An efficient method of distinguishing chaos from noise

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It is an important problem in chaos theory whether an observed irregular signal is deterministic chaotic or stochastic. We propose an efficient method for distinguishing deterministic chaotic from stochastic time series for short scalar time series. We first investigate, with the increase of the embedding dimension, the changing trend of the distance between two points which stay close in phase space. And then, we obtain the differences between Gaussian white noise and deterministic chaotic time series underlying this method. Finally, numerical experiments are presented to testify the validity and robustness of the method. Simulation results indicate that our method can distinguish deterministic chaotic from stochastic time series effectively even when the data are short and contaminated.

Keywords: phase space reconstruction, average false nearest neighbour, chaos detection

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1. Introduction

Chaotic signal is widely present in meteorology, biomedicine, communication, electronic circuits, mechanics, economics, etc. It is a long-debated question whether an observed irregular signal is deterministic chaotic or stochastic for 'noise-like' character of chaotic signal.[1−4] It is not only of theoretical interest but also of practical value to detect chaos from only a scalar time series because of information acquisition restriction (or cost). The most common method of chaos detection is to calculate invariants (such as correlation dimension,[5−7] Lyapunov exponent,[8−10] entropy[11−13] and their extension[2,14−21]). The number of data available and the resolution are important factors to limit the validity of these methods.[2] Moreover, many input parameters (such as search radius and the number of the neighbours) need to be decided subjectively. As a result, different inputs may lead to different results. More seriously, wrong judgment is often made when the data are contaminated by noise.[5,10,22]

Based on the smoothness and differentiability of the phase portrait of chaotic dynamical systems, Kaplan and Glass[4] proposed to measure the average directional vectors in embedding space to detect the determinism. The algorithm is efficient for short time series. Wayland *et al.*[23−26] also used a smoothness detector to test the determinism of a time series. Their algorithms are also valid for short time series, but require the arbitrary choice of input parameters. Jeong *et al.*[27] proposed to compute the angles between two successive tangent vectors in reconstructed phase space to distinguish deterministic chaos from random time series. Though these methods overcome the difficulties in requiring a large number of data and in finite resolution, to some extent, many parameters are determined subjectively and the existence of noise still restricts their application.

To avoid the reconstruction of phase space, Gottwald *et al.*[28−30] constructed a real value function, and then directly applied it to the time series. The input is the time series and the output is 0 or 1. This algorithm seems to be efficient for a moderate number of contaminated data. Djurović *et al.*[31−33] proposed a method to classify signals from chaotic oscillators by using entire spectral domain, and with this method the accurate results for signals with a large number of spectral components in periodic regime can be produced. Zhang *et al.*[34] proposed a method to detect deterministic structure from a pseudoperiodic time series. They segmented the time series into consecutive cycles to compute the correlation coefficient as a measure of the distance between cycles. This method can identify chaos in the presence of noises from different sources. All of these methods avoid the reconstruction of phase space, whereas a large number of data needed are their common shortcoming to some extent.

Kennel *et al.*[35,36] proposed false nearest neighbour (FNN) method in 1992, which was used to determine the optimal embedding dimension of a scalar

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time series. After that, Cao[37] modified the FNN method and proposed an average false nearest neighbour (AFN) method. This method can be used to distinguish deterministic chaotic from stochastic time series. Ramdani *et al.*[38,39] gave the theoretical results obtained by using the AFN method for white Gaussian noise (WGN), and presented some numerical experiments corresponding to various known chaotic processes corrupted by noise. They also analysed the influence of noise and the length of time series on AFN method. Though this method has a good performance when data are lacking and the noise exists, it is needed to search the nearest neighbour in each embedding dimension to exclude the false nearest neighbour. Hence even for short time series it is a time consuming process.

Taking into account all the limitations mentioned above, we propose a new chaos detection method. This method can distinguish deterministic chaotic from stochastic time series effectively even when the data are lacking and contaminated by noise. The anti-noise performance of the new method is discussed by numerical experiments. For the sake of solving the time consuming problem, we construct an algorithm in which only neighbours in initial embedding dimension space need to be searched. The search radius is introduced to cope with finite resolution and short data problem. All points, within a region of radius ε, must be considered as being equally good to describe the changing trend of the distance. Instead of choosing one of them arbitrary we propose to take the arithmetic mean of the individual distance based on these points. The new method overcomes not only the shortage of the subjectivity of parametric choice and time consuming of FNN method, but also the shortcomings of both the requirement for a large number of data and the noise sensitivity of invariant method.

The rest of the present paper is organized as follows. In Section 2 given is the description of the method. In Section 3 described are the numerical experiments. Finally the conclusion is given in Section 4.

2. Description of method

Suppose that we have a time series $x_1, x_2, \ldots, x_N$. The time delay vector can be reconstructed as follows:

$$\begin{aligned} &\boldsymbol{y}(i,d) = (x_i, x_{i+\tau}, \ldots, x_{i+(d-1)\tau}), \\ &i = 1, 2, \ldots, N-(d-1)\tau, \end{aligned} \tag{1}$$

where d is the embedding dimension, τ is the time delay, and $\boldsymbol{y}(i,d)$ denotes the ith reconstructed vector with embedding dimension d.

Actually, any two points which stay close in lower-dimensional phase space (minimum embedding dimension at least) will be still close in higher-dimensional reconstructed space for chaotic time series.[37] To distinguish deterministic time series from nondeterministic one, we discuss the changing trend of the distance between two stay close points in phase space with the increase of embedding dimension.

If $\boldsymbol{y}(j,d)$ is a point whose distance to $\boldsymbol{y}(i,d)$ is $\varepsilon_d (\varepsilon_d \ll 1)$ in d dimensional reconstructed space, then the distance between these two points in $(d+1)$-dimensional reconstructed space is

$$\begin{aligned} \varepsilon_{d+1} &= \|\boldsymbol{y}(i,d+1) - \boldsymbol{y}(j,d+1)\| \\ &= \|(\boldsymbol{y}(i,d), x_{i+d\tau}) - (\boldsymbol{y}(j,d), x_{j+d\tau})\| \\ &\le \|\boldsymbol{y}(i,d) - \boldsymbol{y}(j,d)\| + |x_{i+d\tau} - x_{j+d\tau}| \\ &= \varepsilon_d + |x_{i+d\tau} - x_{j+d\tau}|, \end{aligned} \tag{2}$$

where $\boldsymbol{y}(i,d+1)$ is the ith time delay vector with embedding dimension $d+1$, i.e. $\boldsymbol{y}(i,d+1) = (x_i, x_{i+\tau}, \ldots, x_{i+d\tau})$; j is an integer which makes the distance between $\boldsymbol{y}(j,d)$ and $\boldsymbol{y}(i,d)$ be ε_d. The norm $\|\cdot\|$ is defined as

$$\begin{aligned} &\|\boldsymbol{y}(i,d) - \boldsymbol{y}(j,d)\| \\ &= \left(\sum_{k=0}^{d-1} (x_{i+k\tau} - x_{j+k\tau})^2\right)^{1/2}. \end{aligned} \tag{3}$$

Equation (2) always holds for each phase point with a reasonable embedding dimension. To deal with the finite resolution and the data shortage problem, the search radius ε is introduced. We take the arithmetic mean of the distances between the central phase point and the points in its ε-close neighbourhood for each phase point as follows:

$$\begin{aligned} &r(i,d) = \frac{1}{|U(i,\varepsilon)|} \sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)} \|\boldsymbol{y}(i,d) - \boldsymbol{y}(k,d)\|, \\ &i = 1, 2, \ldots, N-(d-1)\tau, \end{aligned} \tag{4}$$

where $U(i,\varepsilon)$ is the close neighbourhood of $\boldsymbol{y}(i,d)$ with radius ε; $\boldsymbol{y}(k,d)$ is a time delay vector less than or equal to ε for $\boldsymbol{y}(i,d)$; $|U(i,\varepsilon)|$ denotes the number of elements of the ε-close neighbourhood. For each phase point, the choice of ε must ensure that at least one neighbour is included in its ε-close neighbourhood. So the value of ε can be determined by

$$\varepsilon = \max_{1\le i\le N-(d-1)\tau} \{\|\boldsymbol{y}(i,d) - \boldsymbol{y}(n(i,d),d)\|\}, \tag{5}$$

where $n(i,d)$ is the integer which makes $\boldsymbol{y}(n(i,d),d)$ the nearest neighbour of $\boldsymbol{y}(i,d)$.

Remarks

1) Our discussed problem, the changing trend of the average distance with the dimension increasing, starts with the reasonable embedding dimension (minimum embedding dimension at least). The value of k in Eq. (4) only depends on initial reconstructed dimension d and search radius ε. That is to say, its value is determined in initial reconstructed phase space and does not change with dimension. Obviously, the search is needed only once, and the computational time is saved consumedly.

2) The minimum embedding dimension d and time delay τ should be given in advance. The minimum embedding dimension can be determined by invariant method,[5] false nearest neighbour method.[35−37,40], and so on. A good estimate of the time delay is even more difficult to obtain. If τ is small compared with the internal time scales of the system, successive elements of the delay vectors are strongly correlated. If it is too large, successive elements are already almost independent (see Refs. [5] and [41] for more details about the choice of τ).

3) The value of ε in Eq. (5) is related only to the initial embedding dimension, i.e. its value only needs to be determined in the initial embedding space.

In order to describe the changing trend of $r(d)$ from $d-$ to $(d+1)-$ dimensional phase spaces, we define

$$\Omega(d) = \frac{r(d+1)}{r(d)}, \tag{6}$$

where

$$r(d) = \frac{1}{N-(d-1)\tau} \sum_{i=1}^{N-(d-1)\tau} r(i,d). \tag{7}$$

Note that the number of the phase points in d-dimensional reconstructed space is τ more than the number of dimensions, $d+1$. Then, if $\boldsymbol{y}(m,d)(N-(d-1)\tau \geq m > N-d\tau)$ is a neighbour of $\boldsymbol{y}(i,d)$, $\boldsymbol{y}(m,d+1)$ does not exist. So, we truncate the time series to ensure $\boldsymbol{y}(m,d+1)$ existence. At this situation, different dimensional reconstructed phase spaces have the same number of phase points M. In view of Eqs. (4) and (7), we have

$$r(d+1) = \frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}\|\boldsymbol{y}(i,d+1)-\boldsymbol{y}(k,d+1)\|. \tag{8}$$

Note that $y(i,d+1) = (x_i, x_{i+\tau}, \ldots, x_{i+d\tau}) = (y(i,d), x_{i+d\tau})$, and utilize triangle inequality, then we will obtain

$$r(d+1) \leq \frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}\|\boldsymbol{y}(i,d)-\boldsymbol{y}(k,d)\| + \frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}|x_{i+d\tau}-x_{k+d\tau}|. \tag{9}$$

According to Eq. (7), we have

$$r(d) < r(d+1) \leq r(d) + \frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}|x_{i+d\tau}-x_{k+d\tau}|. \tag{10}$$

Inserting Eq. (10) into Eq. (6) and after simplification, we have

$$1 < \Omega(d) \leq 1 + \frac{1}{r(d)}\left(\frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}|x_{i+d\tau}-x_{k+d\tau}|\right). \tag{11}$$

So, the upper bound of $\Omega(d)$ depends on

$$\frac{1}{M}\sum_{i=1}^{M}\frac{1}{|U(i,\varepsilon)|}\sum_{\boldsymbol{y}(k,d)\in U(i,\varepsilon)}|x_{i+d\tau}-x_{k+d\tau}|. \tag{12}$$

The value of Eq. (12) depends on the size of strange attractor. So $\Omega(d)$ keeps in a small area ($1 \leq \Omega(d) \leq 2$) because deterministic chaotic time series is confined within a comparatively small area. But for stochastic time series, the phase points fill entire phase space. Then, two points which stay close in lower-dimensional phase space will be far from each other in higher-dimensional reconstructed space. So the value of Eq. (12) is much bigger than that of deterministic chaos. As a result, the $\Omega(d)$ value is bigger than that of chaos for a small initial value of $r(d)$. The value of $r(d)$, however, gradually increases with the increase of d. Then, the value of $\Omega(d)$ diminishes gradually and approaches to 1. That is to say, the value of $\Omega(d)$ for stochastic time series starts with a large value then gradually diminishes, and finally approaches to 1. In contrast, the value of $\Omega(d)$ for chaos keeps in a small area. So we can use this difference to distinguish deterministic chaotic from stochastic time series.

3. Numerical experiments

In this section, we apply the new algorithm to the Lorenz system corrupted by an additive WGN to assess its validity and robustness. The length of the time series is 1000. As indicated in Ref. [38], the standard deviations of the additive WGN are 1%, 10% and 30% respectively.

The Lorenz equation[42] was given by Lorenz in 1963 as follows:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= -xz + \rho x - y, \\ \dot{z} &= xy - \beta z. \end{aligned} \tag{13}$$

This system yields an irregular chaotic behaviour when $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$. We obtain the time series from the x-component of Lorenz system by using the Runge–Kutta algorithm in integral steps of 0.01. The time delay is set to be 1 and initial reconstructed dimension is set to be 3. Figure 1 shows the changing trend of $\Omega(d)$ for 1000-point time series with different noise intensities. The conclusion drawn from Section 2 is proved greatly. The value of $\Omega(d)$ for WGN rapidly diminishes with the increase of reconstructed dimension, and then approaches to 1. But for chaos and noisy chaos, even if the noise standard deviation is 30%, the values of $\Omega(d)$ always stay in a small area. The difference among chaos, noisy chaos and WGN are clearly shown in Fig. 1. The trend of $\Omega(d)$ is similar to that of pure chaos for a 30% noise level. That is to say, our algorithm is not very sensitive to noise.

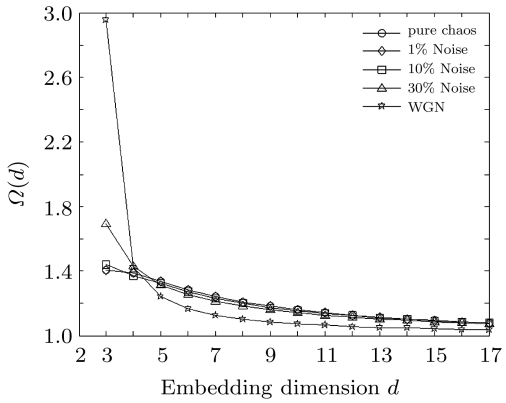
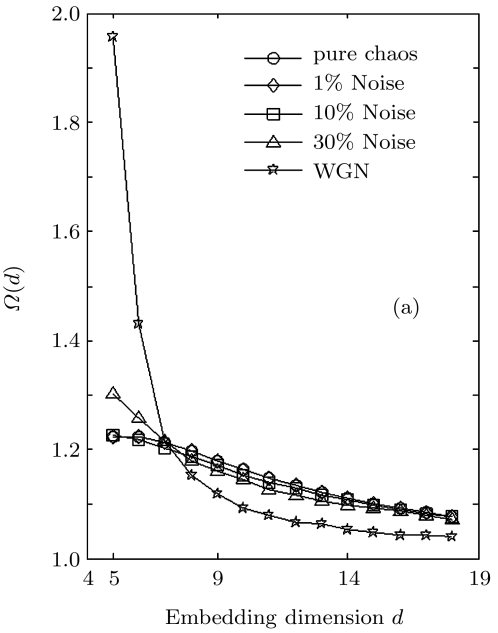


Fig. 1. Changing trends of $\Omega(d)$ for 1000–point time series from x-component of Lorenz system with different noise intensities and WGN. The standard deviations of the additive WGN are 1%, 10% and 30% respectively.

In real application, it is difficult to know the number of optimal reconstructed dimensions accurately. But we know that if a d-dimensional embedding yields a faithful representation of the phase space, every d' dimensional reconstruction with $d' > d$ does so. Figure 2 shows that our method can distinguish chaos from stochastic one when the estimation of minimum reconstructed dimension is inaccurate. The behaviours of $\Omega(d)$ in Fig. 2 are different from in Fig. 1 at the same embedding dimensions, resulting from the different search radii as indicated in Eq. (5).



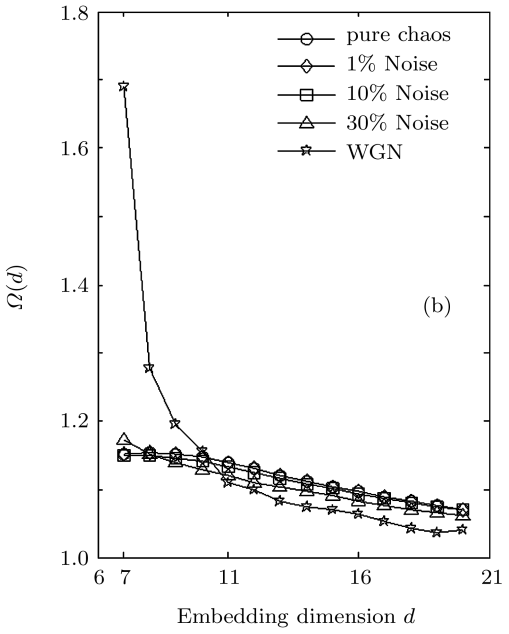


Fig. 2. Changing trends of $\Omega(d)$ for 1000-point time series from x component of Lorenz system with different noise intensities and WGN for different initial embedding dimensions: 5(a) and 7(b).

4. Conclusions

In this paper, we propose a method to distinguish chaos from noise because the close points in reconstructed phase space of deterministic chaotic time series are close to each other as the embedding dimension increases reasonably, however stochastic sequence does not.

Traditional FNN method is time consuming greatly because it needs to search neighbours over again in each reconstructed dimension. But, our method overcomes this disadvantage and needs search only once. Each measurement result of a quantity is only valid for up to some finite resolution in nonlinear time series analysis in application. Then, looking for the single closest state of the phase point is not the best to describe the entire changing trend of the distance. In order to solve this problem, we introduce a parameter ε that makes every phase point have more neighbours to denote the entire changing trend of the distance in phase space. Furthermore, much more points can be investigated for a short time series. The parameter d (reconstructed dimension) is a necessary parameter which must be given prior to identifying the time series. Of course, we can use the phase portrait to identify the time series when the parameter d is known in advance. But the phase portrait cannot be obtained intuitively when the dimension is larger ($d > 3$). In addition, the phase portrait of deterministic chaotic time series is disordered and unsystematic for the influence of additive noise. Hence although the reconstructed dimension is known, we still cannot identify the time series correctly by phase portrait. Optimal embedding dimension d, in a sense, is required but not necessary for our method. Because even if the initial value of d is larger than optimal embedding dimension, our algorithm also holds. Larger initial reconstructed dimension should be chosen when the minimum reconstructed dimension is inaccurate or difficult to know in application.

We present numerical experiments to testify the validity and robustness of the algorithm. WGN with different intensities (at the levels of 1%, 10% and 30%) are added to a pure chaotic time series. One thousand point data from a chaotic system are used to testify the validity of the method for short time series. Simulations indicate that our method can identify the deterministic chaotic time series in the noisy circumstances. In the future, we will apply this method to engineering to detect the chaotic communication signal.

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