Effects of the number of on-ramps on the ring traffic flow*

Tang Tie-Qiao(唐铁桥)[a)b)†], Huang Hai-Jun(黄海军)[b)], and Shang Hua-Yan(尚华艳)[c)]

[a)] *School of Transportation Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing* 100191, *China*
[b)] *School of Economics and Management, Beijing University of Aeronautics and Astronautics, Beijing* 100191, *China*
[c)] *Institute of Transportation Engineering, Tsinghua University, Beijing* 100084, *China*

(Received 21 September 2009; revised manuscript received 27 September 2009)

Since ramps are an important composition of traffic systems and there often exist multi ramps in a traffic system, the number of ramps can have great effects on main road traffic and produce some complex phenomena. In this paper, we employ the model presented by Tang *et al.* [2009 *Communications in Theoretical Physics* **51**(1) 71] to further study the effects of the number of on-ramps on the stability of traffic flow on a ring road. The numerical results show that this model can reproduce some complex traffic phenomena resulting from multi on-ramps on the ring road and the effects of the number of on-ramps on traffic flow, but the phenomena and the effects are both related to the initial density of the main road.

Keywords: traffic flow, on-ramp, stability
PACC: 0550, 0520

1. Introduction

Serious traffic problems have motivated scholars to develop many traffic flow models to study various complex traffic phenomena (e.g. local cluster, stop-and-go, lane-changing, traffic waves, overtaking, etc.).[1−14] However, the above models cannot be used to study the traffic flow on the highway with ramps since they did not consider the factor of ramps. In fact, ramps are an important composition of traffic systems and will produce some complex phenomena traffic phenomena. In order to study ramps, scholars developed many traffic flow models and obtained many important results.[15−27] However, the above models cannot completely describe the various phenomena resulting from ramps. Thus, Tang *et al.*[28] presented a new macro model for traffic flow with the consideration of the friction effect resulting from ramps,[29] and adopted some observed data to study some complex traffic phenomena resulting from ramps during the morning rush hour and evening rush hour. Later, Tang *et al.*[30] further analysed the effects of ramps on the wave properties of traffic flow. But they assumed that there was only one ramp on the traffic system. In fact, there often exist multiple ramps on a road. In the present paper, we use the model[28] to further study some phenomena resulting from multi ramps and the effects of the number of ramps on the traffic flow on a ring road. The numerical results show that the model[28] can describe some complex phenomena induced by multi on-ramps and the effects of the number of on-ramps on traffic flow, but these phenomena and effects are both related to the initial density of the main road.

2. Model

Among the traffic flow models, a simple and useful one is the speed-gradient (SG) model:[3]

$$\begin{cases} \rho_t + (\rho v)_x = s\,(x,t)\,, \\ v_t + v v_x = \dfrac{v_{\rm e}\,(\rho) - v}{T} + c_0 v_x, \end{cases} \tag{1}$$

where ρ is the vehicle density, v the vehicle speed, $v_{\rm e}\,(\rho)$ the vehicle equilibrium speed, T the vehicle reactive time, c_0 the propagation speed of a small perturbation and $s\,(x,t)$ the flow generation rate. Jiang *et al.*[3] proved that their SG model could reproduce the formation and the propagation of traffic wave and small perturbation. And Tang *et al.*[31] recently used the model[3] to further study the evolution of various perturbations and found that the model

*Project supported by the Program for New Century Excellent Talents in University, China (Grant No. NCET-08-0038), the National Natural Science Foundation of China (Grant Nos. 70701002, 70971007 and 70521001), and the National Basic Research Program of China (Grant No. 2006CB705503).
†Corresponding author. E-mail: tieqiaotang@buaa.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

could perfectly describe some complex phenomena induced by various perturbations, which just shows that the model[3] is reasonable and useful. However, the model[3] does not hold true for investigating the phenomena resulting from the ramp since the term $s(x,t)$ is defined as zero. In order to study ramp, Tang *et al.*[15] and Jiang *et al.*[16] defined $s(x,t)$ as

$$s(x,t)=\begin{cases}\dfrac{q_{\rm ramp}}{L_{\rm ramp}}, & \forall x\in\Omega_{\rm ramp},\\ 0, & \text{else},\end{cases} \tag{2}$$

where $\Omega_{\rm ramp}$ is the region of the ramp, $q_{\rm ramp}$ the ramp flow and $L_{\rm ramp}$ the ramp's length. They used Eq. (1) to study some phenomena induced by on-ramp and found that one on-ramp could produce moving synchronized pattern (MSP), widening synchronized pattern (WSP), localized synchronized pattern (LSP) and general pattern (GP). However, they assumed that the ramp flow $q_{\rm ramp}$ was irrelevant to the vehicle density in the weaving region between ramp and main road and that ramp had no effects on the dynamics equation, so the SG model with the term $s(x,t)$ defined by Eq. (2) cannot completely describe some complex phenomena induced by ramp.

In a real traffic system, the quantitative relationship between the ramp flow $q_{\rm ramp}$ and the weaving region density is often complex and should be calibrated by use of many observed data, but it will decrease qualitatively with the weaving region density (shown in Fig. 1).

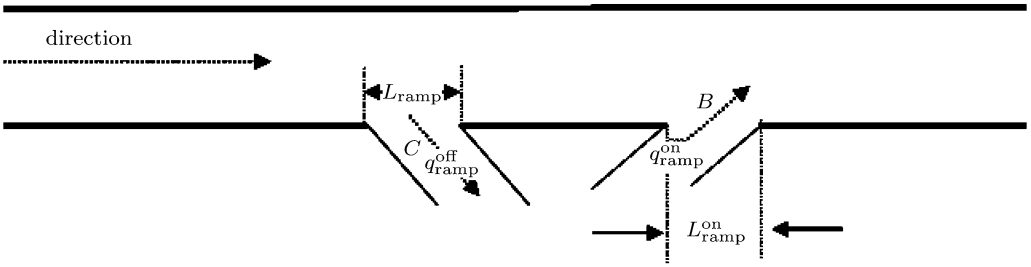


Fig. 1. Scheme of the traffic flow on a highway with ramps.

For simplicity, we here adopt the definition of ramp flow, given by Liu *et al.*[29], i.e.

$$q_{\rm ramp}=q^{\rm on}_{\rm ramp}\left(1-\left(\frac{\rho_{\rm B}}{\rho_{\rm j}}\right)^{\theta}\right),$$

and

$$q_{\rm ramp}=q^{\rm off}_{\rm ramp}\left(1-\left(\frac{\rho_{\rm C}}{\rho_{\rm j}}\right)^{\phi}\right), \tag{3}$$

where $q^{\rm on}_{\rm ramp}$ and $q^{\rm off}_{\rm ramp}$ are respectively the expected inflow flow of the on-ramp and the expected output flow of the off-ramp, $\rho_{\rm B}$ ($\rho_{\rm C}$) is the $B(C)$ region density, $\rho_{\rm j}$ the main road jam density, and θ and ϕ are parameters.

Liu *et al.*[29] found that ramps could often produce friction effect and presented a new dynamic equation as

$$v_t+vv_x=\frac{v_{\rm e}(\rho)-v}{T(\rho)}-\sigma\rho^{\beta}\rho_x+G, \tag{4}$$

where $T(\rho)$ is the vehicle reactive time and determined by the vehicle density, σ and β are parameters and G is the friction term, which can be defined as follows:

$$G=-\mu|s|\rho v, \tag{5}$$

where s is the ramp flow that is defined by Eq. (2) (or Eq. (3)) and μ is the friction coefficient. Equation (5) shows that on-ramp and off-ramp have the same effects on the dynamic equation, which is obviously unreasonable since on-ramp will often have an impeding effect on the main road traffic whereas off-ramp will often produce an active effect under a given condition. The friction effect is very complex, but it often increases with the ramp flow; so we can, for simplicity, set it as follows:

$$G_1=-\mu s\rho v. \tag{6}$$

Based on the foregoing discussion, we obtain a new model for traffic flow on a highway with ramps, i.e.

$$\begin{cases}\rho_t+(\rho v)_x=s(x,t),\\ v_t+vv_x=\dfrac{v_{\rm e}(\rho)-v}{T}+c_0v_x+G_1,\end{cases} \tag{7}$$

where $s(x,t)=q_{\rm ramp}/L_{\rm ramp}$.

Zhang and Wong[32] pointed out that a conservative scheme should be used to discretize the model if dynamic model is used to study the traffic flow, but we here employ the following upwind scheme to discretize Eq. (7) to compare our results with the results presented in Refs. [3], [15] and [16]:

$$\rho_i^{j+1} = \rho_i^j + \frac{\Delta t}{\Delta x}\rho_i^j(v_i^j - v_{i+1}^j) + \frac{\Delta t}{\Delta x}(\rho_{i-1}^j - \rho_i^j) + \Delta t \cdot s_i^j. \tag{8}$$

If $v_i^j < c_0$

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}(c_0 - v_i^j)(v_{i+1}^j - v_i^j) + \frac{\Delta t}{T}(v_e - v_i^j) + \Delta t G_{1,i}^j; \tag{9a}$$

otherwise,

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}(c_0 - v_i^j)(v_i^j - v_{i-1}^j) + \frac{\Delta t}{T}(v_e - v_i^j) + \Delta t G_{1,i}^j, \tag{9b}$$

where i and j represent the road section and the time interval, respectively. Note that we can obtain similar results if a conservative scheme is used to discretize Eq. (7).

3. Simulation

It is difficult to analytically study the complex traffic phenomena induced by ramp, so we use simulations to describe the effects of ramps on main road. Tang *et al.*[28] studied only the effect of one ramp on the ring road. In fact, there exist often multi ramps on a road, so we here focus on studying the effects of multi on-ramps on the traffic flow on a ring road, i.e.

$$\rho(L,t) = \rho(0,t), v(L,t) = v(0,t), \tag{10}$$

where $L = 30$ km is the length of the main road. We use the following equilibrium speed:[33]

$$v_e = v_f\left(1/\left(1 + \exp\left\{\frac{\rho/\rho_j - 0.25}{0.06}\right\}\right) - 3.72 \times 10^{-6}\right), \tag{11}$$

where v_f is the free-flow speed, and ρ_j the jam density. The initial speed is set as follows:

$$v(x,0) = v_e(\rho(x,0)). \tag{12}$$

In fact, there are multi on-ramps and off-ramps in the real traffic. However, few vehicles run out of the off-ramps during the morning rush hour, so we can study only the effects of multi on-ramps on the main road traffic in the present paper. The space between on-ramps often has a great effect on the main road traffic, but it is designed by engineers that different on-ramps have different expected input flows. For simplicity, we here assume that the space between on-ramps is equal to L/N (N is the number of the on-ramps) and that the expected input flows of on-ramps are the same. Based on the observed data,[28] we here define the expected input flow of each on-ramp as follows:

$$q_{\text{ramp}}^{\text{on}} = 120 \text{ veh/h}. \tag{13}$$

And other parameters are as follows:
$v_f = 30$ m/s, $\rho_j = 0.2$ vel/m, $T = 10$ s, $c_0 = 11$ m/s, $\theta = 1$, $\mu = 0.1$, $L_{\text{ramp}} = \Delta x = 100$ m, and $\Delta t = 1$ s. The numerical results under the different numbers of on-ramps are shown in Figs. 2–4, where the vehicle initial densities on a ring road are, respectively, very low, mid and very high. From the three figures, we can obtain the following results.

1) When the vehicle initial density on the ring road is very low, so that (i) the whole system will be relatively stable and local clusters slightly appear only at the upstream of the on-ramps if there are less than 7 on-ramps on the ring road (see Figs. 2(a)–3(c)); (ii) the local clusters which appear at the upstream of the on–ramps will eventually evolve into jam if there are more than 6 on–ramps on the ring road (see Fig. 2(d)); (iii) the density of the local clusters and its upstream will increase with the evolution time (see Fig. 2). The above results show that the ring road will be completely jammed when the number of on–ramps is higher than some critical value even if the vehicle initial density of the ring road on the ring road is very low.

2) When the vehicle initial density of the ring road is higher than some down critical value and less than some up critical value stop-and-go appears on the ring road and becomes more and more serious with the increase of the number of on-ramps. Thus chaotic traffic might appear on the ring road when the number of on-ramps is larger than some critical value (see Fig. 3). In addition, the up critical value often increases with the number of the on-ramps and the down critical value often decreases with the number of on-ramps, which means that the stable region is reduced with the increase of the number of on-ramps.

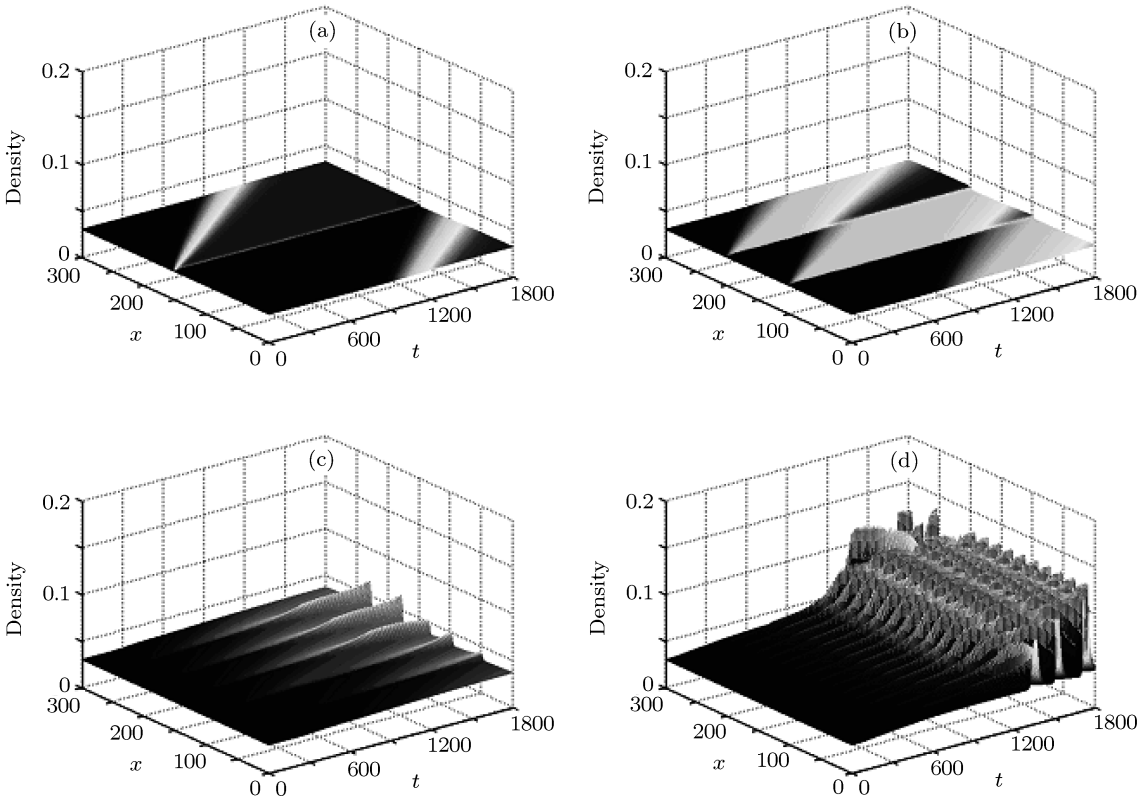


Fig. 2. Evolutions of the vehicle density when the initial one is very low (0.03 veh/m), where the numbers of on-ramps in panels (a)–(d) are respectively 1, 3, 6 and 15.

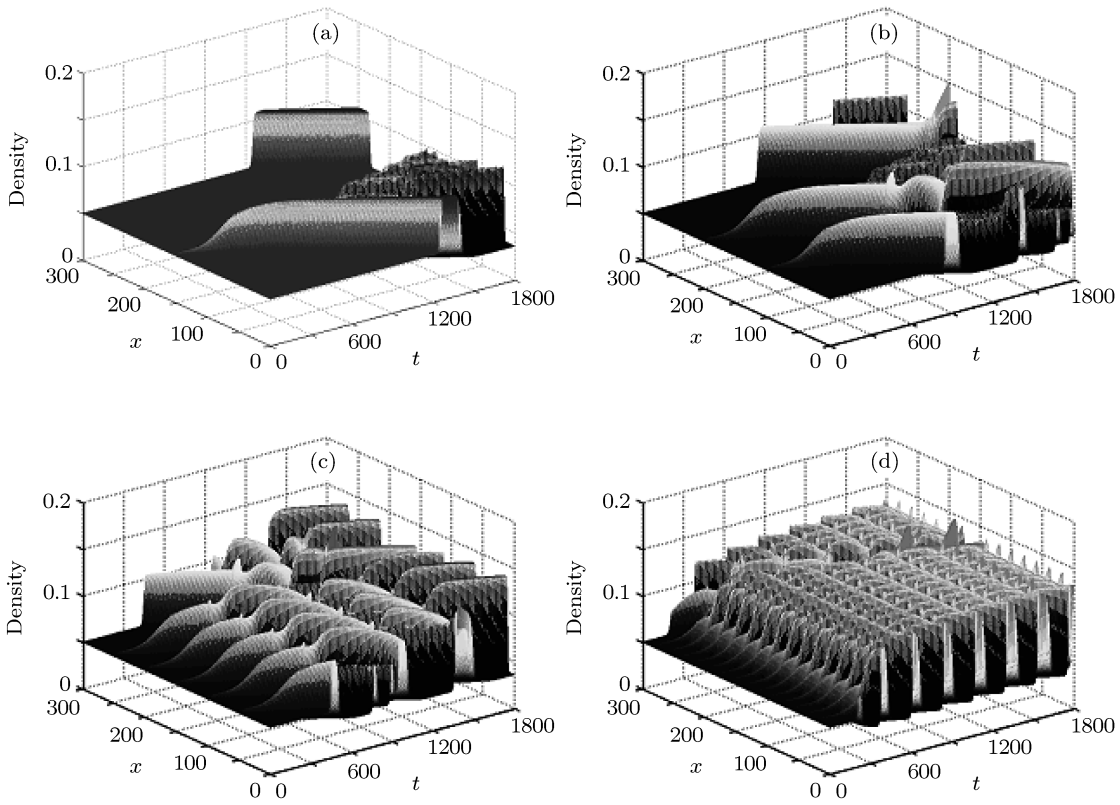


Fig. 3. Evolutions of the vehicle density when the initial one is mild (0.05 veh/m), where the numbers of on-ramps in panels (a)–(d) are respectively 1, 3, 6 and 15.

3) When the vehicle initial density of the ring road is very high, the whole system is relatively stable and

local clusters slightly appear only at the upstream of the on-ramps (see Fig. 4), which shows that the number of on-ramps has little effects on the ring road traffic since the real input flow of each on-ramp is very low at this time even if the expected input flow of each on-ramp is still very high.

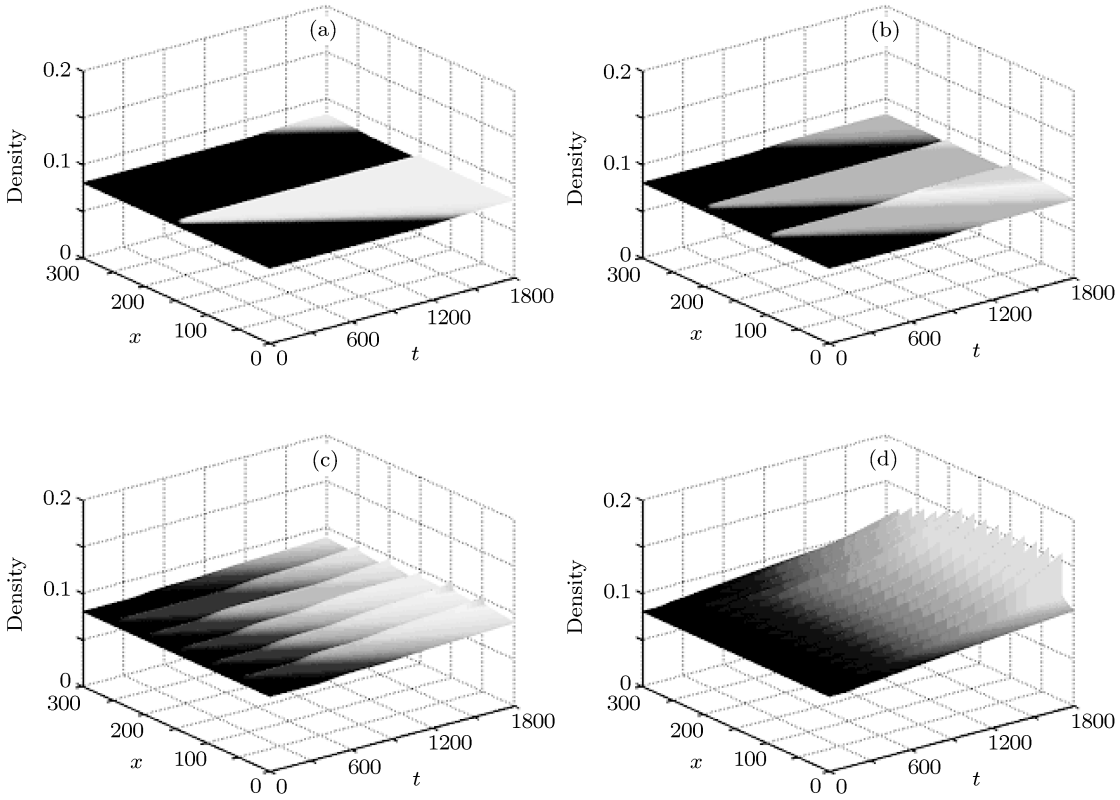


Fig. 4. Evolutions of the vehicle density when the initial one is very high (0.08 veh/m), where the numbers of on-ramps in panels (a)–(d) are respectively 1, 3, 6 and 15.

4) The above results are qualitatively accordant with the real traffic, which can further show that the model[28] can perfectly describe some complex phenomena induced by multi on-ramps.

Note that the critical value of the vehicle initial density on main road is often related to the number of on-ramps and the space between on-ramps, but the inner relationship among them is complex and might be related to other factors, so we should calibrate it by use of many observed data in the future study.

4. Conclusions

Although the existing models can reproduce some complex traffic phenomena, they cannot be used to describe the complex phenomena induced by ramps. In order to describe the effect of ramps, scholars have developed many traffic flow models to study various complex phenomena resulting from ramps and have obtained many important results,[15−30] but they cannot completely describe the various complex phenomena induced by ramps because they did not study many factors related to ramps (e.g. the number of ramps, the space between ramps, etc.). In a real traffic system, there are multi ramps and interactions between ramps. In this paper, we adopt a model[28] to investigate the effects of multi on-ramps on the ring road traffic. The numerical results show that this model can reproduce some phenomena induced by multi on-ramps on the ring road, which further indicates that this model is reasonable. However, we do not study the effects of the space and interaction between ramps, the expected input flow and other relevant factors on the main road traffic, so we will develop more exact models to investigate the various complex phenomena induced by ramps in future study.

References

[1] Chowdhury D, Santen L and Schadschneider A 2000 *Phys. Rep.* **329** 199
[2] Lee H Y, Lee H W and Kim D 2001 *Physica* A **281** 78
[3] Jiang R, Wu Q S and Zhu Z J 2002 *Transp. Res.* B **36** 405
[4] Gupta A K and Kitiyar V K 2006 *Physica* A **371** 674
[5] Huang H J, Tang T Q and Gao Z Y 2006 *Acta Mechanic Sin.* **22** 132
[6] Tang C F, Jiang R and Wu Q S 2007 *Chin. Phys.* **16** 1570
[7] Wang J F, Chen G S and Liu J 2008 *Chin. Phys.* B **17** 2850
[8] Peng G H, Sun D H and He H P 2009 *Chin. Phys.* B **18** 468
[9] Sun D H and Peng G H 2009 *Chin. Phys.* B **18** 3724
[10] Sheng P, Zhao S L, Wang J F, Tang P and Gao L 2009 *Chin. Phys.* B **18** 3347
[11] Zhao J Y, Sun X M and Jia L 2006 *Acta Phys. Sin.* **55** 2306 (in Chinese)
[12] Tang T Q, Huang H J and Xue Y 2006 *Acta Phys. Sin.* **55** 4026 (in Chinese)
[13] Han X L, Jiang C Y, Ge H X and Dai S Q 2007 *Acta Phys. Sin.* **56** 4383 (in Chinese)
[14] Tang T Q, Huang H J, Xu G and Xue Y 2008 *Acta Phys. Sin.* **57** 56 (in Chinese)
[15] Tang C F, Jiang R and Wu Q S 2007 *Physica* A **377** 641
[16] Jiang R, Hu M B, Jia B, Wang R L and Wu Q S 2007 *Phys. Lett.* A **365** 6
[17] Huang D W 2006 *Phys. Rev.* E **73** 016123
[18] Davis L C 2007 *Physica* A **379** 274
[19] Kerner B S and Klenov S L 2006 *J. Phys.* A **39** 1775
[20] Li F, Zhang X Y and Gao Z Y 2007 *Physica* A **374** 827
[21] Jiang R, Hu M B, Jia B, Wang R L and Wu Q S 2008 *Transportmetrica* **4** 51
[22] Zhuang Q, Jia B and Li X G 2009 *Chin. Phys.* B **18** 3271
[23] Teng Y F, Gao Z Y, Jia B and Li F 2008 *Acta Phys. Sin.* **57** 1635 (in Chinese)
[24] Lei L, Dong L Y and Ge H X 2007 *Acta Phys. Sin.* **56** 6874 (in Chinese)
[25] Mei C Q, Huang H J and Tang T Q 2008 *Acta Phys. Sin.* **57** 4786 (in Chinese)
[26] Mei C Q, Huang H J and Tang T Q 2009 *Acta Phys. Sin.* **58** 3014 (in Chinese)
[27] Li S C, Kong L J, Liu M R and Zheng R S 2009 *Acta Phys. Sin.* **58** 2276 (in Chinese)
[28] Tang T Q, Huang H J, Wong S C, Gao Z Y and Zhang Y 2009 *Commu. in Theor. Phys.* **51** 71
[29] Liu G Q, Lyrintzis A S and Michalopoulos P G 1996 *Appl. Math. Modeling* **20** 459
[30] Tang T Q, Huang H J, Xu G and Zhang Y 2008 *Europhysics Letters* **84** 14006
[31] Tang T Q, Huang H J, Zhang Y and Xu X Y 2008 *Int. J. Mod. Phys.* C **19** 1367
[32] Zhang P and Wong S C 2006 *Phys. Rev.* E **74** 026109
[33] Kerner B S and Konhäuser P 1993 *Phys. Rev.* E **48** R2335