Distributed predictive control of spiral wave in cardiac excitable media*

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In this paper, we propose the distributed predictive control strategies of spiral wave in cardiac excitable media. The modified FitzHugh–Nagumo model was used to express the cardiac excitable media approximately. Based on the control-Lyapunov theory, we obtained the distributed control equation, which consists of a positive control-Lyapunov function and a positive cost function. Using the equation, we investigate two kinds of robust control strategies: the time-dependent distributed control strategy and the space-time dependent distributed control strategy. The feasibility of the strategies was demonstrated via an illustrative example, in which the spiral wave was prevented to occur, and the possibility for inducing ventricular fibrillation was eliminated. The strategies are helpful in designing various cardiac devices. Since the second strategy is more efficient and robust than the first one, and the response time in the second strategy is far less than that in the first one, the former is suitable for the quick-response control systems. In addition, our spatiotemporal control strategies, especially the second strategy, can be applied to other cardiac models, even to other reaction-diffusion systems.

Keywords: spiral wave, FitzHugh–Nagumo model, control-Lyapunov function

PACC: 0545, 8700, 8730

1. Introduction

In the fields of reaction–diffusion systems, the dynamics of spiral waves and spatiotemporal chaos are always the notable themes.[1,2] Spiral waves are characteristic structures of excitable media far away from equilibrium, which have been observed in different systems such as catalytic surface oxidation, the Belousov–Zhabotinsky chemical reaction, aggregating colonies of slime mold, and heart tissue, etc.[3−5]

Spiral waves and spatiotemporal chaos are usually harmful and need to be suppressed. For example, many important evidences proved that the appearance of spiral waves is linked to one kind of cardiac arrhythmia. The instability of spiral waves will induce a rapid death of heart, named ventricular fibrillation (VF). Therefore, it is important to find some effective schemes to prevent the appearance of spiral waves and spatiotemporal chaos in many domains. Many scholars have proposed multifarious control schemes.[6−10] Wang and Xie[11] proposed a scheme of spatial perturbations to eliminate the spatiotemporal chaos and spiral waves, and then the scheme was developed as traveling-wave perturbations to suppress turbulent states in cardiac tissue.[12] Zhang *et al.*[13,14] proposed to suppress the spiral waves and spatiotemporal chaos by using periodical forcing in the media. Furthermore, the local self-coupling scheme and local intermittent shock were proposed to suppress spiral waves,[15] and so on.

Many of the schemes are about how to control or eliminate the existing spiral waves. In many cases, the appearance of spiral waves means the appearance of their disaster, too. Furthermore, although many efforts have been done in these schemes, the efficiency and the robustness of the control need to be improved.

In this paper, we propose a practical method to prevent the appearance of spiral waves and spatiotemporal chaos in cardiac excitable media.

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2. Modified FitzHugh–Nagumo model in cardiac excitable media

The modified FitzHugh–Nagumo model was first presented by Barkley according to Ref. [5], which could be employed to describe the spatiotemporal dynamics in excitable media approximately. The equations of cardiac tissue based on the model[16] are given by

$$\frac{\partial u}{\partial t} = D_{\rm u}\nabla^2 u + \frac{1}{\varepsilon}u(1-u)\left(u - \frac{v+b}{a}\right) + i_1, \quad (1)$$

$$\frac{\partial v}{\partial t} = D_{\rm v}\nabla^2 v + u - v, \quad (2)$$

where $a = 0.5$, $b = 0.01$ and $\varepsilon = 0.02$. The values of spatial diffusion coefficients $D_{\rm u}$ and $D_{\rm v}$ are 1 and 0, respectively. The variable u is the fast variable or the excitation variable, which represents the membrane action potential of heart surface, and v plays the role of the slow variable or the recovery variable, which represents an effective membrane conductance. The distributed control variable i_1 is the adjustable external current.

In the following discussions, we use a two-dimensional $L \times L$ square domain with no-flux boundary conditions. In this domain, we consider the eigenfunctions of operator ∇^2, which satisfies $\nabla^2\varphi_{n,\,m}(x,\,y) = -\lambda_{n,\,m}\varphi_{n,\,m}(x,\,y)$ and leads to the eigenfunctions $\varphi_{n,\,m}(x,\,y) = \exp({\rm i}2\pi(nx + my)/L)/L$ with $\int_0^L\int_0^L \varphi^*_{n,m}\varphi_{p,q}{\rm d}x{\rm d}y = \delta_{n,p}\delta_{m,q}$ and the eigenvalues $\lambda_{n,\,m} = (2\pi/L)^2(n^2 + m^2)$, where $n,\, m \in Z$, and Z is an integer set. Using the eigenfunctions φ_m to expand u, v and i_1, we obtain $u = \sum_{n,\,m} C_{1,n,m}(t)\varphi_{n,\,m}(x,y)$, $v = \sum_{n,\,m} C_{2,n,m}(t)\varphi_{n,\,m}(x,y)$ and $i_1 = \sum_{n,\,m} I_{1,n,m}(t)\varphi_{n,\,m}(x,y)$, where $\sum_{n,\,m}$ denotes $\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}$, $C_{1,\,n,\,m}(t) = \int_0^L\int_0^L \varphi^*_{n,m}u{\rm d}x{\rm d}y$, $C_{2,\,n,\,m}(t) = \int_0^L\int_0^L \varphi^*_{n,m}v{\rm d}x{\rm d}y$ and $i_{1,\,n,\,m}(t) = \int_0^L\int_0^L \varphi^*_{n,m}i_1{\rm d}x{\rm d}y$. In a practical problem, $\sum_{n,\,m}$ stands for $\sum_{n=-N}^{N}\sum_{m=-M}^{M}$, where N and M are very large integers. Substituting them into Eqs. (1) and (2), we could deduce the cardiac equations in frequency domain

$$\frac{\rm d}{{\rm d}t}C_{1,\,n,\,m} = F(C_{1,\,n,\,m},\, C_{2,\,n,\,m},\, I_{1,\,n,\,m}), \quad (3)$$

$$\frac{\rm d}{{\rm d}t}C_{2,\,n,\,m} = C_{1,\,n,\,m} - C_{2,\,n,\,m}, \quad (4)$$

where

$$\begin{aligned} F(C_{1,\,n,\,m}) &= -D_1\lambda_{n,\,m}C_{1,\,n,\,m} - \frac{b}{\varepsilon a}C_{1,\,n,\,m} \\ &\quad + \frac{(a+b)}{\varepsilon a}\sum_{k,\,j} C_{1,\,n-k,\,m-j}C_{1,\,k,\,j} \\ &\quad - \frac{1}{\varepsilon a}\sum_{k,\,j} C_{1,\,n-k,\,m-j}C_{2,\,k,\,j} \\ &\quad - \frac{1}{\varepsilon}\sum_{k,\,j}\sum_{p,q} C_{1,\,n-k-p,\,m-j-q}C_{1,\,k,\,j}C_{1,\,p,\,q} \\ &\quad + \frac{1}{\varepsilon a}\sum_{k,\,j}\sum_{p,q} C_{1,\,n-k-p,\,m-j-q}C_{1,\,k,\,j}C_{2,\,p,\,q} \\ &\quad + I_{1,\,n,\,m} \end{aligned}$$

with $n,\, m \in Z$.

3. Control equations

The system is usually in the rest state $u = v = 0$, or equivalently $C_{1,\,n,\,m} = C_{2,\,n,\,m} = 0$. When a perturbation is applied to it, we hope it will eventually return to its rest state by control variables i_1. According to the robust control-Lyapunov theory,[17] we define a control-Lyapunov function $V(u,\, v,\, i_1) > 0$ or $V(u,\, v,\, i_1) = 0$ in the rest state. The control-Lyapunov function is used to test whether the system is feedback stable, that is to say whether or not, for the given controlled variables u and v there exists a control variable i_1 by which the system can be brought back to the rest state. When $t \to \infty$, $\forall u,\, v \neq 0, \exists i_1$, there exists a basic control equation

$$\frac{\rm d}{{\rm d}t}V(u,\, v,\, i_1) = -J(u,\, v,\, i_1), \quad (5)$$

where the cost function $J(u,\, v,\, i_1) > 0$ or $J(u,\, v,\, i_1) = 0$ is in the rest state with $C_{1,\,n,\,m} = C_{2,\,n,\,m} = 0$. In practice, we must optimally select $J(u,\, v,\, i_1)$ to ensure $i_1 = 0$ in the rest state in the system. There has not been a general method to determine the control-Lyapunov function of a system. Fortunately, for the modified FitzHugh–Nagumo model, we could define the control-Lyapunov function V as

$$\begin{aligned} V &= \int_0^L\int_0^L (Au^2 + v^2){\rm d}x{\rm d}y \\ &= \sum_{n,m}(AC^2_{1,n,m} + C^2_{2,n,m}), \end{aligned} \quad (6)$$

where A is a given constant. It is obvious that $V > 0$ or $V = 0$ is in the rest state with $C_{1,\,n,\,m} = C_{2,\,n,\,m} = 0$. The time derivative of Eq. (6) satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}V=\sum_{n,\,m}\left(2AC_{1,\,n,\,m}\frac{\mathrm{d}}{\mathrm{d}t}C_{1,\,n,\,m}+\ 2C_{2,\,n,\,m}\frac{\mathrm{d}}{\mathrm{d}t}C_{2,\,n,\,m}\right). \tag{7}$$

Substituting Eqs. (3) and (4) into the Eq. (7), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}V=-J(C_{1,\,n,\,m},\,C_{2,\,n,\,m}), \tag{8}$$

where the cost function is expressed by

$$\begin{aligned}
&J(C_{1,\,n,\,m},\,C_{2,\,n,\,m},\,I_{1,\,n,\,m})\\
&=2\sum_{n,\,m}\left(A\frac{b}{\varepsilon a}C_{1,n,m}^2-C_{1,n,m}C_{2,n,m}+C_{2,n,m}^2\right)+2AD_1\sum_{n,m}\lambda_{n,m}C_{1,n,m}^2\\
&-2A\sum_{n,m}C_{1,n,m}\left(\frac{a+b}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{1,k,j}-\frac{1}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{2,k,j}\right.\\
&\left.-\frac{1}{\varepsilon}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{1,p,q}+\frac{1}{\varepsilon a}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}+I_{1,n,m}\right).
\end{aligned}$$

According to the basic control Eq. (5), we must determine the control variable i_1 to ensure the cost function $J(C_{1,\,n,\,m},\,C_{2,\,n,\,m})\geq 0$ or $\mathrm{d}/(\mathrm{d}t)V\leq 0$. Hence, we let the control variable i_1 satisfy the control equation in the frequency domain

$$K(C_{1,\,n,\,m},\,C_{2,\,n,\,m},\,I_{1,\,n,\,m})=0, \tag{9}$$

where

$$\begin{aligned}
&K(C_{1,\,n,\,m},\,C_{2,\,n,\,m},\,I_{1,\,n,\,m})\\
&=-\sum_{n,\,m}C_{1,n,m}\left(\frac{a+b}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{1,k,j}+\frac{1}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{2,k,j}\right.\\
&+\left(\frac{1}{\varepsilon}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{1,p,q}-\frac{1}{\varepsilon a}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}\right)\\
&\left.-\sum_{n,m}C_{1,n,m}I_{1,n,m}\right).
\end{aligned}$$

Substituting Eq. (9) into Eq. (8), and selecting $A=\varepsilon a/4b$, we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}V&=-J\\
&=-2\sum_{n,\,m}\left(\frac{1}{2}C_{1,\,n,\,m}-C_{2,\,n,\,m}\right)^2\\
&\quad-2AD_1\sum_{n,\,m}\lambda_{n,\,m}C_{1,\,n,\,m}^2.
\end{aligned} \tag{10}$$

Since the eigenvalues $\lambda_{n,\,m}=(2\pi/L)^2(n^2+m^2)\geq 0$, we could verify $\mathrm{d}V/\mathrm{d}t=-J\leq 0$, which indicates that the control variable i_1 determined by the control Eq. (9) would ensure the system to be brought back to the rest state.

In a practical system, if the perturbations are small enough, we should not employed any control variable that might influence the normal activities of the system. For the rest state, we have $C_{1,\,n,\,m}=C_{2,\,n,\,m}=0$. The cardiac Eq. (3) contains nonlinear term $F(C_{1,\,n,\,m},\,C_{2,\,n,\,m},\,I_{1,\,n,\,m})$. We might expand it in Taylor series around the rest state, and keep only the linear terms

$$F(C_{1,\,n,\,m})=-D_1\lambda_{n,\,m}C_{1,\,n,\,m}-\frac{b}{\varepsilon a}C_{1,\,n,\,m}, \tag{11}$$

where

$$\begin{aligned}
&\left|-D_1\lambda_{n,\,m}C_{1,\,n,\,m}-\frac{b}{\varepsilon a}C_{1,\,n,\,m}\right|\\
&\gg\left|\frac{a+b}{\varepsilon a}\sum_{k,\,j}C_{1,n-k,m-j}C_{1,\,k,\,j}\right.\\
&-\frac{1}{\varepsilon a}\sum_{k,\,j}C_{1,n-k,m-j}C_{2,\,k,\,j}\\
&\left.-\frac{1}{\varepsilon}\sum_{k,\,j}\sum_{p,q}C_{1,\,n-k-p,\,m-j-q}C_{1,\,k,\,j}C_{1,\,p,\,q}\right.
\end{aligned}$$

$$+\frac{1}{\varepsilon a}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}\Big|$$

and $I_{1,n,m}=0$. For the case $n=m=0$, we might define $C_{1\min}$ as a constraint to $C_{1,0,0}$, which satisfies

$$\begin{aligned}&|C_{1,0,0}|\\ \geq\ & C_{1,\min}\\ >\ & \frac{1}{b}\Big|(a+b)\sum_{k,j}C_{1,n-k,m-j}C_{1,k,j}\\ &-\sum_{k,j}C_{1,n-k,m-j}C_{2,k,j}\\ &-a\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{1,p,q}\\ &+\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}\Big|. \quad (12)\end{aligned}$$

In the same way, we can also define a constraint to $C_{2,0,0}$. In a practical system, there also exist some constraints to its control power. In brief, the constraints to control Eq. (9) could be expressed by

$$\begin{aligned}&|C_{1,0,0}|\geq C_{1,\min},\\ &|C_{2,0,0}|\geq C_{2,\min},\\ &P\leq P_{\max}, \quad (13)\end{aligned}$$

where $C_{1,\min}$ and $C_{2,\min}$ are the thresholds of $C_{1,0,0}$ and $C_{2,0,0}$, respectively. The control power $P=\int_0^L\int_0^L i_1^2\mathrm{d}x\mathrm{d}y=\sum_{n,m}I_{1,n,m}^2\leq P_{\max}$, with $P_{\max}$ being the power threshold, which can be evaluated by theoretical or practical ways in an actual system. If the constraints (13) cannot be satisfied, we obtain $I_{1,n,m}=0$. It must be noted that how to determine threshold $P_{\max}$ is usually a problem on the technique realisation of the control. There are fundamental differences between $C_{1,\min}$ or $C_{2,\min}$ and $P_{\max}$. The constraints $C_{1,\min}$ and $C_{2,\min}$ are mainly determined by the normal activities and the stability of the system. To sum up, when the constraints are satisfied, we let $i_1=\sum_{n,m}I_{1,n,m}(t)\varphi_{n,m}(x,y)$ with $I_{1,n,m}$ determined by Eq. (9), otherwise $I_{1,n,m}=0$.

From Eq. (9) we could investigate two suitable control strategies. First, we consider the time-dependent control $i_1=i_1(t)$. In spatial frequency domain, we have $I_{1,0,0}(t)=\int_0^L\int_0^L i_1(t)\mathrm{d}x\mathrm{d}y/L^2=i_1(t)$, and we could easily verify that $I_{1,0,0}$ is the only non-zero coefficient among i_1's frequency coefficients. Therefore, using Eq. (9), we obtain

$$I_{1,0,0}=K_1(C_{1,k,j},\,C_{2,k,j}), \quad (14)$$

where

$$\begin{aligned}&K_1(C_{1,k,j},\,C_{2,k,j})\\ =\ & \frac{1}{C_{1,0,0}}\Big[-\sum_{n,m}C_{1,n,m}\Big(\frac{a+b}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{1,k,j}\\ &+\frac{1}{\varepsilon a}\sum_{k,j}C_{1,n-k,m-j}C_{2,k,j}\\ &+\frac{1}{\varepsilon}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{1,p,q}\\ &-\frac{1}{\varepsilon a}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}\Big)\Big].\end{aligned}$$

When $C_{1,\min}\to 0$ and $C_{1,0,0}\to 0$, we have $K_1(C_{1,k,j},\,C_{2,k,j})\to 0$, and the system could not be controlled by Eq. (14). But the system usually can be well controlled with suitable $C_{1,\min}$, $C_{2,\min}$ and $P_{\max}$. Next, we consider the spatiotemporal-distributed control variable $i_1=i_1(x,\,y,\,t)$. It is convenient to select the distributed control strategy as

$$I_{1,n,m}=K_{2,n,m}(C_{1,k,j},\,C_{2,k,j}), \quad (15)$$

where

$$\begin{aligned}&K_{2,n,m}(C_{1,k,j},\,C_{2,k,j})\\ =\ & \varepsilon^{-1}\Big(-\frac{a+b}{a}\sum_{k,j}C_{1,n-k,m-j}C_{1,k,j}\\ &+\frac{1}{a}\sum_{k,j}C_{1,n-k,m-j}C_{2,k,j}\\ &+\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{1,p,q})\\ &-\frac{1}{a}\sum_{k,j}\sum_{p,q}C_{1,n-k-p,m-j-q}C_{1,k,j}C_{2,p,q}\Big),\end{aligned}$$

and $I_{1,p,q}\sim O(C_{1,n,m}^2)+O(C_{1,n,m}C_{2,n,m})+O(C_{2,n,m}^2)$ with $n,\,m\in Z$. Substituting Eq. (15) into Eqs. (3) and (4), we may directly obtain

$$C_{1,n,m}=C_{1,n,m}(0)\,\mathrm{e}^{-\beta_{n,m}t}, \quad (16)$$

$$C_{2,n,m}=\begin{cases}C_{2,n,m}(0)\,\mathrm{e}^{-t}+C_{1,n,m}(0)t\,\mathrm{e}^{-t}, & \text{for } \beta_{n,m}=1,\\ C_{2,n,m}(0)\,\mathrm{e}^{-t}+C_{1,n,m}(0)(\,\mathrm{e}^{-\beta_{n,m}t}-\,\mathrm{e}^{-t})/(1-\beta_{n,m}), & \text{otherwise,}\end{cases} \quad (17)$$

where $\beta_{n,m} = D_1\lambda_{n,m} + b/\varepsilon a > 0$. Obviously, we have $u(x, y, \infty) = \sum_{n,m} C_{1,n,m}(\infty)\varphi_{n,m}(x, y) \to 0$ and $v(x, y, \infty) = \sum_{n,m} C_{1,n,m}(\infty)\varphi_{n,m}(x) \to 0$. Considering Eq. (11), we could readily verify the stability of the system around the rest state under any given initial conditions, no matter whether we apply control variable i_1 or not. That is to say, the system is stable for $|C_{1,0,0}| < C_{1,\min}$ and $|C_{2,0,0}| < C_{2,\min}$ without control variable. Hence, the second control strategy can be applied to control or prevent the appearance of all kinds of spatiotemporal structures, such as chaos, turbulence and spiral waves, etc. Since the second strategy can directly control the components of high frequencies, the second strategy is more efficient and robust than the first one. As an illustrative example, we demonstrate the effects of both control strategies with the predictive control of spiral waves in the following discussions.

4. Control example

In our practice and numerical simulations, the system is in a two-dimensional square domain in no-flux boundary conditions. The spatial size is $L = 127.5$, and we divide the total space into many lattices with small spatial steps $\Delta x = \Delta y = 0.5$. Naturally, the variables u, v, and i_1 can be changed into $u_{k,j}$, $v_{k,j}$, and $i_{1,k,j}$, where $i, j = 1, 2, \ldots, 256$. The relevant variables in the frequency domain, such as $C_{1,n,m}$, $C_{2,n,m}$, and $I_{1,n,m}$, are derived by using the fast Fourier transform (FFT) algorithm. In addition, the vertical gradient distributed initial conditions produced by the perturbations are shown to be $u_{k,j}(0) = 0.9$, where $k = 126, 127$, $j = 1, 2, \ldots, 128$; $u_{k,j}(0) = 0.6$, $v_{k,j}(0) = 0.6$, where $k = 128, 129$, $j = 1, 2, \ldots, 128$; $v_{k,j}(0) = 0.9$, where $k = 130, 131$, $j = 1, 2, \ldots, 128$; $u_{k,j}(0) = v_{k,j}(0) = 0$, where k and j are other values. Furthermore, we adopt the modified Euler algorithm with changeable time step. The control constraints are given to be $C_{1,\min} = C_{2,\min} = 10^{-4}$ and $P_{\max} = 10^4$.

First, our discrete model is without control. The dynamic pattern is shown in Fig. 1. A spiral wave appears in the whole region. The parameters $C_{1,0,0}$ and $C_{2,0,0}$ are the spatial mean values of u and v, and the variations of $C_{1,0,0}$ and $C_{2,0,0}$ are plotted in Figs. 2(a) and 2(b), respectively. The values of $C_{1,0,0}$ and $C_{2,0,0}$ reach 0.17 at $t = 35$, and begin to fluctuate around 0.17 slightly. The spiral wave appears at $t = 35$ and the spiral period is about 3.82.

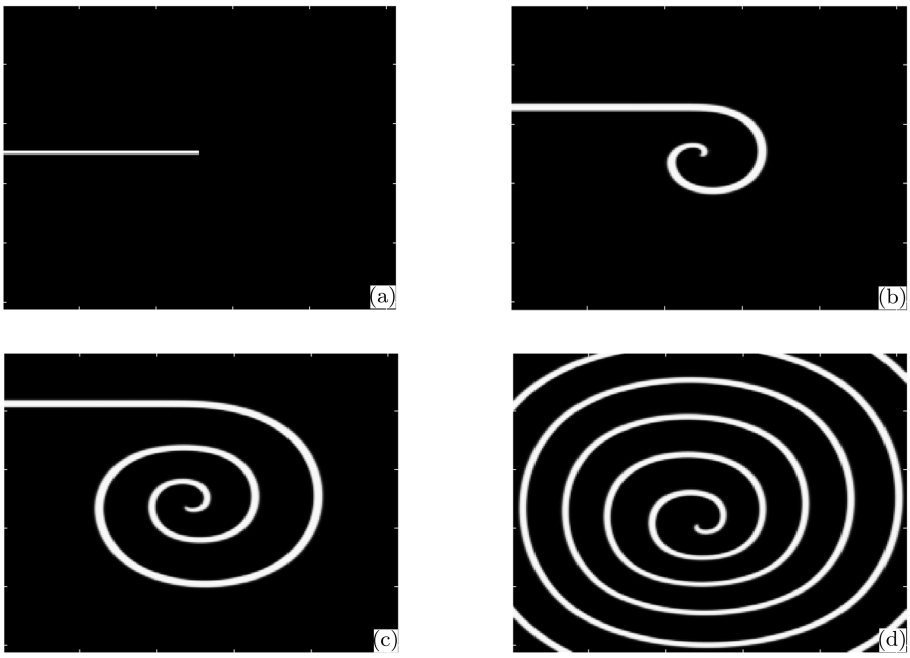


Fig. 1. The evolution of the fast variable u without applying the control variable i_1. (a) The snapshot taken at $t = 0$, (b) $t = 5$, (c) $t = 10$ and (d) $t = 30$.

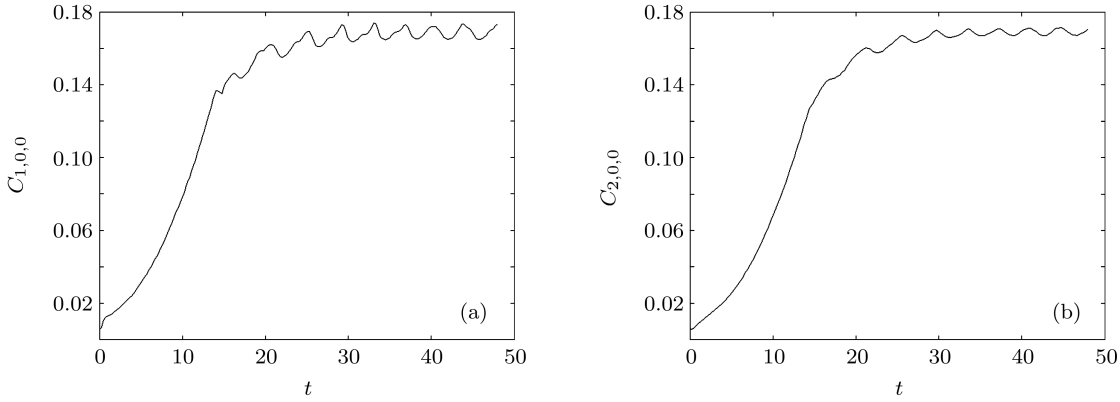


Fig. 2. (a) The evolution of $C_{1,0,0}$ without applying the control variable, where $C_{1,0,0}$ is the spatial mean value of the fast variable u. (b) The evolution of $C_{2,0,0}$ without applying the control variable, where $C_{2,0,0}$ is the spatial mean value of the slow variable v.

Next, under the same initial conditions, we apply the first control strategy to the system. The control constraints are given to be $C_{1,\min} = C_{2,\min} = 10^{-4}$ and $P_{\max} = 10^{-1}$. Figure 3 shows that the pattern is developing towards spiral wave at the beginning, but not long after that, the initial spiral structure was destroyed. Subsequently, from Fig. 4 we could obtain the conclusions with respect to the second control strategy. The control constraints are given to be $C_{1,\min} = C_{2,\min} = 10^{-4}$ and $P_{\max} = 10^{4}$. The patterns in Fig. 4 indicate that they are diffusing gradually and the spiral wave could not exist under the control i_1. The fast variable u would reach its rest state in the end. In each of the two strategies, it is obvious that the spiral wave can be prevented successfully and the possibility of inducing ventricular fibrillation is eliminated, too.

The snapshots of Fig. 5 reveal that the values of $C_{1,0,0}$ decrease quickly from the beginning in the two control strategies. Comparing Fig. 5(a) with Fig. 5(b), we see that they look similar. This is easy to understand. From Eqs. (14) and (15) we can obtain the first control strategy $I_{1,0,0} = K_1(C_{1,k,j}, C_{2,k,j})$ and the second control strategy $I_{1,n,m} = K_{2,n,m}(C_{1,k,j}, C_{2,k,j})$, respectively. For $C_{1,0,0} \gg C_{1,n,m}$ with $n, m \neq 0$, we have $K_{2,0,0}(C_{1,k,j}, C_{2,k,j}) \sim K_1(C_{1,k,j}, C_{2,k,j})$, i.e., the second control strategy is similar to the first one.

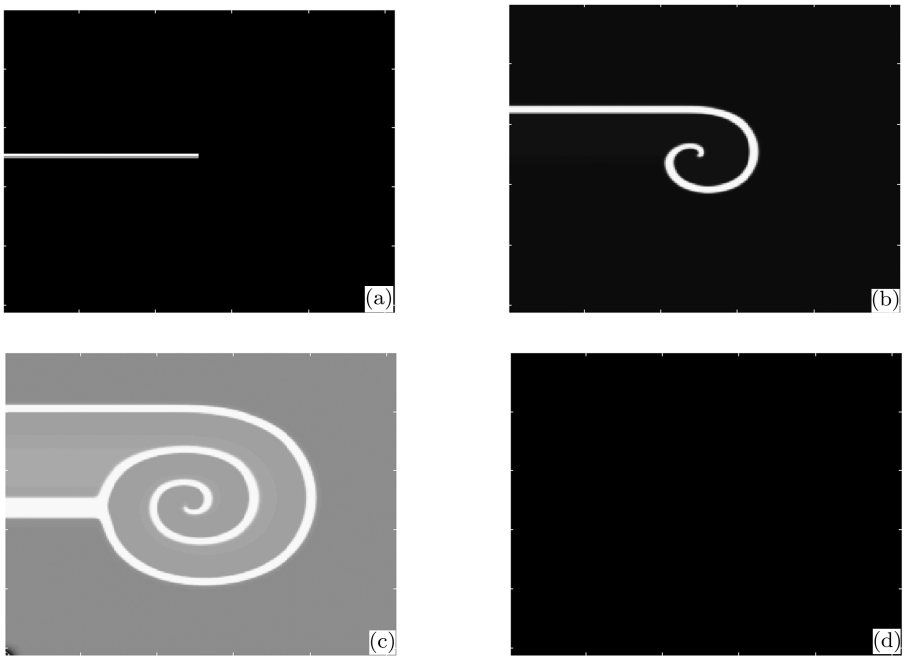


Fig. 3. The evolution of the fast variable u for the first control strategy. The control constraints are given by $C_{1,\min} = C_{2,\min} = 10^{-4}$ and $P_{\max} = 10^{-1}$. (a) The snapshot taken at $t = 0$, (b) $t = 5$, (c) $t = 10$ and (d) $t = 30$.

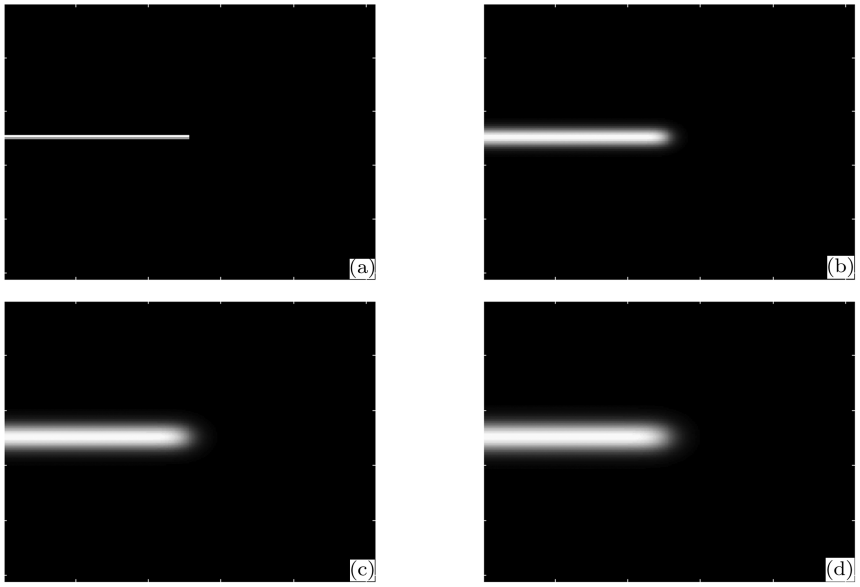


Fig. 4. The evolution of the fast variable u for the second control strategy. The control constraints are given to be $C_{1,\,\min} = C_{2,\,\min} = 10^{-4}$ and $P_{\max} = 10^4$. (a) The snapshot taken at $t = 0$, (b) $t = 5$, (c) $t = 10$ and (d) $t = 30$.

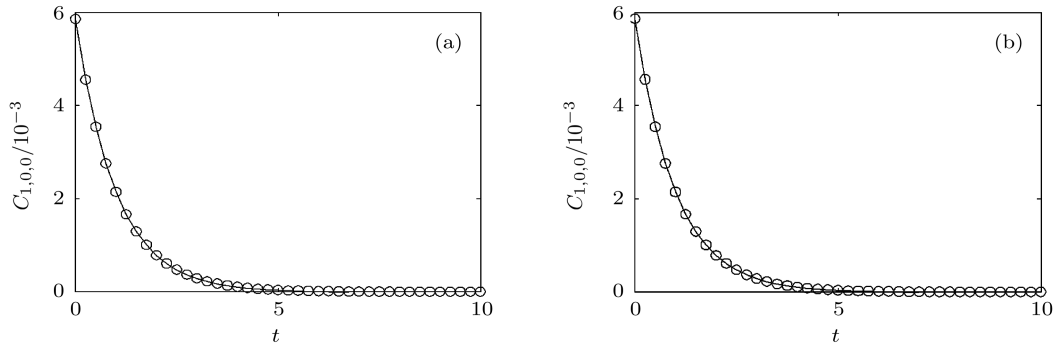


Fig. 5. The evolution of $C_{1,0,0}$, where $C_{1,0,0}$ is the spatial mean value of the fast variable u. (a) For the first control strategy, (b) for the second control strategy.

Finally, the plots in Fig. 6 indicate that the maxima of the control powers in the two control strategies are 5.8×10^{-3} and 8.4×10^3, respectively. Suppose $C_{1,\,0,\,0} \gg C_{1,\,n,\,m}$ with $n,\, m \neq 0$. Substituting Eqs. (14) and (15) into the power expression $P = \int_0^L \int_0^L i_1^2 \mathrm{d}x\mathrm{d}y = \sum_{n,m} I_{1,n,m}^2$, we can readily verify $P_1 \ll P_2$, where P_1 and P_2 are the control powers in the first strategy and in the second strategy, respectively. At the same time, we found that the power-applied time or the response time in the second strategy is only two times unit, which is shorter than that in the first strategy. Hence, the second strategy is suitable for the quick-response control systems.

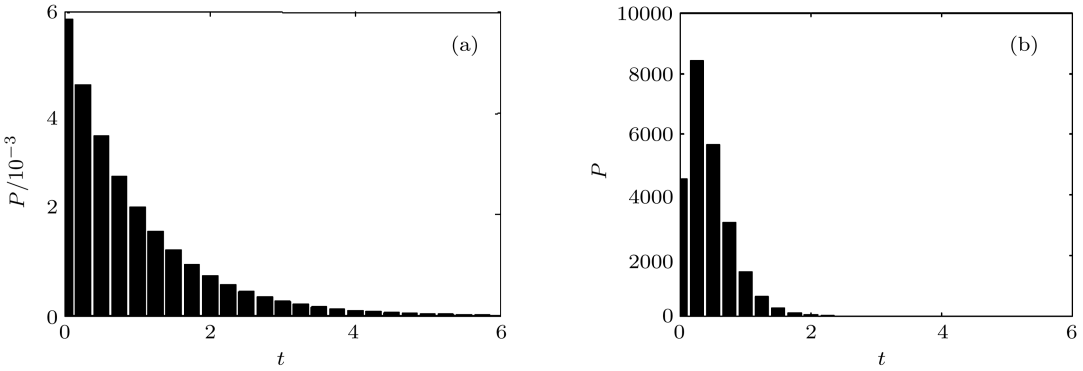


Fig. 6. The evolution of control power P. (a) For the first control strategy, (b) for the second control strategy.

5. Conclusion

In summary, both of the strategies are successful in preventing the appearance of spiral wave or ventricular fibrillation in the cardiac excitable media, and are helpful for designing cardiac devices. Since the second strategy can directly control the components of high frequencies, it is more efficient and robust than the first strategy. Furthermore, the response time in the second strategy is shorter than in the first strategy. Hence, the second strategy is suitable for the quick-response control systems. From the clinical point of view, this strategy is most interesting. For example, the strategy is especially suitable for designing an intelligential quick-response heart pacemaker in human body. In addition, the two control strategies, especially the second one, can be applied to other cardiac models, even to other reaction-diffusion systems.

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