Random-phase-induced chaos in power systems*

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This paper studies how random phase (namely, noise-perturbed phase) effects the dynamical behaviours of a simple model of power system which operates in a stable regime far away from chaotic behaviour in the absence of noise. It finds that when the phase perturbation is weak, chaos is absent in power systems. With the increase of disturbed intensity σ, power systems become unstable and fall into chaos as σ further increases. These phenomena imply that random phase can induce and enhance chaos in power systems. Furthermore, the possible mechanism behind the action of random phase is addressed.

Keywords: random chaos, random phase, stochastic processes, SMIB power systems

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1. Introduction

Phase describes the position of one wave relative to another wave. In the past decades, the significance of phase on the dynamic behaviour of the periodically forced nonlinear systems has received considerable attention. In Ref. [1] the dynamical regime of a single mode laser with modulated losses depended on the phase has been studied experimentally and it was observed that the phase difference between the main driving and the perturbation had a certain influence on the global dynamics of the system; Hu *et al.*[2] showed that phase influences drastically the global dynamics of the Duffing system and proposed a control scheme which makes use of this property where phase acts as the control parameter: phase control of chaos. Kandangath *et al.*[3] investigated how phase induce chaotic attractors in electronic circuits and found that time-varying phase are generally able to drive the circuit into a hierarchy of nonlinear resonant states and eventually into chaos. Zambrano *et al.*[4] studied numerically and experimentally how phase effects the dynamics of periodically driven FitzHugh–Nagumo (FHN) model and showed that phase has two main effects on this model: firstly, it can tame or enhance the spiking regime and, secondly, it can turn chaotic into periodic spiking. More interestingly, very recently, researchers have found that random phase defined by perturbing the phase of system by noise plays a vital role in nonlinear dynamical systems. Lei *et al.*[5] investigated the effects of stochastic phase on two different stochastic systems, namely, the parametrically excited Duffing oscillator and the driven MLC circuit and it is observed that the increase of stochastic phase can either quench chaos or generate chaos in these systems. Yan[6] and Li *et al.*[7] studied the effect of random phase on suppressing chaos in the semiconductor lasers and Duffing's system and showed that the chaotic dynamical behaviour will be suppressed as the noise intensity increases slightly.

Power system is a kind of close coupling, strong non-linear, multivariable dynamic system. With the development of large-scale interconnected power system, electric generation and electric transmission become more and more high-efficient, while the complexity of operation in power system has been increased. Study of chaos in such systems could be of considerable importance from the point of view of avoiding undesirable behaviours such as power blackout. Near these decades around the world, some of the major power systems have taken place losing stabilization even collapse events in succession,[8,9] which incur huge amount of economic loss. So, it is necessary to pay great attention to the reliability and security of power systems. Current researches show that bifurcation and chaos theory are proper and applicable method to study the mechanism of producing cascading failure in power systems. For example, Chiang *et al.*[10] clarified voltage collapse in power systems by static and dynamic models where the static model is employed before a saddle-node bifurcation and the dynamic model is used after the bifurcation; Yu *et al.*[11] studied the routes to chaotic oscillation in power sys-

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tems and found that three routes may cause power systems to chaos, namely, route of cascading period doubling bifurcation (PDB), route of torus bifurcation and route directly initiated by a large disturbance of energy; Ohta and Ueda[12] used limit cycle theory to study bifurcation-induced voltage collapse in an electric power system; Venkatasubramanian and his colleagues investigated complex nonlinear phenomena in a fundamental power system model and showed that when excitation control gains are set high and when the excitation hard-limits are taken into account, this representative power system model undergoes global bifurcations including period-doubling cascades which lead to sustained chaotic behaviour.[13] However, to our best knowledge, in those previous studies, the effect of the random phase on the dynamical behaviours of power systems was not considered. The main aim of this work is to investigate how random phase influences dynamic behaviours in power systems at parameters where chaos does not exist apriori. It is found that when the intensity of random noise σ is small, chaos is absent in power systems. With the increase of noise intensity, power systems become unstable and fall into chaos as σ is further increased. That is, random phase will make it easier to induce and enhance chaos in power systems.

2. The power systems model with random phase

Considering the classical single-machine-infinite bus power systems with random phase, we obtain the equation governing the system (called swing equation) is[14]

$$M\ddot{\theta} + D\dot{\theta} + P_{\max}\sin\theta = P_{\mathrm{m}}\sin(\omega t + \sigma\xi(t)), \quad (1)$$

where θ denotes the angle of generator, M is the moment of inertia, D is the damping constant, P_{m} and ω are amplitude and frequency of the power of the machine, respectively, and $P_{\max}$ is the maximum power of generator, $\xi(t)$ represents a standard Gaussian white noise with Dirac–Delta function as its correlation, namely, $E[\xi(t)] = 0, E[\xi(t)\xi(t+\tau)] = \delta(\tau)$. σ is the strength of the Gaussian white noise. The equation of motion in convenient first order form can be written as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -cx_2 - \beta\sin x_1 + f\sin(\omega t + \sigma\xi(t)), \end{cases} \quad (2)$$

where

$$x_1 = \theta,\ x_2 = \dot{\theta},\ c = D/M,$$
$$\beta = P_{\max}/M,\ f = P_{\mathrm{m}}/M. \quad (3)$$

Parameters c, β and σ are positive constants. The dynamics of the clean power systems (namely, for $\sigma = 0$ in system (1)), which depends on parameter f, has been investigated fully in Ref. [14]. The results of those investigations showed that with the varying parameter f, clean power systems experience complicated behaviours including stability, PDB, chaos, even cascading failure.[14] To make an overall inspection in the dynamical behaviour of the clean power systems, the bifurcation diagram of x_2 versus the parameter f is plotted in Fig. 1. In this paper, in order to study the effect of the random phase on the dynamics of power systems, we take $f = 2.32$ such that the original state of power systems is stable periodic-2 motion as shown in the inset of Fig. 1.

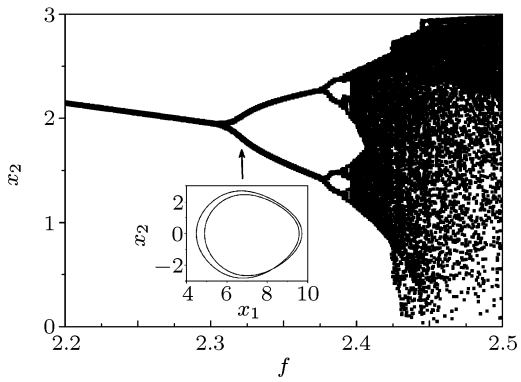


Fig. 1. The bifurcation structure versus the parameter f of power systems without phase perturbations (viz. $\sigma = 0$). The vertical arrow indicates the initial period-2 state of the system brought into chaos by random phase. The systematic parameters are set as $\beta = 1$, $c = 0.5$, and $\omega = 1$.

3. Numerical results

The results discussed hereafter are obtained by varying the disturbed intensity σ. In the numerical analysis, three conventional indexes are adopted to describe the random-phase-induced responses of power systems. The first one is the phase diagram, which is the evolution of a set of trajectories emanating from various initial conditions in the state space. When the solution reaches a stable state, the asymptotic behaviour of the phase trajectories is particularly interesting and the transient behaviour in the systems is neglected. The second one is the power spectrum

(PS), which is utilized to capture the frequency content of a stochastic process and help distinguish between periodic, quasi-periodic and chaotic behaviour for a dynamical system. The last one is the largest Lyapunov exponent (LE),[15] which characterizes the average exponential rate of divergence or convergence of nearby orbits in phase space of the system. Different solutions of a dynamic system, such as fixed points, periodic motions, quasi-periodic motion and chaotic motion can be distinguished by it. If two trajectories start close to one another in phase space, they will move exponentially away from each other for small times on the average. Here, we employ the algorithm presented by Wolf *et al.*[16] to compute the LE. The systematic parameters are set as $f = 2.32$, $\beta = 1$, $c = 0.5$ and $\omega = 1$. We obtain the following results.

Figures 2(a) and 2(b) show the dynamical behaviours of power systems with $\sigma = 0.01$, from which one can clearly see that the systems are still in periodic-2 motion. It is shown that when the phase perturbation is weak, chaos is absent in power systems.

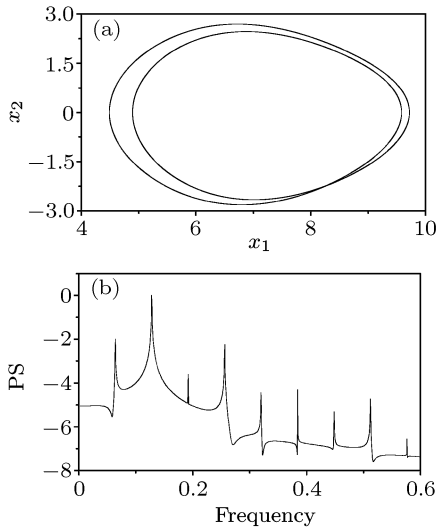


Fig. 2. Period-2 motion in power systems with $f = 2.32$ and $\sigma = 0.01$: (a) phase diagram; (b) the log of power spectra. It is shown that when the intensity of random noise is small, chaos is absent in power systems.

Figures 3(a) and 3(b) show the phase-perturbed-induced dynamics of power systems with $\sigma = 0.08$, where the PS change from period to chaos. It is shown that with the increase of phase perturbation intensity, the chaotic responses appear. Figures 4(a) and 4(b) show the phase-perturbed-induced dynamics of power systems with $\sigma = 0.5$, from which it can be seen that PS transfer to the noise-like spectrum. That is to say, with phase perturbation intensity σ is further increased, phase-perturbed-induced chaos in power systems become stronger. To view globally the dynamical behaviour of power systems over a range of parameter σ and compare simultaneously different types of motion, the largest LE (LLE) of the power systems versus the parameter σ is shown in Fig. 5, where the curve is mostly well above zero. That is, random phase will make it easier to induce and enhance chaos in power systems.

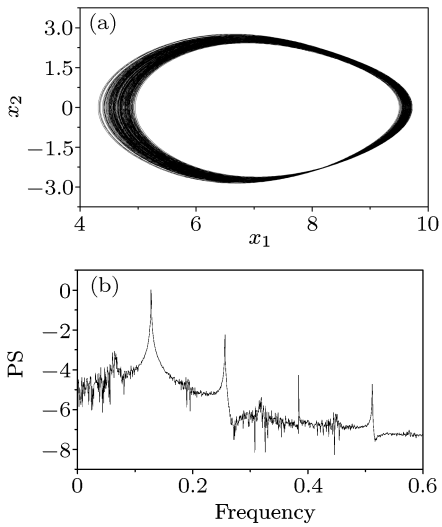


Fig. 3. Random- phase-induced chaotic responses in power systems with $f = 2.32$ and $\sigma = 0.08$: (a) phase diagram; (b) the log of power spectra.

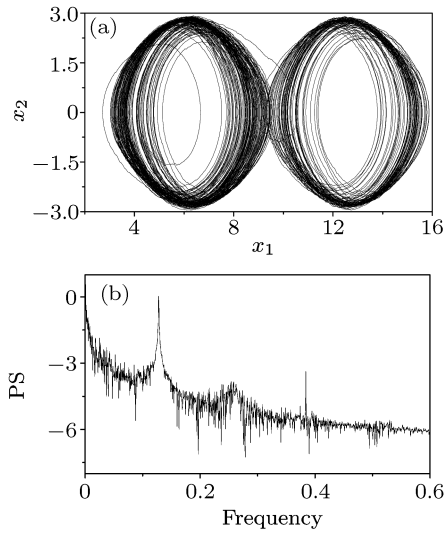


Fig. 4. Random-phase-induced chaotic responses in power systems with $f = 2.32$ and $\sigma = 0.5$: (a) phase diagram; (b) the log of power spectra.

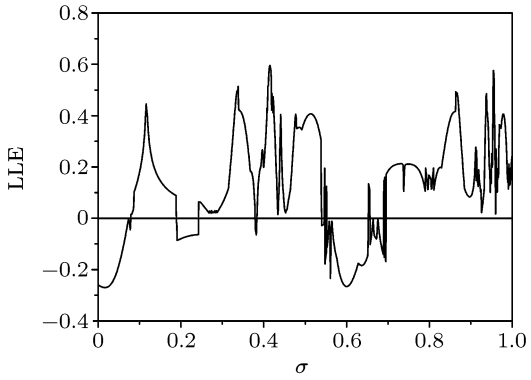


Fig. 5. The LLE of power systems vs σ with $f = 2.32$, where the curve is mostly above zero.

4. The mechanism of random-phase-induced chaos

As it is shown in previous literature, the non-autonomous system with phase perturbation is one of the simplest dynamical systems exhibiting chaos.[17,18] The perturbation destroys integrals of motion and causes emergence of unstable area in phase space. If the criterion of Chirikov[19] holds, then the unstable area has form of chaotic sea with fast diffusion therein. Basing on this knowledge, the possible mechanism of random-phase-induced chaos in power system is addressed as follows. We consider a random phase-perturbed non-autonomous system with the Hamiltonian of the generic form

$$H = H_0(x,p) + \eta H_1(x,p,t), \tag{4}$$

where $0 < \eta \ll 1$, the unperturbed term can be expressed as the sum of kinetic and potential parts:

$$H_0(x,p) = \frac{p^2}{2} + U(x), \tag{5}$$

and the perturbed term is

$$H_1 = \sin(\omega x + \sigma\xi(t)). \tag{6}$$

A trajectory satisfies the coupled equations of motion:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\partial H}{\partial p} = p, \\ \dfrac{\mathrm{d}p}{\mathrm{d}t} = -\dfrac{\partial H}{\partial x} = -\dfrac{\partial U}{\partial x} + \eta\omega\cos\phi, \end{cases} \tag{7}$$

where

$$\phi = \omega x + \sigma\xi(t) \tag{8}$$

is the phase of the perturbation. There exists a resonances in the system (7), when the stationary phase satisfies the resonant condition:

$$\dot{\phi} = \omega p + \sigma \simeq 0. \tag{9}$$

Using Eqs. (7) and (9) we derive the equation

$$\ddot{\phi} + \omega({U_x}' - \eta\omega\cos\phi) = 0, \tag{10}$$

which describes variation of the phase ϕ in the neighbourhood of the resonance. When the resonance occurs, the oscillations of ϕ stays near the stable fixed point

$${U_x}' = \eta\omega\cos(\phi). \tag{11}$$

Confining ϕ by the interval $[-\pi,\pi]$ and solving Eq. (11) we find the unstable fixed point

$$\phi_1 = \arccos\left(\frac{{U_x}'}{\eta\omega}\right), \tag{12}$$

and the stable one

$$\phi_2 = \pi - \arccos\left(\frac{{U_x}'}{\eta\omega}\right). \tag{13}$$

Thus influence of the resonance depends on the duration of the oscillating regime for ϕ. One can distinguish regimes of strong and weak capturing. The strong capturing takes place if ω is large and the perturbation causes additional minima of the potential. Averaging these oscillations yields:

$$x = -\frac{\sigma}{\omega}t. \tag{14}$$

Consequently one expects the strongest resonant response near the points of the stable or unstable equilibrium of the oscillator. The resonance between spatial and temporal oscillations induces bounded chaotic layers in the ranges of the energy, given by Eq. (14). It is shown that the layers formed can be conjuncted, that leads to the emergence of the wide chaotic sea.

5. Conclusion

To gain new insights into the effects of phase perturbations on a dynamical system, in this paper we study the phase-perturbed-induced response in power systems with parameters admitting stable operation. Three indexes are employed to assess the effect of noise intensity σ upon the dynamical behaviours of power systems: the phase diagram as a measure of the evolution of response trajectories; the power spectra as a measure of the periodicities of response signals; the LLE as a measure of chaos. It is found that when the phase perturbation is weak, chaos is absent in power systems. With the increase of disturbed intensity σ, power systems become unstable and fall into chaos as σ further increases. It is well-known that every physical system is subjected to noise in the real world. The study of the nonlinear dynamic system with random perturbed phase is significant. Although this study focuses on a theoretical model of a simple power system, the conclusions might well be of relevance to practical power systems and provide a useful tip for understanding power system's security operation.

References

[1] Meucci R, Gadomski W, Ciofini M and Arecchi F T 1994 *Phys. Rev.* E **49** R2528

[2] Qu Z, Hu G, Yang G and Qin G 1995 *Phys. Rev. Lett.* **74** 1736

[3] Kandangath A, Krishnamoorthy S, Lai Y C and Gaudet J A 2007 *IEEE Trans. Circ. Sys.*-I **54** 1109

[4] Zambrano S, Seoane J M, Marino I P, Sanjuan M A F, Euzzor S, Meucci R and Arecchi F T 2008 *New J. Phys.* **10** 073030

[5] Lei Y, Xu W, Xu Y and Fang T 2004 *Chaos, Solitons and Fractals* **21** 1175

[6] Yan S L 2006 *Acta Phys. Sin.* **55** 5109 (in Chinese)

[7] Li S, Xu W and Li R H 2006 *Acta Phys. Sin.* **55** 1049 (in Chinese)

[8] Yu Y N 1983 *Electric Power System Dynamics* (New York: Academic Press)

[9] Carreras B A, Lynch V E and Dobson I 2002 *Chaos* **12** 985

[10] Chiang H D, Dobson I, Thomas R J, Thorp J S and Fekih-Ahmed L 1990 *IEEE Trans. on Power Systems* **5** 601

[11] Yu Y, Jia H, Li P and Su J 2003 *Electric Power Systems Research* **65** 187

[12] Ohta H and Ueda Y 2002 *Chaos, Solitons and Fractals* **14** 1227

[13] Ji W and Venkatasubramanian V 1999 *IEEE Trans. Circ. Sys.*-I **46** 405

[14] Yamayee Z A and Bala J L 1990 *Electromechanical Energy Devices Power Systems* (New York: John Wiley and Sons)

[15] Gan C 2006 *Nonlinear Dynamics* **45** 305

[16] Wolf A, Swift J R, Swinney H L and Vastano J A 1985 *Physica* D **16** 285

[17] Zaslavsky G M 1998 *Physics of Chaos in Hamiltonian Systems* (Oxford: Academic Press)

[18] Makarov D V and Uleysky M Y 2008 *Communications in Nonlinear Science and Numerical Simulation* **13** 400

[19] Chirikov B V 1979 *Phys. Rep.* **52** 265