Fault tolerant synchronization of chaotic systems based on T–S fuzzy model with fuzzy sampled-data controller*

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In this paper the fault tolerant synchronization of two chaotic systems based on fuzzy model and sample data is investigated. The problem of fault tolerant synchronization is formulated to study the global asymptotical stability of the error system with the fuzzy sampled-data controller which contains a state feedback controller and a fault compensator. The synchronization can be achieved no matter whether the fault occurs or not. To investigate the stability of the error system and facilitate the design of the fuzzy sampled-data controller, a Takagi–Sugeno (T–S) fuzzy model is employed to represent the chaotic system dynamics. To acquire good performance and produce a less conservative analysis result, a new parameter-dependent Lyapunov–Krasovksii functional and a relaxed stabilization technique are considered. The stability conditions based on linear matrix inequality are obtained to achieve the fault tolerant synchronization of the chaotic systems. Finally, a numerical simulation is shown to verify the results.

Keywords: fault tolerant synchronization, fuzzy sampled-data controller, Takagi–Sugeno (T–S) fuzzy model, linear matrix inequality (LMI)

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1. Introduction

Chaotic dynamics can be found in various real processes, for instance, weather, turbulences in liquids, human heart beating, brain activities, etc.[1] Synchronization in chaotic systems has received a great deal of interest of scientists from various research fields since Pecora and Carroll[2] introduced a method to synchronize two identical chaotic systems with different initial conditions. Various control approaches have been used theoretically and experimentally to synchronize the systems, such as adaptive control,[3−6] impulsive control,[7−11] fuzzy-model-based control,[12−17] state feedback control,[18−21] etc. In general, the idea of synchronization is to modify the chaotic behaviour of one chaotic system with the controller to track the chaotic behaviour of the other chaotic system.

Among the control methods, the control of nonlinear systems is a challenging problem because no systematic mathematical tools are available to acquire the necessary and sufficient conditions for guaranteeing their stability and performance. The fuzzy-model-based control approach is a promising approach to the treatment of complex nonlinear systems, because fuzzy-model can express a nonlinear system as a weighted sum of some simple linear subsystems. Recently, the fuzzy-model based control approach has been employed to synchronize chaotic systems,[12−16] which is a useful application in secure communication systems. Based on Takagi–Sugeno (T–S) fuzzy model, in Ref. [12] a new fuzzy observer to achieve the generalized projective synchronization of Chua chaotic system was presented. By using a fuzzy state feedback controller, the synchronization of chaotic systems based on the T–S fuzzy model was investigated in Refs. [13] and [14]. Two fuzzy controllers were designed via parallel distributed compensation (PDC) and exact linearization techniques to synchronize two identical hyperchaotic systems which are represented by the T–S fuzzy model.[15]

But when a fault happens in chaotic systems, the methods of synchronizing chaotic systems in Refs. [3]–[21] are invalid. In order to increase the safety and the reliability of synchronization systems when a fault happens, fault tolerant synchronization is essential to promise the synchronization of chaotic systems.[22] The method of fault tolerant synchronization is to design a fault tolerant controller to achieve the synchro-

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nization no matter whether a fault happens or not. To the best of the authors' knowledge, the problem of fault tolerant synchronization based on fuzzy model is still an open problem and seldom researched.

In practical and modern control systems, computers which play an important role in our lives are usually used as digital controllers which are used to produce a discrete-time control signal to control continuous-time systems.[23−25] So it is essential to use sampled-data controllers to achieve the goal of control. Because when a sampled-data controller is employed, compared with a continuous-time controller, the sampled-data controller can be implemented by a microcontroller or a digital computer to lower the implementation cost and time. However, discontinuity introduced by the sampling activity complicates the dynamics of the system compared with that of the pure continuous-time control system.

In this paper, the fault tolerant synchronization of chaotic systems is handled based on the fuzzy-model-based approach with a fuzzy sampled-data controller. More specifically, a T–S fuzzy model with simple rules is employed to describe the dynamical behaviours of the chaotic systems, and the fuzzy sampled-data controller which contains a state feedback controller and a fault compensator is designed to promise that the fault tolerant synchronization can be acquired. The fuzzy sample-data controller can be solved efficiently by linear matrix inequality (LMI) and the LMI-based fault tolerant synchronization conditions have less conservatism for using a new parameter-dependent Lyapunov–Krasovskii functional and relaxed stabilization technique.

The rest of the paper is organized as follows. In Section 2, the fuzzy model and the fuzzy sampled-data controller are presented to implement the fault tolerant synchronization for the chaotic systems. In Section 3, LMI-based conditions are derived to guarantee the global asymptotical stability and performance of the error system. In Section 4, a simulation example is given to illustrate the effectiveness of the proposed approach. A conclusion is drawn in Section 5.

2. Preliminaries and problem formulations

2.1. Fuzzy chaotic model

A kind of chaotic system can be described by the following fuzzy IF–THEN rules. The ith rule is in the following form:

Rule i: If $\theta_1(x(t))$ is M_1^i, and, ... , and $\theta_m(x(t))$ is M_m^i

$$\text{Then} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i \boldsymbol{x}(t), \tag{1}$$

where i is the number of IF–THEN rules, $i = 1, 2, \ldots, p$, $\theta_\alpha(x(t))$ is premise variable, M_α^i is the fuzzy term of rule i corresponding to function $\theta_\alpha(x(t))$, $\alpha = 1, 2, \ldots, m$, $\boldsymbol{x}(t) \in R^{n\times 1}$ is the system state. $\boldsymbol{A}_i$ is a known constant matrix with an appropriate dimension.

By using a centre-average defuzzier, product inference and singleton fuzzier, the overall master chaotic systems can be described by

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{p} w_i(x(t)) \boldsymbol{A}_i \boldsymbol{x}(t), \tag{2}$$

where

$$w_i(x(t)) = \frac{\beta_i(\theta(x(t)))}{\sum_{j=1}^{p} \beta_j(\theta(x(t)))},$$

$$\beta_i(\theta(x(t))) = \prod_{j=1}^{m} M_j^i(\theta_j(x(t))),$$

and

$$\boldsymbol{\theta}(\boldsymbol{x}(t)) = \left[\, \theta_1(x(t)) \;\; \theta_2(x(t)) \;\; \ldots \;\; \theta_m(x(t)) \,\right],$$

with $M_j^i(\theta_j(x(t)))$ being the degree of membership of $\theta_j(x(t))$ in M_j^i. Then we can know $\beta_i(\theta(x(t))) \geq 0$ and $\sum_{j=1}^{p} \beta_j(\theta(x(t))) > 0$. Therefore, for all values of t, we have the fuzzy weighting function $w_i(x(t)) \in [\,0\;\; 1\,]$ and $\sum\limits_{i=1}^{p} w_i(x(t)) = 1$.

Similarly, the dynamics of the slave chaotic systems can be represented by a fuzzy model with q fuzzy rules in the form of Eq. (2). The dynamics of the slave chaotic system can be represented as

$$\dot{\hat{\boldsymbol{x}}}(t) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))(\hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t) + \boldsymbol{u}(t)), \tag{3}$$

where $\hat{\boldsymbol{x}}(t) \in R^{n\times 1}$ is the system state vector; $\hat{\boldsymbol{A}}_j$ is a known constant system matrix with an appropriate dimension; $\hat{w}_j(\hat{x}(t))$ is the fuzzy weighting function: $\hat{w}_j(\hat{x}(t)) \in [\,0\;\; 1\,]$ and $\sum\limits_{j=1}^{q} \hat{w}_j(\hat{x}(t)) = 1$ for all values of t ; $\boldsymbol{u}(t)$ is the input vector.

2.2. Fuzzy sampled-data controller

A fuzzy sampled-data controller with q fuzzy rules is designed based on the fuzzy model of nonlinear plant for the purpose of master–slave synchronization.

Rule j: If $\hat{\theta}_1(\hat{x}(t_{\rm sd}))$ is N_1^j, and, ..., and $\hat{\theta}_n(\hat{x}(t_{\rm sd}))$ is N_n^j, then

$$\begin{aligned} &\boldsymbol{u}(t) = \boldsymbol{K}_j \boldsymbol{e}(t_{\rm sd}), \quad j = 1, 2, \ldots, q, \\ &t_{\rm sd} \le t \le t_{\rm sd+1}, \end{aligned} \tag{4}$$

where $\boldsymbol{K}_j \in R^{m\times n}$ is the controller gain to be designed; $\boldsymbol{e}(t_{\rm sd}) = \boldsymbol{x}(t_{\rm sd}) - \hat{\boldsymbol{x}}(t_{\rm sd})$; $t_{\rm sd}$ is the sampling time; N_β^j is the fuzzy term of rule j corresponding to function $\hat{\theta}_\beta(\hat{x}(t_{\rm sd}))$, $\beta = 1, 2, \ldots, n$. For the above sampled-data fuzzy controller, we have assumed that a zero-order holder is placed behind each subcontroller.

Throughout this paper we make the following assumptions.

Assumption 1 It is assumed that the distance between any two sampling instants is bounded by h that is,

$$t_{\rm sd+1} - t_{\rm sd} \le h, \quad \forall t_{\rm sd} \ge 0; \tag{5}$$

Assumption 2 For Eqs. (2) and (3), the system states $\boldsymbol{x}(t)$ and $\hat{\boldsymbol{x}}(t)$ are bounded in a bounded compact set.

Then the sampled-data fuzzy controller is given as

$$\boldsymbol{u}(t) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t_{\rm sd}))(\boldsymbol{K}_j \boldsymbol{e}(t_{\rm sd})) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t_{\rm sd}))\boldsymbol{K}_j \boldsymbol{e}(t-(t-t_{\rm sd})) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t_{\rm sd}))\boldsymbol{K}_j \boldsymbol{e}(t-\tau(t)), \tag{6}$$

where $\tau(t) = t - t_{\rm sd} \le h$ for $t_{\rm sd} \le t \le t_{\rm sd+1}$.

Now from Eqs. (2), (3) and (6), the error system can be represented as

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t) \\ &= \sum_{i=1}^{p} w_i(x(t))\boldsymbol{A}_i \boldsymbol{x}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))(\hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t) + \boldsymbol{u}(t)) \\ &= \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\hat{\boldsymbol{A}}_j \boldsymbol{x}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t) + \sum_{i=1}^{p} w_i(x(t))\boldsymbol{A}_i \boldsymbol{x}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\hat{\boldsymbol{A}}_j \boldsymbol{x}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\boldsymbol{u}(t) \\ &= \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t)) \left(\hat{\boldsymbol{A}}_j \boldsymbol{e}(t) - \boldsymbol{u}(t)\right) + \boldsymbol{m}_{\rm e}(t) = \sum_{i=1}^{q} \hat{w}_i(\hat{x}(t)) \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t_{\rm sd})) \left(\hat{\boldsymbol{A}}_i \boldsymbol{e}(t) - \boldsymbol{K}_j \boldsymbol{e}(t-\tau(t)\right) + \boldsymbol{m}_e(t) \\ &= \boldsymbol{A}(t)\boldsymbol{e}(t) - \boldsymbol{K}(t)\boldsymbol{e}(t-\tau(t)) + \boldsymbol{m}_{\rm e}(t), \end{aligned} \tag{7}$$

where $\boldsymbol{A}(t) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\hat{\boldsymbol{A}}_j$, $\boldsymbol{K}(t) = \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t_{\rm sd}))\boldsymbol{K}_j$ and $\boldsymbol{m}_{\rm e}(t) = \sum_{i=1}^{p} w_i(x(t))\boldsymbol{A}_i \boldsymbol{x}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{x}(t))\hat{\boldsymbol{A}}_j \boldsymbol{x}(t)$.

The objective of master–slave synchronization is to drive the states of the slave chaotic system (3) to follow those of the master chaotic system (2). If $\boldsymbol{e}(t) = 0$ as $t \to \infty$, we can achieve the master–slave synchronization.

2.3. Fault tolerant master–slave synchronization

In the case of the component failure in a circuit or the changes of model parameters caused by malfunctions in sensors, the fault will happen. Here we consider the case where the fault happens in the master system as described below:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{p} w_i(x(t))\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_{\rm f} f(t), \tag{8}$$

where $f(t) \in R^{n_1}$ is the fault, $\boldsymbol{B}_{\rm f}$ is a constant matrix with an appropriate dimension.

In this paper, we use a general model to represent fault $f(t)$. It is assumed that fault $f(t)$ is the output of the following system:[22]

$$\dot{\boldsymbol{x}}_{\rm f}(t) = \boldsymbol{A}_{\rm wf}\boldsymbol{x}_{\rm f}(t) + \boldsymbol{B}_{\rm wf} f_0(t), \tag{9}$$

$$f(t) = \boldsymbol{C}_{\rm wf}\boldsymbol{x}_{\rm f}(t), \quad x_{\rm f}(0) = 0, \tag{10}$$

where $\boldsymbol{x}_{\rm f}(t) \in R^{n_{\rm f}}$ is the state of fault system; $\boldsymbol{A}_{\rm wf}$, $\boldsymbol{B}_{\rm wf}$ and $\boldsymbol{C}_{\rm wf}$ are matrices obtained from the prior knowledge about the possible fault; $f_0(t) = \sum_{i=1}^{n} \beta_i \delta(t - t_{\rm fi})$ with $\delta(t)$ being the impulse function, $t_{\rm fi}$ being the time when the fault happens, β_i being the parameter that can determine the value of fault, and n being a constant.

Remark 1 From Ref. [22] it follows that the fault model described by Eqs.(9) and (10) can be used to model stepwise fault, slow-drifting fault, and sine wave fault.

If we use the original sampled-data fuzzy controller to acquire the master–slave synchronization, we fail to do so for the fault $f(t)$. So we introduce a new sampled-data fuzzy controller as follows:

$$\boldsymbol{u}(t) = \boldsymbol{K}(t)\boldsymbol{e}(t-\tau(t)) + \boldsymbol{B}_{\rm f}\hat{f}(t). \tag{11}$$

The first term on the right-hand side of the above expression part of $\boldsymbol{u}(t)$ represents a state feedback controller and the second term refers to a fault compensator, where $\hat{f}(t)$ is an estimation of fault $f(t)$. $\hat{f}(t)$ is given by the following observer-based fault estimator:

$$\dot{\hat{\boldsymbol{x}}}_{\rm f}(t) = \boldsymbol{A}_{\rm wf}\hat{\boldsymbol{x}}_{\rm f}(t) + \boldsymbol{v}(t), \tag{12}$$

$$\hat{f}(t) = \boldsymbol{C}_{\rm wf}\hat{\boldsymbol{x}}_{\rm f}(t), \tag{13}$$

where $\hat{\boldsymbol{x}}_{\rm f}(t) \in R^{n_{\rm f}}$ is an estimation of the fault state $\boldsymbol{x}_{\rm f}(t)$, and $\boldsymbol{v}(t)$ is a sampled-data controller and given as follows:

$$\begin{aligned}\boldsymbol{v}(t) &= \bar{\boldsymbol{K}}\boldsymbol{e}(t_{\rm sd}) = \bar{\boldsymbol{K}}\boldsymbol{e}(t-(t-t_{\rm sd}))\\ &= \bar{\boldsymbol{K}}\boldsymbol{e}(t-\tau(t)),\end{aligned} \tag{14}$$

where $\bar{\boldsymbol{K}}$ is the gain of sampled-data controller $\boldsymbol{v}(t)$ to be designed.

Define $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$, and $\boldsymbol{e}_{\rm f}(t) = \boldsymbol{x}_{\rm f}(t) - \hat{\boldsymbol{x}}_{\rm f}(t)$

From Eqs. (7)–(14), we can know

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \boldsymbol{A}(t)\boldsymbol{e}(t) - \boldsymbol{K}(t)\boldsymbol{e}(t-\tau(t))\\ &\quad + \boldsymbol{B}_{\rm f}\boldsymbol{C}_{\rm wf}\boldsymbol{e}_{\rm f}(t) + \boldsymbol{m}_{\rm e}(t),\end{aligned} \tag{15}$$

$$\dot{\boldsymbol{e}}_{\rm f}(t) = \boldsymbol{A}_{\rm wf}\boldsymbol{e}_{\rm f}(t) - \bar{\boldsymbol{K}}\boldsymbol{e}(t-\tau(t)) + \boldsymbol{B}_{\rm wf}f_0(t). \tag{16}$$

When $f_0(t) \neq 0$, if Eqs. (15) and (16) are globally asymptotically stable, as $\lim\limits_{t\to\infty}\boldsymbol{e}(t) = 0$ and $\lim\limits_{t\to\infty}\boldsymbol{e}_{\rm f}(t) = 0$, we can know that the chaotic system can realize the fault tolerant synchronization. So in this paper, our main purpose is to design the sampled-data fuzzy controller (11) to guarantee that the state of slave chaotic system (3) can follow those of master chaotic system of Eq. (8) when the fault happens in the master chaotic system.

Remark 2 According to Eq. (7), we can know that the $\boldsymbol{m}_{\rm e}(t)$ is bounded as $\sum\limits_{i=1}^{p} w_i(x(t))$, $\sum\limits_{j=1}^{q} \hat{w}_j(\hat{x}(t))$, $\boldsymbol{x}(t)$ and $\hat{\boldsymbol{x}}(t)$. And there exists a positive scalar $\tilde{\rho}$ that satisfies the following optimization problem:

$$\tilde{\rho} = \min_{\boldsymbol{x}(t),\hat{\boldsymbol{x}}(t)\in R} \frac{\boldsymbol{e}^{\rm T}(t)\boldsymbol{e}(t)}{\boldsymbol{m}_{\rm e}^{\rm T}(t)\boldsymbol{m}_{\rm e}(t)}, \tag{17}$$

where R denotes the real space. In practice, the value of $\tilde{\rho}$ can be estimated by the prior knowledge of experience.

We define a cost function J to measure the influence of fault $f(t)$ on both the state error of chaotic systems and the state error of estimation fault as follows:

$$J = \int_0^\infty \boldsymbol{Z}^{\rm T}(t)\boldsymbol{Z}(t){\rm d}t = \|\boldsymbol{Z}(t)\|_2, \tag{18}$$

where $\boldsymbol{Z}(t) = \boldsymbol{F}[\,\boldsymbol{e}^{\rm T}(t)\ \boldsymbol{e}_{\rm f}^{\rm T}(t)\,]^{\rm T}$, $F > 0$ is a diagonal matrix.

Let $\boldsymbol{\eta}(t) = [\,\boldsymbol{e}^{t}(t)\ \boldsymbol{e}_{\rm f}^{\rm T}(t)\,]^{\rm T}$, we can have the following equation:

$$\begin{aligned}\dot{\boldsymbol{\eta}}(t) &= \bar{\boldsymbol{A}}\boldsymbol{\eta}(t) - \boldsymbol{B}\boldsymbol{H}\boldsymbol{\eta}(t-\tau(t))\\ &\quad + \bar{\boldsymbol{B}}_{\rm wf}f_0(t) + \boldsymbol{D}\boldsymbol{m}_{\rm e}(t),\end{aligned} \tag{19}$$

where

$$\bar{\boldsymbol{A}} = \begin{bmatrix}\boldsymbol{A}(t) & \boldsymbol{B}_{\rm f}\boldsymbol{C}_{\rm wf}\\ 0 & \boldsymbol{A}_{\rm wf}\end{bmatrix},\quad \boldsymbol{B} = \begin{bmatrix}\boldsymbol{K}(t)\\ \bar{\boldsymbol{K}}\end{bmatrix},$$

$$\bar{\boldsymbol{B}}_{\rm wf} = \begin{bmatrix}0\\ \boldsymbol{B}_{\rm wf}\end{bmatrix},\quad \boldsymbol{D} = \begin{bmatrix}\boldsymbol{I}\\ 0\end{bmatrix},$$

and

$$\boldsymbol{H} = [\,\boldsymbol{I}\ 0\,].$$

The following work is to design the sampled-data fuzzy controller which contains two parts: the state feedback controller and the fault compensator. The fuzzy sampled-data controller can guarantee that system (19) is globally asymptotically stable for $f_0(t) = 0$ and for a prescribed $\gamma > 0$ arrives at the level of given performance for $f_0(t) \neq 0$ as follows:

$$\|\boldsymbol{Z}(t)\|_2 \leq \gamma\,\|f_0(t)\|_2 \quad \text{for} \quad f_0(t) \in L_2[\,0\ \infty\,). \tag{20}$$

The control method is shown in Fig. 1.

Remark 3 In this paper, the fault that we have considered is kept within a bound and happens only in an interval time $[\,t_{\rm f}\ t\,]$ where $t < \infty$. Using the above technique, the problem of fault tolerant synchronization of master–slave chaotic system is transformed into the problem of stabilizing the underlying error system (19) with bounded model error signals. Both the parameter-dependent Lyanpunov–Krasovskii functional and the new relaxed quadratic stabilization technique for the fuzzy error system will be proposed in the process of system synthesis to acquire the fault tolerant synchronization.

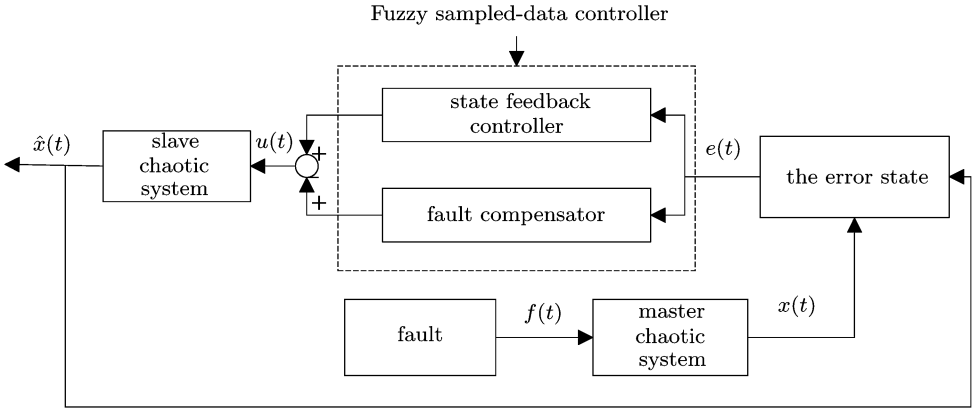


Fig. 1. Block diagram of the fault tolerant synchronization.

The following Lemma will be useful for deriving our main results.

Lemma 1[26] For any matrices $\boldsymbol{X}$, $\boldsymbol{Y}$, each with an appropriate dimension, and a real symmetric positive definite matrix $\boldsymbol{R}$, the following inequality holds:

$$\boldsymbol{X}\boldsymbol{Y}+\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}}\leq\boldsymbol{X}\boldsymbol{R}\boldsymbol{X}^{\mathrm{T}}+\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{Y}. \tag{21}$$

3. Main results

In this section, we are concerned with designing the sampled-date fuzzy controller to make system (19) globally asymptotically stable when the fault is equal to zero and arrive at the level of given performance to achieve the fault tolerant synchronization when a fault happens. The following results are obtained.

Theorem 1 Given the maximum sampling interval h, the performance level $\gamma>0$ and positive scalar $\rho>\tilde{\rho}$, if there exist the symmetric positive definite matrices $\boldsymbol{P}>0$, $\boldsymbol{N}(t)>0$, $\boldsymbol{Q}(t)>0$, $\boldsymbol{Z}(t)>0$ and matrices $\boldsymbol{M}(t)$, such that the following inequalities hold:

$$\begin{bmatrix}\boldsymbol{\Theta} & \boldsymbol{G}^{\mathrm{T}} & \boldsymbol{M}(t)\\ * & -h^{-1}\boldsymbol{Q}(t) & 0\\ * & * & -h^{-1}\boldsymbol{Z}(t)\end{bmatrix}<0, \tag{22}$$

$$\boldsymbol{Z}(t)<\boldsymbol{Q}(t), \tag{23}$$

where

$$\boldsymbol{\Theta}(t)=\begin{bmatrix}\boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} & \boldsymbol{\Theta}_{13} & \boldsymbol{\Theta}_{14}\\ * & \boldsymbol{\Theta}_{22} & \boldsymbol{\Theta}_{23} & 0\\ * & * & \boldsymbol{\Theta}_{33} & 0\\ * & * & * & -\gamma^2\boldsymbol{I}\end{bmatrix},$$

$$\begin{aligned}
\boldsymbol{\Theta}_{11}&=\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}+\boldsymbol{M}_1^{\mathrm{T}}(t)+\boldsymbol{M}_1(t)\\
&\quad+\boldsymbol{H}^{\mathrm{T}}\boldsymbol{N}(t)\boldsymbol{H},\\
\boldsymbol{\Theta}_{12}&=-\boldsymbol{P}\boldsymbol{B}\boldsymbol{H}-\boldsymbol{M}_1(t)+\boldsymbol{M}_2^{\mathrm{T}}(t),\\
\boldsymbol{\Theta}_{13}&=\boldsymbol{P}\boldsymbol{D}+\boldsymbol{M}_3^{\mathrm{T}}(t),\quad \boldsymbol{\Theta}_{14}=\boldsymbol{P}\bar{\boldsymbol{B}}_{\mathrm{wf}},\\
\boldsymbol{\Theta}_{22}&=-\boldsymbol{M}_2(t)-\boldsymbol{M}_2^{\mathrm{T}}(t),\quad \boldsymbol{\Theta}_{23}=-\boldsymbol{M}_3^{\mathrm{T}}(t),\\
\boldsymbol{\Theta}_{33}&=-\rho\boldsymbol{N}(t),\\
\boldsymbol{G}&=\begin{bmatrix}\boldsymbol{Q}(t)\bar{\boldsymbol{A}} & -\boldsymbol{Q}(t)\boldsymbol{B}\boldsymbol{H} & \boldsymbol{Q}(t)\boldsymbol{D} & \boldsymbol{Q}(t)\bar{\boldsymbol{B}}_{\mathrm{wf}}\end{bmatrix},
\end{aligned}$$

and

$$\boldsymbol{M}(t)=\left(\boldsymbol{M}_1^{\mathrm{T}}(t),\boldsymbol{M}_2^{t}(t),\boldsymbol{M}_3^{\mathrm{T}}(t),0\right)^{\mathrm{T}},$$

then system (19) is globally asymptotically stable and reaches the level of given performance.

Proof

Under the condition of Theorem 1, we first show that system (19) is globally asymptotically stable with $f_0(t)=0$. For this purpose, we define the following parameter-dependent Lyapunov–Krasovskii candidate for erroneous system (19):

$$\begin{aligned}
V(t)&=\boldsymbol{\eta}(t)^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta}(t)\\
&\quad+\int_{-h}^{0}\int_{t+\beta}^{t}\dot{\boldsymbol{\eta}}(s)^{\mathrm{T}}\boldsymbol{Q}(s)\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\mathrm{d}\beta.
\end{aligned} \tag{24}$$

The time derivative of $V(t)$ along the solution of Eq. (24) with $f_0(t)=0$ is given as follows:

$$\begin{aligned}
\dot{V}(t)&=2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{\eta}}(t)+h\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\dot{\boldsymbol{\eta}}(t)\\
&\quad-\int_{t-h}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\boldsymbol{Q}(s)\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\\
&\leq 2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{\eta}}(t)+h\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\dot{\boldsymbol{\eta}}(t)\\
&\quad-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\boldsymbol{Q}(s)\dot{\boldsymbol{\eta}}(s)\mathrm{d}s.
\end{aligned} \tag{25}$$

According to Newton–Leibniz formula, we have

$$2\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}(t)\left(\boldsymbol{\eta}(t)-\boldsymbol{\eta}(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\right)=0, \tag{26}$$

where $\tilde{\boldsymbol{M}}(t)=\left(\boldsymbol{M}_1^{\mathrm{T}}(t),\ \boldsymbol{M}_2^{\mathrm{T}}(t),\ \boldsymbol{M}_3^{\mathrm{T}}(t)\right)^{\mathrm{T}}$ and $\tilde{\boldsymbol{\xi}}(t)=[\,\boldsymbol{\eta}^{\mathrm{T}}(t)\ \boldsymbol{\eta}(t-\tau(t))^{\mathrm{T}}\ \boldsymbol{m}_{\mathrm{e}}^{t}(t)\,]^{\mathrm{T}}$.

From Remark 2, the following inequality holds:

$$\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{N}(t)\boldsymbol{e}(t)-\rho\boldsymbol{m}_{\mathrm{e}}^{\mathrm{T}}(t)\boldsymbol{N}(t)\boldsymbol{m}_{\mathrm{e}}(t)>0. \tag{27}$$

From inequality (27), we can know the following inequality:

$$\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}}\boldsymbol{N}(t)\boldsymbol{H}\boldsymbol{\eta}(t)-\rho\boldsymbol{m}_{\mathrm{e}}^{\mathrm{T}}(t)\boldsymbol{N}(t)\boldsymbol{m}_{\mathrm{e}}(t)>0. \tag{28}$$

Combining expressions (25), (26) and (28), we can obtain

$$\begin{aligned}\dot{V}(t)<{}&2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\Big(\bar{\boldsymbol{A}}\boldsymbol{\eta}(t)-\boldsymbol{B}\boldsymbol{H}\boldsymbol{\eta}(t-\tau(t))+\boldsymbol{D}\boldsymbol{m}_{\mathrm{e}}(t))+h\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\dot{\boldsymbol{\eta}}(t)-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\boldsymbol{Q}(s)\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\\&+\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}}\boldsymbol{N}(t)\boldsymbol{H}\boldsymbol{\eta}(t)-\rho\boldsymbol{m}_{\mathrm{e}}^{\mathrm{T}}(t)\boldsymbol{N}(t)\boldsymbol{m}_{\mathrm{e}}(t)+2\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}(t)\left(\boldsymbol{\eta}(t)-\boldsymbol{\eta}(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\right)\Big)\\<{}&\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\left(\tilde{\boldsymbol{\Theta}}+\tilde{\boldsymbol{\pi}}(t)+h\tilde{\boldsymbol{M}}(t)\boldsymbol{Z}(t)^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}(t)\right)\tilde{\boldsymbol{\xi}}(t)-\int_{t-h}^{t}\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}(t)\boldsymbol{Z}(t)^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}(t)\tilde{\boldsymbol{\xi}}(t)\mathrm{d}s\\&+\int_{t-\tau(t)}^{t}\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}(t)\boldsymbol{Q}^{-1}(s)\tilde{\boldsymbol{M}}^{\mathrm{T}}(t)\tilde{\boldsymbol{\xi}}(t)\mathrm{d}s\\&-\int_{t-\tau(t)}^{t}[\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}(t)+\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{Q}(s)]\boldsymbol{Q}^{-1}(s)[\tilde{\boldsymbol{M}}^{\mathrm{T}}(t)\tilde{\boldsymbol{\xi}}(t)+\boldsymbol{Q}(s)\boldsymbol{\eta}(t)]\mathrm{d}s,\end{aligned} \tag{29}$$

where $\tilde{\boldsymbol{\Theta}}=\begin{bmatrix}\tilde{\boldsymbol{\Theta}}_{11}&\boldsymbol{\Theta}_{12}&\boldsymbol{\Theta}_{13}*&\boldsymbol{\Theta}_{22}&\boldsymbol{\Theta}_{23}*&*&\boldsymbol{\Theta}_{33}\end{bmatrix}$, $\tilde{\boldsymbol{\pi}}(t)=\begin{bmatrix}\bar{\boldsymbol{A}}^{\mathrm{T}}\\\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\\\boldsymbol{D}^{\mathrm{T}}\end{bmatrix}h\boldsymbol{Q}(t)\begin{bmatrix}\bar{\boldsymbol{A}}^{\mathrm{T}}\\\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\\\boldsymbol{D}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, and $\tilde{\boldsymbol{\Theta}}_{11}=\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{M}_1^{\mathrm{T}}(t)+\boldsymbol{M}_1(t)+\boldsymbol{H}^{\mathrm{T}}\boldsymbol{N}(t)\boldsymbol{H}$. We can know that the last part on the right-hand side in inequality (29) is less than 0, and according to inequality (23), we have

$$\dot{V}(t)<\tilde{\boldsymbol{\xi}}^{\mathrm{T}}(t)(\tilde{\boldsymbol{\Theta}}+\tilde{\boldsymbol{\pi}}(t)+\boldsymbol{h}\tilde{\boldsymbol{M}}(t)\boldsymbol{Z}(t)^{-1}\tilde{\boldsymbol{M}}^{t}(t))\tilde{\boldsymbol{\xi}}(t). \tag{30}$$

Now, applying the Schur complement formula to inequality (22), we have

$$\tilde{\boldsymbol{\Theta}}+\tilde{\boldsymbol{\pi}}(t)+\boldsymbol{h}\tilde{\boldsymbol{M}}(t)\boldsymbol{Z}(t)^{-1}\tilde{\boldsymbol{M}}^{\mathrm{T}}(t)<0. \tag{31}$$

This together with (30) implies $\dot{V}(t)<0$ for all $\tilde{\boldsymbol{\xi}}(t)\neq 0$. Therefore, system (19) with $f_0(t)=0$ is globally asymptotically stable.

Now the performance of the system $\|\boldsymbol{Z}(t)\|_2\leq\gamma\|f_0(t)\|_2$ is considered for all $f_0(t)\neq 0\in L_2[\,0\ \infty\,)$. Note that

$$\begin{aligned}&\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{Z}(t)-\gamma^2f_0^{\mathrm{T}}(t)f_0(t)+\dot{V}(t)\\\leq{}&\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{\Theta}(t)+\boldsymbol{\pi}(t)+h\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}^{\mathrm{T}}(t))\boldsymbol{\xi}(t)-\int_{t-\tau(t)}^{t}\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}^{\mathrm{T}}(t)\boldsymbol{\xi}(t)\mathrm{d}s\\&+\int_{t-\tau(t)}^{t}\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Q}^{-1}(s)\boldsymbol{M}^{\mathrm{T}}(t)\boldsymbol{\xi}(t)\mathrm{d}s\\&-\int_{t-\tau(t)}^{t}[\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)+\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{Q}(s)]\boldsymbol{Q}^{-1}(s)[\boldsymbol{M}^{\mathrm{T}}(t)\boldsymbol{\xi}(t)+\boldsymbol{Q}(s)\boldsymbol{\eta}(t)]\mathrm{d}s,\end{aligned} \tag{32}$$

where $\boldsymbol{\xi}(t) = [\,\boldsymbol{\eta}^{\mathrm{T}}(t) \;\; \boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t)) \;\; \boldsymbol{m}_{\mathrm{e}}^{\mathrm{T}}(t) \;\; f_0^{\mathrm{T}}(t)\,]$, and $\boldsymbol{\pi}(t) = \begin{bmatrix} \bar{\boldsymbol{A}}^{\mathrm{T}} \\ -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} \\ \boldsymbol{D}^{\mathrm{T}} \\ \bar{\boldsymbol{B}}_{\mathrm{wf}}^{\mathrm{T}} \end{bmatrix} h\boldsymbol{Q}(t) \begin{bmatrix} \bar{\boldsymbol{A}}^{\mathrm{T}} \\ -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} \\ \boldsymbol{D}^{\mathrm{T}} \\ \bar{\boldsymbol{B}}_{\mathrm{wf}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$.

Applying the schur complement formula to inequality (22), we can obtain

$$\boldsymbol{\Theta}(t) + \boldsymbol{\pi}(t) + h\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}^{\mathrm{T}}(t) < 0. \tag{33}$$

So we can know that

$$\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{Z}(t) - \gamma^2 f_0^{\mathrm{T}}(t)f_0(t) + \dot{V}(t) \le 0. \tag{34}$$

The performance of system (19) can be presented as follows:

$$\begin{aligned} J_1 &= \int_0^\infty (\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{Z}(t) - \gamma^2 f_0^{\mathrm{T}}(t)f_0(t))\mathrm{d}t \\ &= \int_0^\infty (\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{Z}(t) - \gamma^2 f_0^{\mathrm{T}}(t)f_0(t) \\ &\quad + \dot{V}(t))\mathrm{d}t - V(t)\,|_{t=\infty} + V(t)\,|_{t=0} \\ &\le \int_0^\infty (\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{Z}(t) - \gamma^2 f_0^{\mathrm{T}}(t)f_0(t) \\ &\quad + \dot{V}(t))\mathrm{d}t. \end{aligned} \tag{35}$$

From inequality (34), we can know that $J_1 \le 0$, which implies $\|\boldsymbol{Z}(t)\|_2 \le \gamma \|f_0(t)\|_2$. Thus the proof is completed.

Now we can see that the derived condition of the sampled-data fuzzy controller is a nonlinear matrix inequality and cannot be solved by the tool box of the Matlab. In the following, we will present a feasible condition in which it can be solved by the linear matrix inequality technique.

Theorem 2 Given the maximum sampling interval h, the performance level $\gamma > 0$ and positive constant scalar $\rho > \tilde{\rho}$, if there exist symmetric positive definite matrices $\boldsymbol{P} > 0$, $\boldsymbol{N}_i > 0$, $\boldsymbol{Q}_j > 0$, $\boldsymbol{Z}_j > 0$, matrices $\boldsymbol{Y}_k, \boldsymbol{M}_{li}$ and $\boldsymbol{\Lambda}_{ijk}$, where $\boldsymbol{\Lambda}_{ijk} = \boldsymbol{\Lambda}_{jik}^{\mathrm{T}}$ with the appropriate dimensions $i, j, k = 1, 2, \ldots, q$, $l = 1, 2, 3$, such that the following LMI hold:

$$\boldsymbol{\Pi}_{iik} < \boldsymbol{\Lambda}_{iik}, \tag{36}$$

$$\boldsymbol{\Pi}_{ijk} + \boldsymbol{\Pi}_{\mathrm{jik}} < \boldsymbol{\Lambda}_{ijk} + \boldsymbol{\Lambda}_{jik}, \tag{37}$$

$$\begin{bmatrix} \boldsymbol{\Lambda}_{11k} & \boldsymbol{\Lambda}_{12k} & \ldots & \boldsymbol{\Lambda}_{1qk} \\ \boldsymbol{\Lambda}_{21k} & \boldsymbol{\Lambda}_{22k} & \ldots & \boldsymbol{\Lambda}_{2qk} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Lambda}_{q1k} & \boldsymbol{\Lambda}_{q2k} & \ldots & \boldsymbol{\Lambda}_{qqk} \end{bmatrix} < 0, \tag{38}$$

$$\boldsymbol{Z}_j < \boldsymbol{Q}_j, \tag{39}$$

where

$$\boldsymbol{\Pi}_{ijk} = \begin{bmatrix} \boldsymbol{\Pi}_{11} & \boldsymbol{\Pi}_{12} & \boldsymbol{\Pi}_{13} & \boldsymbol{\Pi}_{14} & \boldsymbol{\Pi}_{15} & \boldsymbol{\Pi}_{16} & \boldsymbol{\Pi}_{17} \\ * & \boldsymbol{\Pi}_{22} & \boldsymbol{\Pi}_{23} & 0 & \boldsymbol{\Pi}_{25} & \boldsymbol{\Pi}_{26} & 0 \\ * & * & \boldsymbol{\Pi}_{33} & 0 & \boldsymbol{\Pi}_{35} & \boldsymbol{\Pi}_{36} & 0 \\ * & * & * & \boldsymbol{\Pi}_{44} & \boldsymbol{\Pi}_{45} & 0 & 0 \\ * & * & * & * & \boldsymbol{\Pi}_{55} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Pi}_{77} \end{bmatrix},$$

with $\boldsymbol{\Pi}_{11} = \boldsymbol{P}\bar{\boldsymbol{A}}_i + \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{M}_{1i}^{\mathrm{T}} + \boldsymbol{M}_{1\mathrm{i}} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{N}_i\boldsymbol{H}$, $\boldsymbol{\Pi}_{12} = -\boldsymbol{Y}_k\boldsymbol{H} - \boldsymbol{M}_{1i} + \boldsymbol{M}_{2i}^{\mathrm{T}}$, $\boldsymbol{\Pi}_{13} = \boldsymbol{P}\boldsymbol{D} + \boldsymbol{M}_{3i}^{\mathrm{T}}$, $\boldsymbol{\Pi}_{14} = \boldsymbol{P}\bar{\boldsymbol{B}}_{\mathrm{wf}}$, $\boldsymbol{\Pi}_{15} = \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}$, $\boldsymbol{\Pi}_{16} = \boldsymbol{M}_{1i}$, $\boldsymbol{\Pi}_{17} = \boldsymbol{F}^{\mathrm{T}}$, $\boldsymbol{\Pi}_{22} = -\boldsymbol{M}_{2i} - \boldsymbol{M}_{2i}^{\mathrm{T}}$, $\boldsymbol{\Pi}_{23} = -\boldsymbol{M}_{3i}^{\mathrm{T}}$, $\boldsymbol{\Pi}_{25} = -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{Y}_k^{\mathrm{T}}$, $\boldsymbol{\Pi}_{26} = \boldsymbol{M}_{2i}$, $\boldsymbol{\Pi}_{35} = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P}$, $\boldsymbol{\Pi}_{36} = \boldsymbol{M}_{3i}$, $\boldsymbol{\Pi}_{44} = -\gamma^2\boldsymbol{I}$, $\boldsymbol{\Pi}_{45} = \bar{\boldsymbol{B}}_{\mathrm{wf}}^{\mathrm{T}}\boldsymbol{P}$, $\boldsymbol{\Pi}_{55} = -h^{-1}(2\varepsilon\boldsymbol{P} - \varepsilon^2\boldsymbol{Q}_j)$, $\boldsymbol{\Pi}_{66} = -h^{-1}\boldsymbol{Z}_j$, and $\boldsymbol{\Pi}_{77} = -\boldsymbol{I}$, then error system (19) is globally asymptotically stable and reaches the level of given performance under the sampled-data fuzzy controller $\boldsymbol{u}(t)$. Moreover, the sampled-data fuzzy controller gain matrices can be obtained from $\boldsymbol{B}_k = \boldsymbol{P}^{-1}\boldsymbol{Y}_k$.

Proof Now reordering expression (22), we can obtain the following expression:

$$\begin{aligned} &\boldsymbol{\Pi} = \sum_{k=1}^{q} \hat{w}_k(\hat{\boldsymbol{x}}(t_{\mathrm{sd}}))\Big(\sum_{i=1}^{q}\sum_{j=1}^{q} \hat{w}_i(\hat{x}(t))\hat{w}_j(\hat{x}(t))\tilde{\boldsymbol{\Pi}}_{ijk}\Big), \\ &1 \le i \le j \le q, \end{aligned} \tag{40}$$

where

$$\tilde{\boldsymbol{\Pi}}_{ijk} = \begin{bmatrix} \boldsymbol{\Pi}_{11} & \tilde{\boldsymbol{\Pi}}_{12} & \boldsymbol{\Pi}_{13} & \boldsymbol{\Pi}_{14} & \tilde{\boldsymbol{\Pi}}_{15} & \boldsymbol{\Pi}_{16} & \boldsymbol{\Pi}_{17} \\ * & \boldsymbol{\Pi}_{22} & \boldsymbol{\Pi}_{23} & 0 & \tilde{\boldsymbol{\Pi}}_{25} & \boldsymbol{\Pi}_{26} & 0 \\ * & * & \boldsymbol{\Pi}_{33} & 0 & \tilde{\boldsymbol{\Pi}}_{35} & \boldsymbol{\Pi}_{36} & 0 \\ * & * & * & \boldsymbol{\Pi}_{44} & \tilde{\boldsymbol{\Pi}}_{45} & 0 & 0 \\ * & * & * & * & \tilde{\boldsymbol{\Pi}}_{55} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Pi}_{77} \end{bmatrix},$$

$$\tilde{\boldsymbol{\Pi}}_{12} = -\boldsymbol{P}\boldsymbol{B}_k\boldsymbol{H} - \boldsymbol{M}_{1\mathrm{i}} + \boldsymbol{M}_{2\mathrm{i}}^{\mathrm{T}}, \quad \tilde{\boldsymbol{\Pi}}_{15} = \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{Q}_j,$$

$$\tilde{\boldsymbol{\Pi}}_{25} = -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{Q}_j, \quad \tilde{\boldsymbol{\Pi}}_{35} = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{Q}_j,$$

$$\tilde{\boldsymbol{\Pi}}_{45} = \bar{\boldsymbol{B}}_{\mathrm{wf}}^{\mathrm{T}}\boldsymbol{Q}_j, \quad \text{and} \quad \tilde{\boldsymbol{\Pi}}_{55} = -h^{-1}\boldsymbol{Q}_j.$$

Now pre- and post-multiply both sides of $\tilde{\boldsymbol{\Pi}}_{ij}$ by $\mathrm{diag}[\boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{P}\boldsymbol{Q}_j^{-1}, \boldsymbol{I}, \boldsymbol{I}]$ and take their transposes, respectively, we can obtain

$$\bar{\boldsymbol{\Pi}}_{ij} = \begin{bmatrix} \boldsymbol{\Pi}_{11} & \tilde{\boldsymbol{\Pi}}_{12} & \boldsymbol{\Pi}_{13} & \boldsymbol{\Pi}_{14} & \boldsymbol{\Pi}_{15} & \boldsymbol{\Pi}_{16} & \boldsymbol{\Pi}_{17} \\ * & \boldsymbol{\Pi}_{22} & \boldsymbol{\Pi}_{23} & 0 & \bar{\boldsymbol{\Pi}}_{25} & \boldsymbol{\Pi}_{26} & 0 \\ * & * & \boldsymbol{\Pi}_{33} & 0 & \boldsymbol{\Pi}_{35} & \boldsymbol{\Pi}_{36} & 0 \\ * & * & * & \boldsymbol{\Pi}_{44} & \boldsymbol{\Pi}_{45} & 0 & 0 \\ * & * & * & * & \bar{\boldsymbol{\Pi}}_{55} & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \boldsymbol{\Pi}_{77} \end{bmatrix}, \tag{41}$$

where $\bar{\boldsymbol{\Pi}}_{25} = -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{Q}_j$, and $\bar{\boldsymbol{\Pi}}_{55} = -h^{-1}\boldsymbol{P}\boldsymbol{Q}_j^{-1}\boldsymbol{P}$.
Using lemma 1 yields that

$$-h^{-1}\boldsymbol{P}\boldsymbol{Q}_j^{-1}\boldsymbol{P} \leq -h^{-1}(2\varepsilon\boldsymbol{P} - \varepsilon^2\boldsymbol{Q}_{\boldsymbol{j}}). \tag{42}$$

From inequalities (36)–(38), we can have the following inequality:

$$\begin{aligned} \sum_{i=1}^{q}\sum_{j=1}^{q}\hat{w}_i(\hat{x}(t))\hat{w}_j(\hat{x}(t))\boldsymbol{\Pi}_{ijk} &= \sum_{i=1}^{q}\hat{w}_i(\hat{x}(t))^2\boldsymbol{\Pi}_{iik} + \sum_{i=1}^{q-1}\sum_{j>i}^{q}\hat{w}_i(\hat{x}(t))\hat{w}_j(\hat{x}(t))(\boldsymbol{\Pi}_{ijk} + \boldsymbol{\Pi}_{jik}) \\ &\leq \sum_{i=1}^{q}\hat{w}_i(\hat{x}(t))^2\boldsymbol{\Lambda}_{iik} + \sum_{i=1}^{q-1}\sum_{j>i}^{q}\hat{w}_i(\hat{x}(t))\hat{w}_j(\hat{x}(t))(\boldsymbol{\Lambda}_{\boldsymbol{i}jk} + \boldsymbol{\Lambda}_{jik}) \\ &= \begin{bmatrix} \hat{w}_1(\hat{x}(t)) \\ \hat{w}_2(\hat{x}(t)) \\ \vdots \\ \hat{w}_q(\hat{x}(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{\Lambda}_{11k} & \boldsymbol{\Lambda}_{12k} & \dots & \boldsymbol{\Lambda}_{1qk} \\ \boldsymbol{\Lambda}_{21k} & \boldsymbol{\Lambda}_{22k} & \dots & \boldsymbol{\Lambda}_{2qk} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Lambda}_{q1k} & \boldsymbol{\Lambda}_{q2k} & \dots & \boldsymbol{\Lambda}_{qqk} \end{bmatrix} \begin{bmatrix} \hat{w}_1(\hat{x}(t)) \\ \hat{w}_2(\hat{x}(t)) \\ \vdots \\ \hat{w}_q(\hat{x}(t)) \end{bmatrix} < 0. \end{aligned} \tag{43}$$

Letting $\boldsymbol{Y}_k = \boldsymbol{P}\boldsymbol{B}_k$ and combining expressions (41)–(43), we can know that $\boldsymbol{\Pi} < 0$. Equality (39) can imply inequality (23): we can obtain inequalities (22)–(23) from inequalities (36)–(39). So the error system is globally asymptotically stable and arrives at the level of given performance γ. The sampled-data controller gain can be obtained from $\boldsymbol{B}_k = \boldsymbol{P}^{-1}\boldsymbol{Y}_k$. The proof is completed.

4. Simulation

In this section we consider an example of the Rossler's system.[27] The fault tolerant synchronization of two Rossler's systems with using the proposed sampled-data controller is proposed to illustrate the effectiveness of the proposed results.

The dynamics of the slave Rossler's system is described as follows:

$$\dot{\hat{\boldsymbol{x}}}(t) = \hat{\boldsymbol{A}}(\hat{\boldsymbol{x}}(t))\hat{\boldsymbol{x}}(t) + \boldsymbol{u}(t), \tag{44}$$

where $\hat{\boldsymbol{x}}(t) = \begin{bmatrix} \hat{x}_1^{\mathrm{T}}(t) & \hat{x}_2^{\mathrm{T}}(t) & \hat{x}_3^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, $\hat{\boldsymbol{A}}(\hat{\boldsymbol{x}}(t)) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \hat{a} & 0 \\ \hat{b} & 0 & -(\hat{c}(t) - \hat{x}_1(t)) \end{bmatrix}$, $\hat{a} = 0.34$, $\hat{b} = 0.4$,

$$\hat{c}(t) = \frac{\hat{c}_{\max} + \hat{c}_{\min}}{2} + \frac{\hat{c}_{\max} - \hat{c}_{\min}}{2} \times \sin(t) \in \begin{bmatrix} \hat{c}_{\min} & \hat{c}_{\max} \end{bmatrix} > 0$$

which is an uncertain value, $\hat{c}_{\max} = 7.7$, and $\hat{c}_{\min} = 4.5$. Then it is assumed that $\hat{x}_1(t) \in \begin{bmatrix} \hat{c}_{\min} - d, & \hat{c}_{\max} + d \end{bmatrix}$ and $d = 25$. The exact fuzzy model of the slave Rossler's system is represented by the following fuzzy rules:
Rule i: IF $\hat{x}_1(t)$ is N^i

$$\text{Then} \quad \dot{\hat{\boldsymbol{x}}}(t) = \hat{\boldsymbol{A}}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{u}(t), \quad i = 1, 2. \tag{45}$$

The slave Rossler's system is defined as

$$\dot{\hat{\boldsymbol{x}}}(t) = \sum_{i=1}^{2}\hat{w}_i(\hat{x}_1(t))(\hat{\boldsymbol{A}}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{u}(t)), \tag{46}$$

where $\hat{\boldsymbol{A}}_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \hat{a} & 0 \\ \hat{b} & 0 & -d \end{bmatrix}$, $\hat{\boldsymbol{A}}_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \hat{a} & 0 \\ \hat{b} & 0 & d \end{bmatrix}$, $\hat{w}_1(\hat{x}_1(t) = \frac{1}{2}(1 + (c(t) - \hat{x}_1(t))/d)$ and $\hat{w}_2(\hat{x}_1(t)) = 1 - w_1(\hat{x}_1(t))$.

On the other hand, the dynamics of the master Rossler's system is given as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x}(t))\boldsymbol{x}(t), \tag{47}$$

where $\boldsymbol{x}(t) = \begin{bmatrix} x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t) & x_3^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{A}(\boldsymbol{x}(t)) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -(c(t) - x_1(t)) \end{bmatrix}$, $a = 0.34$, $b = 0.4$ and

$$c(t) = \frac{c_{\max} + c_{\min}}{2} + \frac{c_{\max} - c_{\min}}{2} \times \cos(t) \in \begin{bmatrix} c_{\min} & c_{\max} \end{bmatrix} > 0,$$

which is an uncertain value, $c_{\max} = 7.7$, $c_{\min} = 4.5$ and $x_1(t) \in \begin{bmatrix} c_{\min} - d, & c_{\max} + d \end{bmatrix}$, then the master Rossler's system can be represented by the fuzzy model with the following rules:

Rule i: IF $x_1(t)$ is M^i

$$\text{THEN} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t), \quad i = 1, 2. \tag{48}$$

So, the master Rossler's system is defined as

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{2} w_i(x_1(t))\boldsymbol{A}_i\boldsymbol{x}(t), \tag{49}$$

where $w_1(x_1(t) = \frac{1}{2}(1 + (c(t) - x_1(t))/d)$ and $w_2(x_1(t)) = 1 - w_1(x_1(t))$.

Figure 2 shows the phase trajectory of master Rossler's system and we can see what Eq. (49) describes is a chaotic system.

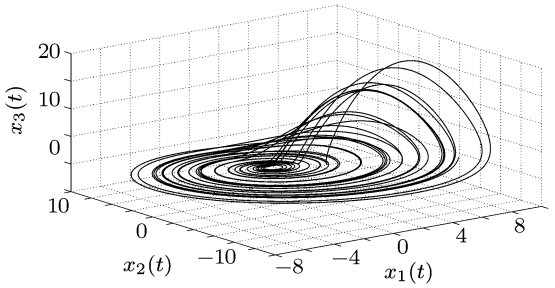


Fig. 2. Phase trajectory of master Rossler's system.

Here we assume that a ramp fault $f(t)$, which can be characterized by Eqs. (9) and (10), happens in the master system, the master Rossler's system becomes

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{2} w_i(x_1(t))\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_{\mathrm{f}}f(t), \tag{50}$$

where $\boldsymbol{B}_{\mathrm{f}} = [10;\ 0;\ 0]$ and $f(t)$ can be characterized as follows:

$$\dot{\boldsymbol{x}}_{\mathrm{f}}(t) = \boldsymbol{A}_{\mathrm{wf}}\boldsymbol{x}_{\mathrm{f}}(t) + \boldsymbol{B}_{\mathrm{wf}}f_0(t), \tag{51}$$

$$f(t) = \boldsymbol{C}_{\mathrm{wf}}\boldsymbol{x}_{\mathrm{f}}(t), \tag{52}$$

where $\boldsymbol{A}_{\mathrm{wf}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $\boldsymbol{B}_{\mathrm{wf}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\boldsymbol{C}_{\mathrm{wf}} = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $f_0(t) = \beta\delta(t - t_{\mathrm{f}})$, $\beta = 0.1$, and $t_{\mathrm{f}} = 75$.

Fault signal $f(t)$ is shown in Fig. 3. From Fig. 3, we can know that the fault happens when $t > 75$ s. Figure 4 shows the phase trajectory of master Rossler's system with the ramp fault.

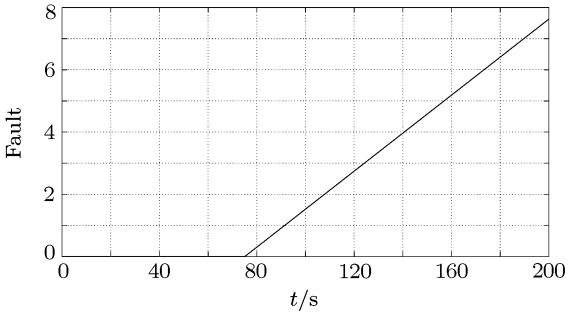


Fig. 3. Fault signal $f(t)$.

We design a sampled-data controller to achieve the fault tolerant synchronization of the master–slave Rossler's system. The sampled-data controller is defined as

$$\boldsymbol{u}(t) = \boldsymbol{K}(t)\boldsymbol{e}(t_{\mathrm{sd}}) + \boldsymbol{B}_{\mathrm{f}}\hat{f}(t). \tag{53}$$

Now we choose some parameters: $h = 0.01$, $\varepsilon = 1$, $\gamma = 0.5$, and $\rho = 0.06$. Then we use Theorem 2 to acquire the controller $\boldsymbol{u}(t)$ and the controller gain of $\boldsymbol{u}(t)$ is given as follows:

$$\boldsymbol{K} = \begin{bmatrix} 25.1680 & -0.1528 & -0.5617 \\ 0.6552 & 16.5996 & 0.0006 \\ -0.0562 & 0.0016 & 38.0373 \end{bmatrix},$$

and

$$\bar{\boldsymbol{K}} = \begin{bmatrix} 5.9186 & 0.1686 & 0.1322 \\ 10.9356 & 0.4786 & 0.2457 \end{bmatrix}.$$

For the purpose of testing the controller's performance, we did not add any control signal to the slave chaotic system when $t < 50$ s. When 50 s $\leq t \leq$ 150 s, we apply only the state feedback controller to the slave

chaotic system. Both the state feedback controller and the fault compensator are used when $t \geq 150$ s. We set the initial conditions of two chaotic systems to be $\boldsymbol{x}(0) = [0.5\ 1\ -0.5]^{\mathrm{T}}$ and $\hat{\boldsymbol{x}}(0) = [1\ 0.5\ -1]^{\mathrm{T}}$ respectively.

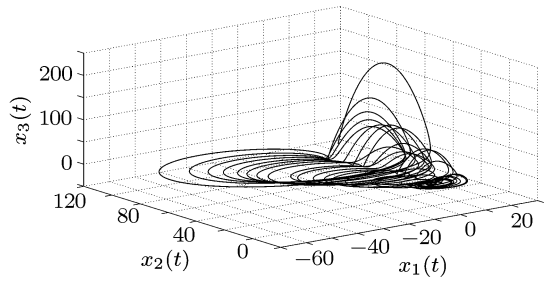


Fig. 4. Phase trajectory of master Rossler's system with a ramp fault.

Figures 5 and 6 show the estimation of the fault and the error between the fault and the estimation of the fault, respectively.

From Figs. 5 and 6, we can know that when the fault compensator's signal is added to the slave system, the estimation of fault can track the fault that happens in the master system as quickly as possible.

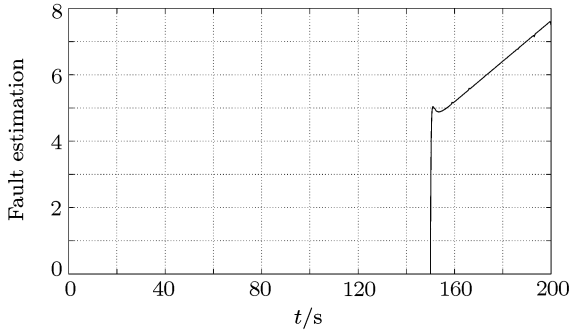


Fig. 5. Estimation of ramp fault.

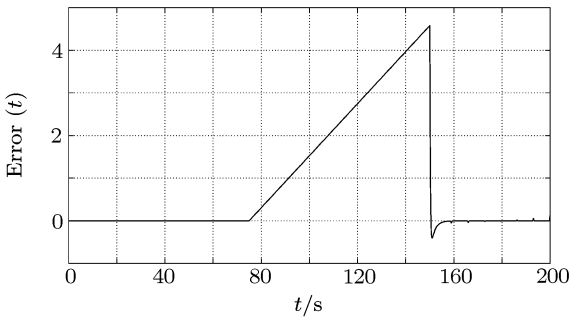


Fig. 6. Error between real fault and estimation of the fault.

Figure 7 shows the state errors of the erroneous system. After the state feedback controller is applied at $t = 50$ s the synchronization is achieved immediately. But when the ramp fault occurs in the master system at $t = 75$ s, the state error of the erroneous system is not kept at zero. The synchronization cannot be achieved because of the existence of the ramp fault. For achieving the fault tolerant synchronization, we use both the state feedback controller and the fault compensator at $t = 150$ s. Referring to Fig. 7, we can see that the state errors between master and slave chaotic systems become sufficiently small at $t \geq 150$ s which indicates that the fault tolerant synchronization is successfully achieved.

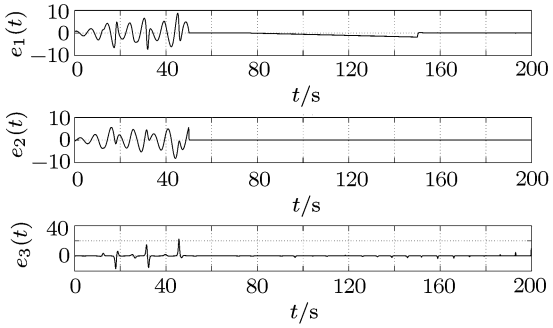


Fig. 7. Tracking errors between master and slave Rossler's systems.

5. Conclusions

In this paper, we study the problem of fault tolerant synchronization of two chaotic systems with using a sampled-data fuzzy controller which contains two parts: the state feedback controller and the fault compensator. The sampled-data fuzzy controller with using the system state of the master chaotic system and the slave chaotic system is proposed to realize the fault tolerant synchronization and can be implemented efficiently by using microcontrollers or digital computers to lower the implementation cost and time. The stability conditions of the erroneous system based on the LMI are derived by using a new parameter-dependent Lyapunov–Krasovskii functional and relaxed quadratic stabilization techniques. A simulation example is given to illustrate the effectiveness of the proposed approach.

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