s\to\infty$, the continuous phase probability distribution can now be obtained as

$$\begin{aligned}&P(\theta)\\ &= \lim_{s\to\infty}\frac{s+1}{2\pi}|\langle\theta_m|\psi_N\rangle|^2\\ &= \frac{1}{2\pi}\Bigg\{1+2|\lambda_N|^2\exp\left(-r^2\right)\sum_{j,k=1}^{N}\sum_{n>n'=0}^{\infty}C_kC_j^*\\ &\quad\times\frac{r^nr^{n'}}{\sqrt{n!n'!}}\cos\left[(n\phi_k-n'\phi_j)-(n-n')\theta\right]\Bigg\}.\end{aligned} \tag{17}$$

The phase distribution $P(\theta)$ is normalised such that

$$\int_{\theta_0}^{\theta_0+2\pi}P(\theta)\,\mathrm{d}\theta = 1. \tag{18}$$

For a large r^2, the Poissonian distribution may be approximated as a Gaussian:[23]

$$\exp\left(-r^2\right)\frac{r^{2n}}{n!} \approx (2r^2\pi)^{-1/2}\exp\left[-\frac{(r^2-n)^2}{2r^2}\right]. \tag{19}$$

We can now substitute the square root of Eq. (19) into Eq. (16) and perform the integration over n

$$\begin{aligned}\langle\theta_m|\psi_N\rangle &= (s+1)^{-1/2}(2r^2\pi)^{-1/4}\lambda_N\sum_{k=1}^{N}C_k\int\exp\left[-\frac{1}{4r^2}(r^2-n)^2\right]\exp\left[-\mathrm{i}n\left(\theta_m-\phi_k\right)\right]\mathrm{d}n\\ &= \left(\frac{2\pi}{s+1}\right)^{1/2}\left(\frac{4r^2}{2\pi}\right)^{1/4}\lambda_N\sum_{k=1}^{N}C_k\exp\left[-r^2(\theta_m-\phi_k)^2\right]\exp\left[-\mathrm{i}r^2(\theta_m-\phi_k)\right].\end{aligned} \tag{20}$$

The resultant phase probability distribution is

$$P(\theta) = \left(\frac{2r^2}{\pi}\right)^{1/2}|\lambda_N|^2\sum_{j,k=1}^{N}C_kC_j^*\exp\left[\mathrm{i}r^2(\phi_k-\phi_j)\right]\exp\left[-r^2(\theta-\phi_k)^2-r^2(\theta-\phi_j)^2\right]. \tag{21}$$

For comparison purposes we also calculate the phase properties of the statistical mixture states described by the density operator

$$\hat{\varrho} = \frac{1}{N}\sum_{k=1}^{N}|\alpha_k\rangle\langle\alpha_k|, \tag{22}$$

for which we find

$$P(\theta) = \left(\frac{2r^2}{\pi}\right)^{1/2} \frac{1}{N} \sum_{k=1}^{N} \exp\left[-2r^2(\theta - \phi_k)^2\right]. \tag{23}$$

We can see clearly that this phase distribution does not contain the quantum interference term and therefore does not describe non-classical effects.

For $N = 1$ Eq. (21) goes over into the result for one peak of a coherent state and $N = 2$ constitutes the structure of two peaks of even or odd coherent state.[31,32] The Pegg–Barnett phase distributions of the odd coherent state, the even coherent state and the statistical mixture states are almost uniform, and therefore rather insensitive to quantum interference between component states. The fringe structure, however, gets more complicated as the number of the coherent states N taking part in the superposition increases.

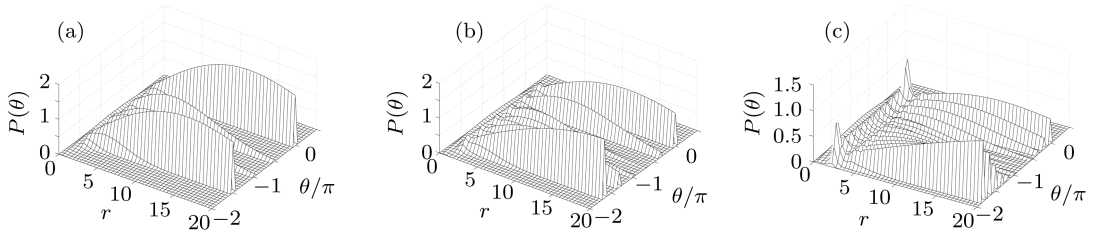


Fig. 1. The PPD of the ECS for various values of the number of superposed coherent states and $\varphi_0 = -2\pi$: (a)$N = 4$, (b)$N = 6$, and (c)$N = 12$.

In Figs. 1–4, we have plotted Pegg–Barnett PPD $P(\theta)$ for the even and odd circular states and compared with statistical mixture states for various values of θ, r and ϕ_0. Figure 1 displays the three-dimensional plot of the PPDs of the ECS, where we can simultaneously appreciate the modifications in the distribution as a function of the moduli of coherent state r and θ phases. Here we have taken the phase of the amplitude $\phi_0 = -2\pi$. It is evident that the PPD of the ECS splits into N separate peaks with different heights and these peaks appear at the phase angles $\theta = \phi_0 + 2k\pi/N$. These splittings of the distribution $P(\theta)$ suggest the number of evolved superposition of distinguishable coherent states. It is seen that with increasing N, the amplitudes of the peaks of the PPD become smaller and the widths of these peaks broadened. The structures of peaks weaken obviously and quantum phase information is lost partially. As r is increased, the amplitude values of the distribution become large, but do not increase monotonically. Hence we can obtain some phase information. From Fig. 2 we can see that the distribution for the ECS (dashed line) and the OCS (dot-dashed line) is oscillatory and for the statistical mixture states (solid line) increases monotonically.

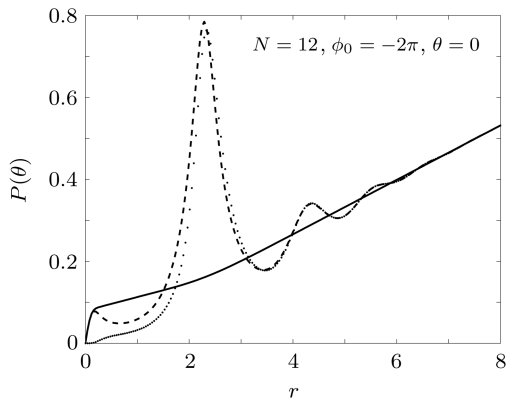


Fig. 2. The PPD of the ECS (dashed line), of the OCS (dotted line), and of the statistical mixture states (solid line) versus r.

Figure 3 shows the phase distributions for ECS, OCS and statistical mixture states against θ the mean number of photons $\bar{n}$ and the different number of superposed coherent states. It is seen that as the number of components in the superposition increases the interference terms play an increasing role and the differences between the phase distributions of the ECS, OCS and statistical mixture states become obvious. The amplitude of the PPD for the OCS is the smallest and for the statistical mixture state is the largest. When $N = 14$, the phase of the superposition state becomes randomized and the PPD of the mixture state is equal to $1/2\pi$. From this discussion we can conclude that using the Pegg–Barnett formalism we can distinguish between the quantum superposition states

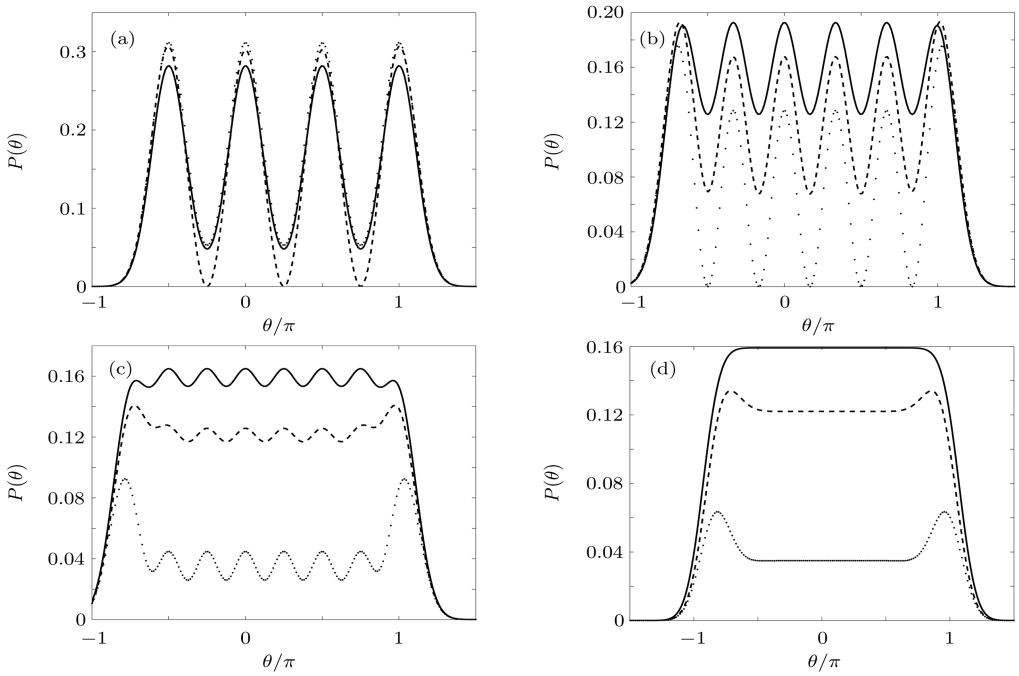


Fig. 3. The PPD of the ECS (dashed line), of the OCS (dotted line), and of the statistical mixture states (solid line) with average photon number $\bar{n} = 2$, $\phi_0 = -\pi$, for (a) $N = 4$, (b) $N = 6$, (c) $N = 8$, (d) $N = 14$.

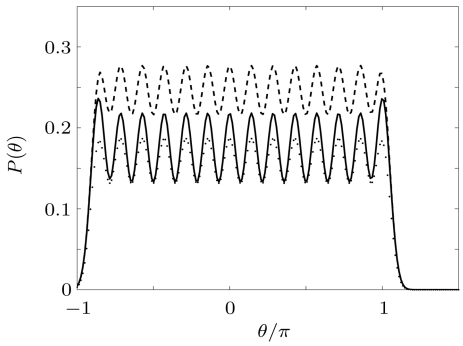


Fig. 4. The phase probability distributions of the ECS (solid line), the OCS (dashed line), and the statistical mixture state (dotted line) with average photon number $\bar{n} = 10$, $\phi_0 = -\pi$ and $N = 14$.

and the corresponding statistical mixture states when the number of components is larger than two. Moreover, in Fig. 4 a larger amplitude of coherent state can slow down the process of randomization. Therefore for large r we can obtain some quantum information, and the quantum interference effect of the state is prominent.

3.2. Phase fluctuations

The phase probability distribution illustrates the phase structure of the field while the variance provides a good understanding of the field phase fluctuations. Note that the phase distribution (21) allows us to calculate the variance of the phase operator (13) with this function in a classical-like manner. For circular states the phase variance and the photon-number variance are given by, respectively,

$$\left\langle \left(\Delta\hat{N}\right)^2 \right\rangle = |\lambda_N|^2 \sum_{j,k=1}^{N} C_k C_j^* \alpha_j^* \alpha_k \left(1 + \alpha_j^* \alpha_k\right) \exp\left(-r^2 + \alpha_j^* \alpha_k\right) - \left[|\lambda_N|^2 \sum_{j,k=1}^{N} C_k C_j^* \alpha_j^* \alpha_k \exp\left(-r^2 + \alpha_j^* \alpha_k\right) \right]^2, \quad (24)$$

$$\left\langle \left(\Delta\hat{\Phi}_\theta\right)^2 \right\rangle = \frac{\pi^2}{3} + 4|\lambda_N|^2 \sum_{n>n'=0}^{\infty} \sum_{j,k=1}^{N} C_k C_j^* \exp\left(-r^2\right) \times \frac{r^{n+n'}}{\sqrt{n!n'!}} \frac{(-1)^{n+n'}}{(n-n')^2} \exp\left[\mathrm{i}\frac{2\pi}{N}\left(nk - n'j\right)\right]. \quad (25)$$

The value $\pi^2/3$ is the variance for the uniformly distributed phase, as in the case of a single number state.

In Figs. 5 and 6 we have displayed the phase variance and photon-number variance against the mean photon number for different values of N. From the two figures one can observe that the variation of $\langle(\Delta\hat{\Phi}_\theta)^2\rangle$ is smooth. It starts from the initial value $\pi^2/3$ and goes to its minimum values. In other words, as $\bar{n}$ increases, the phase variance decreases. Further, we can observe that when the number of superposed coherent states N increases, the phase variance asymptotically approaches to $\pi^2/3$. For a finite value of $\bar{n}$, $\langle(\Delta\hat{\Phi}_\theta)^2\rangle$ goes to randomly distributed phase more rapidly. This fact can be realised by comparing the behaviour of the different curves in Fig. 5. However, photon-number variance $\langle(\Delta\hat{N})^2\rangle$ is just the opposite, which increases with $\bar{n}$ increasing. For a given $\bar{n}$, it will decrease with N getting larger. The OCS has the similar variance behaviour.

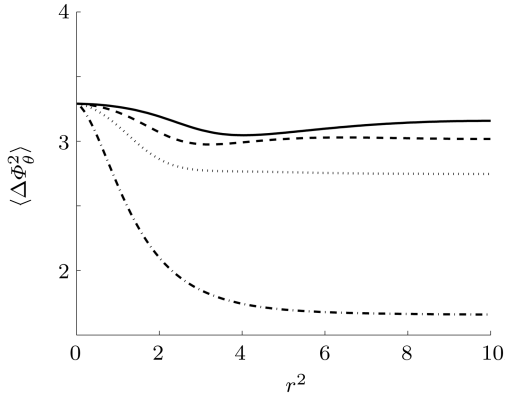


Fig. 5. Phase variance of the ECS for $N = 2$ (dot-dashed line), $N = 4$ (dotted line), $N = 6$ (dashed line), and $N = 10$ (solid line).

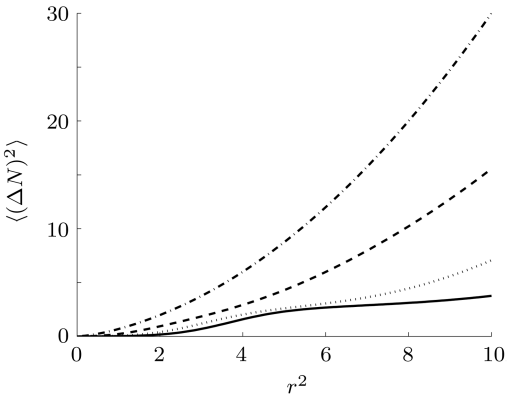


Fig. 6. Photon umber variance of the ECS for $N = 2$ (dot-dashed line), $N = 4$ (dashed line), $N = 8$ (dotted line), and $N = 12$ (solid line).

3.3. Phase squeezing

Since the number operator $\hat{N}$ and the phase operator $\hat{\Phi}_\theta$ are conjugate operators, they obey the following number-phase uncertainty relation[23,24]

$$\left\langle\left(\Delta\hat{N}\right)^2\right\rangle\left\langle\left(\Delta\hat{\Phi}_\theta\right)^2\right\rangle \geq \frac{1}{4}\left|\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle\right|^2, \quad (26)$$

where

$$\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle = \mathrm{i}\left[1 - 2\pi P(\theta_0)\right]. \quad (27)$$

Therefore, both sides of the uncertainty relation (26) can be calculated independently for a given state of the field. The uncertainty relation itself can be tested for finding, for example, the minimum uncertainty states.

To describe the relative quantum fluctuation, with respect to the minimum uncertainty, it is convenient to introduce the notion of the number and phase squeezing defined by

$$S^N = \frac{2\left\langle\left(\Delta\hat{N}\right)^2\right\rangle - \left|\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle\right|}{\left|\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle\right|}, \quad (28)$$

$$S^\Phi = \frac{2\left\langle\left(\Delta\hat{\Phi}_\theta\right)^2\right\rangle - \left|\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle\right|}{\left|\left\langle\left[\hat{N},\hat{\Phi}_\theta\right]\right\rangle\right|}. \quad (29)$$

If $-1 \leq S^{N(\Phi)} < 0$ then there will be number (or phase) squeezing; and $S^{N(\Phi)} = -1$ is the maximum 100% squeezing of the variances. Using the terminology introduced by Glauber and Lewenstein[33] we can say that the variable $\hat{\Phi}_\theta$ or $\hat{N}$ is subfluctuant (superfluctuant) when $S^{N(\Phi)} < 0$ ($S^{N(\Phi)} > 0$).

In Figs. 7 and 8 we have plotted the squeezing parameters S^N and S^Φ for the even and odd circular state against r, while keeping θ and φ_0 constant. From the figures it is clear that both the ECS and the OCS exhibit number squeezing and phase squeezing and their squeezing properties are very similar. In the limit of $r \to 0$ the parameter S^N tends to -1, which means that the vacuum state is 100% squeezed with respect to the operator $\hat{N}$. In the limit of high intensities the parameter S^Φ leads to -1, which means that these states describe a precisely defined phase. As r increases, the degree of squeezing in $\hat{\Phi}_\theta$ increases and this operator becomes subfluctuant, and the degree of squeezing in $\hat{N}$ decreases and becomes superfluctuant. Moreover, we find that phase squeezing not only depends on the intensity of the coherent field and the number of superposed coherent states but also on the actual value of the reference phase θ_0.

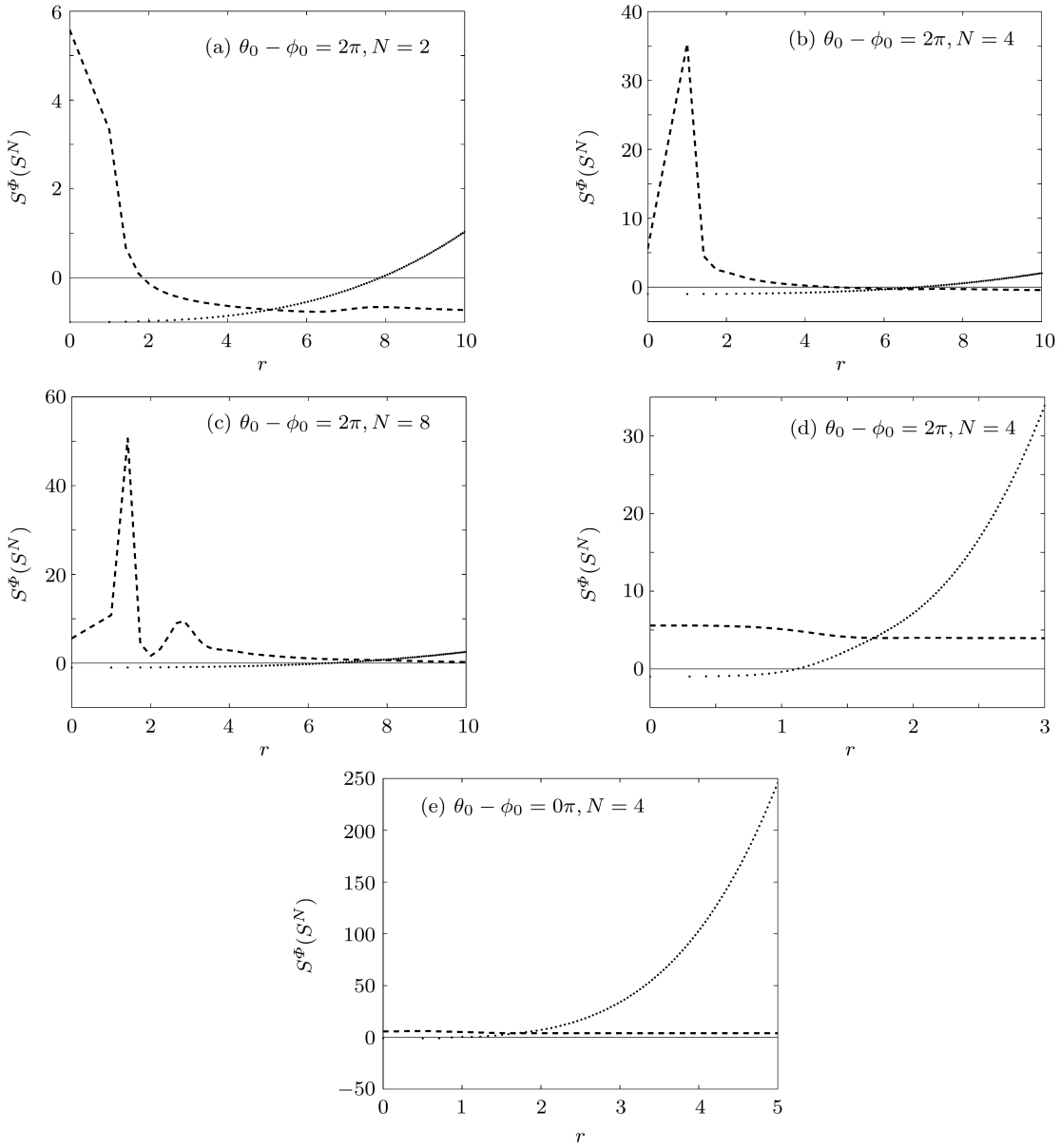


Fig. 7. Photon-number squeezing parameter S^N (dashed curve) and phase squeezing parameter S^{Φ} (dotted curve) of the ECS versus r.

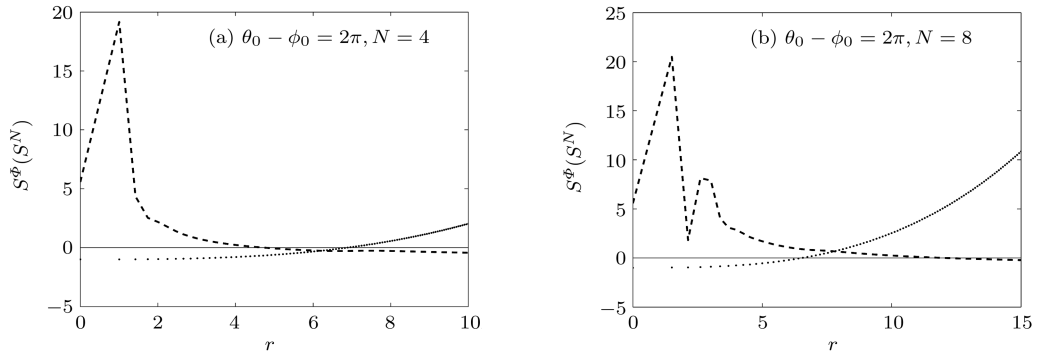


Fig. 8. Photon-number squeezing parameter S^N (dashed curve) and phase squeezing parameter S^{Φ}(dotted curve) of the OCS versus r.

4. Conclusion

We have investigated in detail the phase properties of the even and odd circular states using the Hermitian phase formalism of Pegg and Barnett. The exact analytical formula for the phase probability distribution of the circular states has been obtained and illustrated graphically. The variance and phase squeezing of the Hermitian phase operator is calculated for different values of the parameters. With the increase of the number of coherent states, we found that the Pegg–Barnett PPD is more sensitive to the characteristics of quantum interference between the different components of the states. The odd circular states can exhibit more pronounced nonclassical oscillations arising from the superposition. We demonstrate that the quantum mechanical superposition states and the corresponding statistical mixture states can be distinguishable by using the Pegg–Barnett formalism. Moreover, the phase becomes less uncertain when the mean photon number increases. Both the even and odd circular states have been shown to exhibit photon-number and phase squeezing. Our results can be applied to the investigations of phase properties of physical systems having a finite number of states.

References

[1] Glauber R J 1963 *Phys. Rev.* **131** 2766
[2] Klauder J R and Sudarshan E C G 1968 *Fundamentals of Quantum Optics* (New York: Benjamin)
[3] Yurke B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13
[4] Schleich W, Pernigo M and Fam Le Kien 1991 *Phys. Rev.* A **44** 2172
[5] Gamal M and Abd Al-Kader 1999 *Acta Phys. Sin.* **48** 754 (in Chinese)
[6] Muhammad A A, Ren M, Ma A Q and Liu S T 2008 *Chin. Phys.* B **17** 1777
[7] Janszky J, Domokos P and Adam P 1993 *Phys. Rev.* A **48** 2213
[8] Gagen M J 1995 *Phys. Rev.* A **51** 2715
[9] Janszky J, Domokos P, Szabo S and Adam P 1995 *Phys. Rev.* A **51** 4191
[10] Ragi R, Baseia B and Mizrahi S S 2000 *J. Opt.* B: *Quantum Semiclass. Opt.* **2** 299
[11] Ye Y H and Zeng G J 2007 *Chin. Phys.* **16** 1554
[12] Szabo S, Adam P, Janszky J and Domokos P 1996 *Phys. Rev.* A **53** 2698
[13] Wolinsky M and Carmichael H J 1988 *Phys. Rev. Lett.* **60** 1836
[14] Brune M, Haroche S, Lefevre V, Raimond J M and Zagury N 1990 *Phys. Rev. Lett.* **65** 976
[15] Brune M, Haroche S, Raimond J M, Davidovich L and Zagury N 1992 *Phys. Rev.* A **45** 5193
[16] Domokos P, Janszky J and Adam P 1994 *Phys. Rev.* A **50** 3340
[17] Jose W D and Mizrahi S S 2000 *J. Opt.* B: *Quantum Semiclass. Opt.* **2** 306
[18] Smithey D T, Beck M, Cooper J and Raymer M G 1993 *Phys. Rev.* A **48** 3159
[19] Buzek V, Gantsog Ts and Kim M S 1993 *Phys. Scr.* **T48** 131
[20] Dirac P A M 1927 *Proc. Roy. Soc.* (London) A **114** 243
[21] Susskind L and Glogower J 1964 *Physics* **1** 49
[22] Pegg D T and Barnett S M 1988 *Europhys. Lett.* **6** 483
[23] Barnett S M and Pegg D T 1989 *J. Mod. Opt.* **36** 7
[24] Pegg D T and Barnett S M 1989 *Phys. Rev.* A **39** 1665
[25] Zhou P 1993 *Acta Phys. Sin.* (Overseas Edition) 401
[26] Fan A F 1996 *Acta Phys. Sin.* (Overseas Edition) 737
[27] Tanas R, Miranowicz A and Gantsog Ts In: Wolf E (Ed) 1996 *Progress in Optics* **35** (Amsterdam: Elsevier)
[28] Tian Y H, Peng J S, Xu D H and Han L B 1999 *Acta Phys. Sin.* **48** 252 (in Chinese)
[29] Wang J S, Liu T K, Feng J and Sun J Z 2004 *Acta Phys. Sin.* **53** 3729 (in Chinese)
[30] Qian Y, Ma A Q, Ma Z M, Liu Z J and Liu S T 2007 *Acta Phys. Sin.* **56** 4571 (in Chinese)
[31] Guo H and Guo G C 1993 *Acta Phys. Sin.* **42** 918 (in Chinese)
[32] Gerry C C and Knight P L 2005 *Introductory Quantum Optics* (UK: Cambridge University Press) **46-47** 177
[33] Glauber R J and Lewenstein M 1991 *Phys. Rev.* A **43** 467