Reflection and refraction of waves in oscillatory media*

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(Received 24 July 2009; revised manuscript received 24 August 2009)

This paper uses the two-dimensional Brusselator model to study reflection and refraction of chemical waves. It presents some boundary conditions of chemical waves, with which occurence of observed phenomena at interface as refraction and reflection of chemical waves can be interpreted. Moreover, the angle of reflection may be calculated by using the boundary conditions. It finds that reflection and refraction of chemical waves can occur simultaneously even if plane wave goes from a medium with higher speed to a medium with lower speed, provided the incident angle is larger than the critical angle.

Keywords: reflection, refraction, chemical wave
PACC: 0547, 0340K

1. Introduction

The reflection and refraction of nonlinear waves have attracted the interest of scientists since reflection and refraction of chemical waves were found experimentally by Zhabotinsky *et al.* in 1993.[1] Many related results have been obtained.[2−20] The results show that the reflection and refraction of nonlinear waves occur not only in excitable media but also in oscillatory media.[4−7] Refraction of nonlinear waves obeys Snell's law, and reflection of the wave occurs when the wave goes from a medium with lower speed to a medium of higher speed, provided the angle of incidence is larger than the critical angle. The reflection of nonlinear waves does not obey the usual law of light wave reflection, but rather exhibits a single angle of reflection, which is equal to the critical angle. Moreover, the reflection is a back refraction.[6] However, many problems still remain so far. For example, whether there is the boundary condition of wave vectors at the interface of two media is unknown. Whether the reflection of the wave can occur is not clear when the wave goes from a medium that supports higher wave speed to a medium with lower wave speed.

In this paper we study numerically the reflection and refraction of chemical waves at the interface of two media with different wave speeds by using the Brusselator model. The boundary conditions of the wave vectors are presented. The boundary conditions can interpret the observed phenomena at the interface as reflection and refraction of chemical waves occurs. The reflection of chemical waves has been observed when the wave goes from a medium of a higher wave speed to a medium with a lower wave speed.

2. Boundary conditions of the wave vectors

Now we consider how to obtain the boundary conditions of the wave vectors. Suppose that two infinite two-dimensional media are characterized by propagation speeds $v_{\rm i}$ and $v_{\rm r}$ for $y<0$ and $y>0$, respectively. The speeds of the incident, refracted and reflected wave are $v_{\rm i}$, $v_{\rm r}$ and $v_{\rm s}$, respectively. Corresponding wave vectors are $\boldsymbol{k}_{\rm i}$, $\boldsymbol{k}_{\rm r}$ and $\boldsymbol{k}_{\rm s}$, respectively. It is obvious that $v_{\rm s}$ is equal to $v_{\rm i}$ since incident wave and reflected wave propagate in the same medium. Thus $k_{\rm i}$ is equal to $k_{\rm s}$. It is well-known that the light wave vectors obey the following boundary conditions:

$$k_{{\rm i}x}\,|_{y=0} = k_{{\rm s}x}\,|_{y=0} = k_{{\rm r}x}\,|_{y=0}\,, \tag{1a}$$

$$\sigma = |k_{{\rm i}y} - k_{{\rm r}y}|_{y=0} = |k_{{\rm s}y} - k_{{\rm r}y}|_{y=0} = \sigma'. \tag{1b}$$

According to these boundary conditions, we can obtain the refractive and reflective law of optics.

It is obvious that we may suppose that the chemical wave vectors obey similar boundary conditions defined by Eq. (1) since the refraction of chemical waves obeys Snell's law of refraction. Considering that the chemical wave does not obey the principle of superposition, i.e., the reflected wave and the incident wave can not exist in the same region. The chemical wave exhibits a single angle of reflection. The reflection is a back refraction. Therefore, we suppose that the wave

*Project supported by National Natural Science Foundation of China (Grant No. 10765002).
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vectors of chemical waves obey the following boundary conditions:

(1) If the interface of two media is occupied by incident and refracted waves, the boundary conditions are

$$k_{\rm i}\sin\theta_{\rm i} = k_{\rm r}\sin\theta_{\rm r}, \tag{2a}$$

$$|k_{\rm i}\cos\theta_{\rm i} - k_{\rm r}\cos\theta_{\rm r}| = \sigma < \sigma_{\rm m}. \tag{2b}$$

(2) If the interface of two media is occupied by reflected and refracted waves, the boundary conditions are

$$k_{\rm s}\sin\theta_{\rm s} = k_{\rm r}\sin\theta_{\rm rc}, \tag{3a}$$

$$|k_{\rm s}\cos\theta_{\rm s} - k_{\rm r}\cos\theta_{\rm rc}| = \sigma' = \sigma_{\rm m}, \tag{3b}$$

where $\theta_{\rm i}$, $\theta_{\rm r}$ and $\theta_{\rm s}$ are angles of incidence, refraction and reflection, respectively. The angle of reflection has single value, so that $\theta_{\rm s}$ and $\sigma_{\rm m}$ are constants. The conditions (2a) and (3a) show that the refraction and the reflection of chemical waves obey Snell's law due to $k = \omega/v$.

According to the above boundary conditions, we can obtain

$$\frac{{\rm d}\sigma}{{\rm d}\theta_{\rm i}} = k_{\rm i}\cos\theta_{\rm i}\,|\tan\theta_{\rm i} - \tan\theta_{\rm r}| > 0, \tag{4a}$$

$$\frac{{\rm d}\sigma'}{{\rm d}\theta_{\rm i}} = \begin{cases} -k_{\rm i}\cos\theta_{\rm i}\tan\theta_{\rm r} < 0, & \text{if } \ k_{\rm r} > k_{\rm i},\ k_{\rm r}\cos\theta_{\rm r} > k_{\rm s}\cos\theta_{\rm s}, \\ k_{\rm i}\cos\theta_{\rm i}\tan\theta_{\rm r} > 0, & \text{if } \ k_{\rm r} < k_{\rm i},\ k_{\rm r}\cos\theta_{\rm r} < k_{\rm s}\cos\theta_{\rm s}. \end{cases} \tag{4b}$$

When $\theta_{\rm s}$ is given and $\theta_{\rm i}$ increases, the corresponding changes of σ and σ' are demonstrated in Fig. 1. It is observed that σ is equal to σ' when the angle of reflection is equal to the critical angle of incidence $\theta_{\rm ic}$. Reflection takes place when the angle of incidence is larger than the critical value $\theta_{\rm ic}$. The angle of refraction reaches 90° for $k_{\rm i} > k_{\rm r}$ (i.e., the incident wave comes from the medium with lower wave speed) because σ' reaches a maximum constant $\sigma_{\rm m}$. The angle of reflection can thus be given by

$$\theta_{\rm s} = \cos^{-1}\frac{\sigma_{\rm m}}{k_{\rm i}} = \sin^{-1}\frac{k_{\rm r}}{k_{\rm i}} = \sin^{-1}\frac{v_{\rm i}}{v_{\rm r}}, \quad \text{for} \ \ k_{\rm i} > k_{\rm r}. \tag{5}$$

The result is the same as the experimental result of Zhabotinsky *et al.*[1]

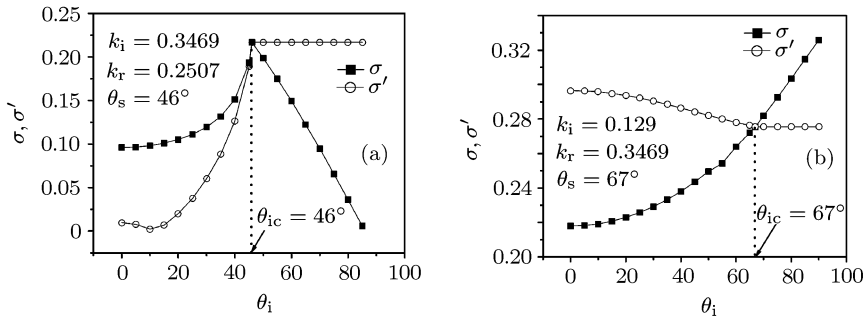


Fig. 1. The σ and σ' defined by Eq. (1) versus $\theta_{\rm i}$ with different values of $k_{\rm i}$, $k_{\rm r}$ and $\theta_{\rm s}$. (a) $k_{\rm i} > k_{\rm r}$, (b) $k_{\rm r} > k_{\rm i}$.

When incident wave comes from the medium with higher speed (i.e., $k_{\rm i}$ is smaller than $k_{\rm r}$), the angle of reflection is given by the following expression:

$$\theta_{\rm s} = \cos^{-1}\frac{k_{\rm r}^2 - k_{\rm i}^2 - \sigma_{\rm m}^2}{2k_{\rm i}\sigma_{\rm m}}, \quad \text{for} \quad k_{\rm i} < k_{\rm r}, \tag{6}$$

where $\sigma_{\rm m}$ depends on the parameters of system. Given $\sigma_{\rm m}$, we can obtain the angle of reflection.

3. Model

Now let us use Brusselator model[6] to study the reflection and the refraction of chemical waves. The two-dimensional Brusselator model is represented by the evolution equations,

$$\frac{\partial X}{\partial t} = 1 - (2.3a + 1)X + aX^2Y + \nabla^2 X, \tag{7a}$$

$$\frac{\partial Y}{\partial t} = 2.3aX - aX^2Y, \tag{7b}$$

which has the steady state solution

$$X_0 = 1, Y_0 = 2.3. \tag{8}$$

In order to obtain stable plane wave, we apply a periodic force

$$F(x,y,t) = \begin{cases} \cos(\omega t), & (x,y) \in \Omega, \\ 0, & (x,y) \notin \Omega, \end{cases} \tag{9}$$

to the right-hand side of Eq. (7a), where ω is the frequency of external force. The periodic force F is applied to a local region Ω, which located at the left boundary stripe of $x = 0$. Equation (7) is integrated by the Euler scheme together with a second order accurate finite-difference method, with fixed time step $\Delta t = 0.005$ and spatial step $\Delta x = \Delta y = 0.5$, i.e., we divide the $L_x \times L_y$ rectangular physical domain into $N_x \times N_y = 800 \times 400$ grid points. No-flux boundary conditions are used in all simulations. The initial state is the steady state solution defined by Eq. (8).

In our simulation, frequency ω is fixed at $\omega = 1.06$. We vary a in [1.0, 1.32]. The medium is divided into two uniform regions, with $a = a_{\rm i}$ and $a = a_{\rm r}$, respectively, by a straight borderline inclined at angle θ to the x axis. A plane wave generated at left edge of incident region ($a_{\rm i}$) goes from incident region to refractive region ($a_{\rm r}$). The simulations were performed in the two regions.

4. Simulation results

We study firstly the reflection and refraction of chemical waves when the incident wave goes from a medium with higher speed to a medium with lower speed. $a_{\rm i} = 1.28$ and $a_{\rm r} = 1.00$ are applied. The wave numbers determined numerically in two regions are $k_{\rm i} = 0.129$ and $k_{\rm r} = 0.3469$.

Figure 2 presents the effects of refraction and reflection. In Figs. 2(a) and 2(b) the incident angles are 30° and 90°, respectively. One can observe that only refraction occurs when the incident angle is equal to 30°. The angle of refraction measured in simulation is 11°. Incidence and refraction obey the boundary condition (2a). Reflection and refraction occur simultaneously when $\theta_{\rm i} = 90°$. The angles of reflection and refraction measured in simulation are 67° and 20°, respectively. So, the refraction and reflection obey the boundary condition (3a).

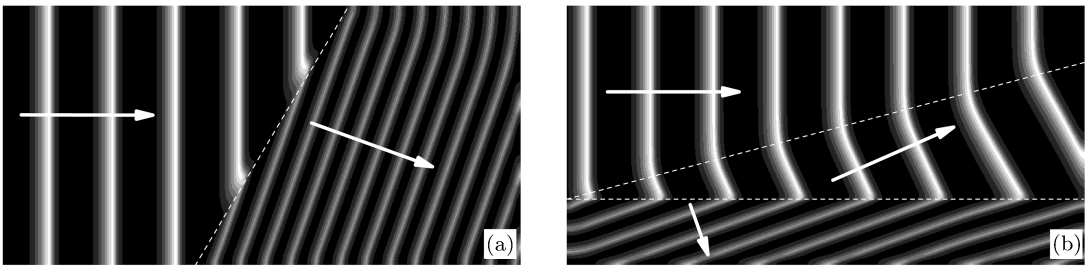


Fig. 2. The phenomena of reflection and refraction with $k_{\rm i} < k_{\rm r}$, $a_{\rm i} = 1.28$ and $a_{\rm r} = 1.0$ are applied: (a) $\theta_{\rm i} = 30°$, (b) $\theta_{\rm i} = 90°$. The arrows represent the direction of wave propagation. The dotted lines represent the interface of two regions.

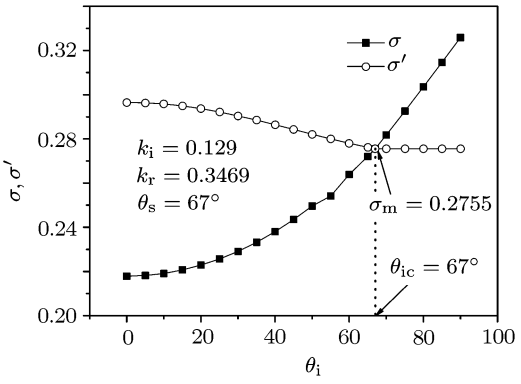


Fig. 3. The σ and σ' measured in the simulation vs $\theta_{\rm i}$ for $k_{\rm i} < k_{\rm r}$.

In order to obtain $\sigma_{\rm m}$ and test boundary conditions, we plot in Fig. 3 σ and σ' vs $\theta_{\rm i}$ with given $\theta_{\rm s} = 67°$. Where σ and σ' are calculated by using refractive angle measured in the simulation. We get $\sigma_{\rm m} \approx 0.2755$ and $\theta_{\rm ic} = 67°$. According to Eq. (6), the predicted reflective angle is $\theta_{\rm s} = 67°$. So, reflection angle is in agreement with the prediction.

Now we study the reflection and refraction of waves when the incident wave comes from the medium with lower speed, $a_{\rm i} = 1.0$ and $a_{\rm r} = 1.1$ are applied. The wave numbers determined numerically in two regions are $k_{\rm i} = 0.3469$ and $k_{\rm r} = 0.2507$. Figure 4 presents the effects of refraction and reflection. In

Figs. 4(a) and 4(b) the incident angles are 30° and 75°, respectively. It is observed that reflection does not occur for $\theta_i = 30°$. The angle of refraction measured in simulation is 44°. Incidence and refraction obey the boundary condition (2a). Reflection and refraction occur simultaneously when $\theta_i = 75°$. The angles of reflection and refraction measured in simulation are 46.5° and 90°, respectively. So, the refraction and reflection obey the boundary condition (3a).

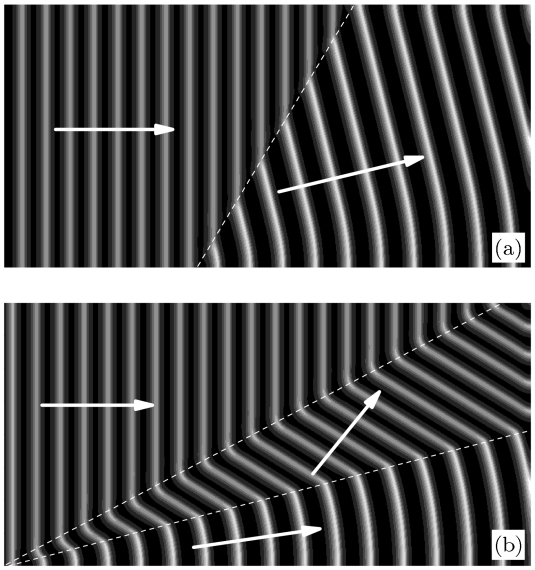


Fig. 4. The phenomena of reflection and refraction with $k_i > k_r$, $a_i = 1.0$ and $a_r = 1.1$ are applied. (a) $\theta_i = 30°$, (b) $\theta_i = 75°$. The arrows represent the direction of wave propagation. The dotted lines represent the interface of two regions.

Figure 5 refers to the case with $a_i = 1.0$, $a_r = 1.1$ and $\theta_s = 46.5°$. We get $\sigma_m \approx 0.2386$ and $\theta_{ic} = 46.5°$. According to Eq. (5), the predicted reflection angle is $\theta_s = 46.5°$. So, reflection angle is in agreement with the prediction.

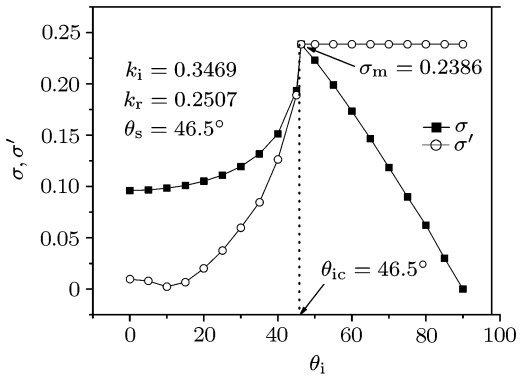


Fig. 5. The σ and σ' measured in the simulation vs θ_i for $k_i > k_r$.

5. Conclusion

The above results indicate that the refraction and the reflection of chemical waves obey the boundary conditions (2) and (3). Reflection and refraction can occur simultaneously when the wave goes from a medium with a higher speed to a medium with a lower speed, provided the incident angle is larger than the critical angle. The concrete value of σ_m depends on parameters k_i, k_r and ω. It thus cannot be computed analytically. We expect that the problem can be solved in the future.

References

[1] Zhabotinsky A M, Eager M D and Epstein I R 1993 *Phys. Rev. Lett.* **71** 1526

[2] Kosek J and Marek M 1995 *Phys. Rev. Lett.* **74** 2134

[3] Hwang S C and Timothy H H 1996 *Phys. Rev.* E **54** 3009

[4] Brazhnik P K and Tyson J J 1996 *Phys. Rev.* E **54** 1958

[5] Pechenik L and Levine H 1998 *Phys. Rev.* E **58** 2910

[6] Sainhas J and Dilão R 1998 *Phys. Rev. Lett.* **80** 5216

[7] Rabinovitch A, Gutman M and Aviram I 2003 *Phys. Rev.* E **67** 036212

[8] Remhof A, Wijngaarden R J and Griessen R 2003 *Phys. Rev. Lett.* **90** 145502

[9] Gutman M, Aviram I and Rabinovitch A 2004 *Phys. Rev.* E **69** 016211

[10] Skaar J 2006 *Phys. Rev.* E **73** 026605

[11] Zhang R, Yang L, Zhabotinsky A M and Epstein I R 2007 *Phys. Rev.* E **76** 016201

[12] Aranson I S and Kramer L 2002 *Rev. Mod. Phys.* **74** 99

[13] Hu Y H, Fu X Q, Wen S C, Su W H and Fan D Y 2006 *Chin. Phys.* **15** 2970

[14] Lv Y P, Gu G F, Lu H C, Dai Y and Fang G N 2009 *Acta Phys. Sin.* **58** 2996 (in Chinese)

[15] Xiao J H, Hu G and Hu B B 2004 *Chin. Phys. Lett.* **21** 1224

[16] Wang X, Tian X, Wang H L, Ouyang Q and Li H 2004 *Chin. Phys. Lett.* **21** 2365

[17] Shao X, Ren Y and Ouyang Q 2006 *Chin. Phys.* **15** 0513

[18] Tang Z H, Yan J R and Liu L H 2006 *Chin. Phys.* **15** 2638

[19] Shao X, Wu Y B, Zhang J Z, Wang H L and Ouyang Q 2008 *Phys. Rev. Lett.* **100** 198304

[20] Lv Y P, Gu G F, Lu H C, Dai Y and Tang G N 2009 *Acta Phys. Sin.* **58** 7573 (in Chinese)