Exact analytical solutions to the mean-field model depicting microcavity containing semiconductor quantum wells*

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By using a two-mode mean-field approximation, we study the dynamics of the microcavities containing semiconductor quantum wells. The exact analytical solutions are obtained in this study. Based on these solutions, we show that the emission from the microcavity manifests periodic oscillation behaviour and the oscillation can be suppressed under a certain condition.

Keywords: mean-field model, microcavity, exact analytical solutions
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1. Introduction

Recently, various properties of the semiconductor microcavity systems have attracted extensive attention.[1−7] It is known that the interaction between the semiconductor structures and the microcavities occurs via the exciton which is an electron–hole pair bound by the Coulomb interaction.[8,9] In the strong-coupling dynamical regime, the coherent exciton–cavity coupling becomes dominant compared with the dissipative decay processes.[10−12] Weisbuch *et al.* have observed a key signature of 'atom–cavity' strong coupling, namely, normal mode splitting in multiple-quantum-well systems.[13] When the collective dipole coupling rate between the cavity mode and the exciton mode is larger than the decay rates of excitons and cavity, coherent exciton–photon interaction leads to an oscillatory energy exchange between the exciton and cavity which shows the Rabi-like oscillating behaviour.[14−17] Other effects such as dynamical Stark shift, bleaching of the exciton resonance with increasing excitation intensity, plasma screening and band-filling phenomena have also been predicted and observed in microcavities containing atoms.[18] In this coupled system, if the density of excitons is relatively large, the excitons can no longer be treated as ideal bosons due to the residual Coulomb interaction among the excitons. Solving this problem, one can put these deviations into the effective interaction between the hypothetical ideal bosons and still give the form of bosonic operators.[19,20] Using rotating wave approximation,[21] the analytical expressions of the light intensity show that the nonlinear exciton–exciton interaction will give rise to the collapses and revivals in the novel oscillating emission.[22−25] Furthermore, at low temperature, the exciton–photon scattering is shown to destroy the non-classical effects and reduce excess noise in the output field.[26]

In this paper, our model which is a high finesse microcavity containing a semiconductor quantum well is presented. The non-linear exciton–exciton interaction is taken into account. In contrast with previous full quantized treatment,[22] we present a theoretical study of the dynamics of the exciton–photon system under the two-mode mean-field approximation which means we study the system's behaviours by including the next order corrections to this model. The explicit analytical solutions are given and it can satisfy the relative parameters and possible initial conditions. Also, the main property of the system is presented

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and it shows a periodic oscillatory feature. We point out that under certain conditions, the oscillation between the two modes can be rather uncompleted. The remainder of this paper is organized as follows. In Section 2, our basic model of the exciton–photon system under the mean-field approximation by virtue of Bloch representation is presented. In Section 3, the explicit analytical solutions of the system, which are complete and universal are given. In Section 4, we analyze the solutions obtained in Section 3 and show that the emission from the microcavity manifests periodic oscillation behaviour, whereas the oscillation can be suppressed under a certain condition. Conclusion is given in Section 5.

2. The quantum model

We consider a microcavity made of two highly reflecting planar mirrors separated by a distance of the order of the wavelength. A thin layer of semiconductor quantum well parallel to the mirrors is embedded in the middle of the microcavity. The electromagnetic field can excite an electron from the filled valence band to the conduction band, leading to create a hole in the valence band. The electron–hole system possesses bound state, the excitonic states. The exciton and photon modes are quantized along the direction normal to the microcavity. We will focus the lowest-order mode in this direction.[26] To further simplify the model, we consider only one photon mode of the microcavity with wave-vector $\boldsymbol{k}$ and frequency ω very close to the lowest $\nu = 1$s exciton energy level. The 1s exciton has the strongest oscillator strength and is most actively coupled with the photon. Because of the smallness of the photon wave vector, $\boldsymbol{k}$ can then be put equal to zero.[26,27] And if only $\boldsymbol{k} = 0$, $\nu = 1$s excitons exist. Due to the translational invariance in the plane of the microcavity, photon with in-plane wavevector $\boldsymbol{k}$ can only be coupled with an exciton having the same in-plane wavevector $\boldsymbol{K} = \boldsymbol{k}$. In view of the above disscussion, we can consider the interaction of a cavity mode with one exciton mode only, giving rise to strong coupling.[28] And all the other exciton modes weakly coupled to the resonant mode can be regarded as a reservoir.

Neglecting the spin degrees of freedom, an effective interaction Hamiltonian for the coupled exciton–photon system in the cavity is as follows[19,27]

$$H_{\rm eff} = \omega_{\rm c} a^{\dagger} a + \omega_{\rm ex} b^{\dagger} b + g(a^{\dagger} b + b^{\dagger} a) + A b^{\dagger} b^{\dagger} b b, \quad (1)$$

where a and $a^{\dagger}$ are the annihilation and creation operators of the cavity field with frequency $\omega_{\rm c}$, respectively and b and $b^{\dagger}$ are the annihilation and creation operators of exciton with frequency $\omega_{\rm ex}$. We consider both of them bosonic operators here. g corresponds to the strength of the coupling between the field mode and the exciton mode. A stands for the strength of exciton–exciton coupling which arose from the Coulomb interaction. We may see that it is similar to the model describing a single-mode field coupled with a nonlinear medium[29] or a Bose–Einstein condensate (BEC) in a tilted double well.[30−33] The energy eigenvalues and eigenstates for the fully quantized Hamiltonian depicting these systems can be obtained by using proper methods.[29] We shall pay our attention to the dynamics under mean-field approximation. Using the well-known Schwinger representation of the angular momentum,[34]

$$\begin{aligned} J_x &= \frac{b^{\dagger} a + a^{\dagger} b}{2}, \\ J_y &= \frac{b^{\dagger} a - a^{\dagger} b}{2{\rm i}}, \\ J_z &= \frac{b^{\dagger} b - a^{\dagger} a}{2}. \end{aligned} \quad (2)$$

The Hamiltonian can be written as

$$H_{\rm eff} = \Theta J_z + 2g J_x + A J_z^2 + f(j), \quad (3)$$

where $\Theta = \omega_{\rm ex} - \omega_{\rm c} + A(2j-1)$, and $f(j) = Aj^2 + (\omega_{\rm c} + \omega_{\rm ex} - A)j$, $j = N/2$, and $N = a^{\dagger} a + b^{\dagger} b$ is the total number of the system. For simplification, in this study, we set $\omega_{\rm ex} = \omega_{\rm c} = 1$. Since we focus on the dynamics of the system and, item $f(j)$ would not change with time, and we assume the total number is conserved, so $f(j)$ can be dropped safely. The Hamiltonian is rewritten as:

$$H = \Theta J_z + 2g J_x + A J_z^2. \quad (4)$$

Starting from the Heisenberg equations of motion

$$\frac{{\rm d}F}{{\rm d}t} = -\frac{\rm i}{\hbar}[F, H], \quad (5)$$

we give the time evolution equations of the three angular momentum operators as follows,

$$\begin{aligned} \frac{{\rm d}J_x}{{\rm d}t} &= -\Theta J_y - A J_y J_z - A J_y J_z, \\ \frac{{\rm d}J_y}{{\rm d}t} &= \Theta J_x - 2g J_z + A J_x J_z + A J_z J_x, \\ \frac{{\rm d}J_z}{{\rm d}t} &= 2g J_y. \end{aligned} \quad (6)$$

We may define three real variables,

$$S_k = 2\langle J_k\rangle/N, k = x, y, z, \tag{7}$$

where N is the total particle number. It is known that S_x is the particle number difference between the two single-particle energy eigenstates of exciton–photon system, S_y represents the mean momentum of the model, while S_z corresponds to the occupation difference of the two modes. Rewriting the Eq. (6) in terms of the formula above via taking a second-order approximation $\langle J_i J_k\rangle \approx \langle J_i\rangle\langle J_k\rangle$ which can be called a mean field approximation,[35] one is able to obtain,

$$\frac{\mathrm{d}S_x}{\mathrm{d}t} = -\Theta S_y - ANS_yS_z, \tag{8}$$

$$\frac{\mathrm{d}S_y}{\mathrm{d}t} = \Theta S_x - 2gS_z + ANS_zS_x, \tag{9}$$

$$\frac{\mathrm{d}S_z}{\mathrm{d}t} = 2gS_y. \tag{10}$$

Our task is to solve the two-mode mean-field model for the possible initial conditions.

3. Exact analytical solution

In this section, we shall obtain the exact analytical solution to the mean-field model. Starting from Eqs. (8)–(10), we have,

$$S_x(t) = -\frac{AN}{4g}[S_z^2(t) - S_z^2(0)] - \frac{\Theta}{2g}[S_z(t) - S_z(0)] + S_x(0), \tag{11}$$

$$S_y(t) = \frac{1}{2g}\frac{\mathrm{d}S_z(t)}{\mathrm{d}t}, \tag{12}$$

$$\frac{\mathrm{d}^2S_z(t)}{\mathrm{d}t^2} = \alpha S_z^3(t) + \beta S_z^2(t) + \gamma S_z(t) + \delta. \tag{13}$$

Multiplying Eq. (13) by $2\mathrm{d}S_z(t)/\mathrm{d}t$ and integrating it once, one can obtain

$$\left(\frac{\mathrm{d}S_z(t)}{\mathrm{d}t}\right)^2 = aS_z^4(t) + bS_z^3(t) + cS_z^2(t) + \mathrm{d}S_z(t) + e, \tag{14}$$

where

$$\begin{aligned} a &= -\frac{1}{4}A^2N^2,\\ b &= -AN\Theta,\\ c &= \frac{A^2N^2}{2}S_z^2(0) + AN\Theta S_z(0)\\ &\quad + 2ANgS_x(0) - \Theta^2 - 4g^2,\\ d &= AN\Theta S_z^2(0) + 2\Theta^2 S_z(0)\\ &\quad + 4g\Theta S_x(0),\\ e &= -aS_z^4(0) - bS_z^3(0) - cS_z^2(0)\\ &\quad - dS_z(0) + 4g^2S_y^2(0). \end{aligned} \tag{15}$$

In order to get the analytical solutions for Eq. (14), we shall do some algebraic manipulation and use the famous Jacobi elliptic functions[36] at the meantime. See Appendix, we find two solutions,

$$\begin{aligned} S_x(t) &= -\frac{AN}{4g}\left[\left(\frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1}\right)^2 - S_z^2(0)\right] - \frac{\Theta}{2g}\left[\frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1} - S_z(0)\right] + S_x(0),\\ S_y(t) &= \frac{1}{2g}\frac{\mathrm{d}S_z(t)}{\mathrm{d}t} = -\frac{(\lambda_2 - \lambda_1)s(0)}{2g}\frac{\mathrm{sn}(\zeta t,k)\mathrm{dn}(\zeta t,k)}{(s(0)\mathrm{cn}(\zeta t,k) - 1)^2},\\ S_z(t) &= \frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} S_x(t) &= -\frac{AN}{4g}\left[\left(\frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{dn}(\zeta t,k) - 1}\right)^2 - S_z^2(0)\right] - \frac{\Theta}{2g}\left[\frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{dn}(\zeta t,k) - 1} - S_z(0)\right] + S_x(0),\\ S_y(t) &= \frac{1}{2g}\frac{\mathrm{d}S_z(t)}{\mathrm{d}t} = -\frac{(\lambda_2 - \lambda_1)k^2 s(0)}{2g}\frac{\mathrm{sn}(\zeta t,k)\mathrm{cn}(\zeta t,k)}{(s(0)\mathrm{dn}(\zeta t,k) - 1)^2},\\ S_z(t) &= \frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{dn}(\zeta t,k) - 1}. \end{aligned} \tag{17}$$

The expressions of the variables above and the scope of the parameter for each solution are also given in the Appendix.

4. Dynamics

In this section, we will illustrate the main features of the dynamics of the system under the two-mode mean-field approximation. In Fig. 1, we display the main features of the three angular momentum components by plotting $S_x(t), S_y(t), S_z(t)$ as a function of dimensionless time ζt of different initial value $S_x(0), S_y(0), S_z(0)$. We find that there has no existence for expression Eq. (17) corresponding to the meaningful initial conditions.

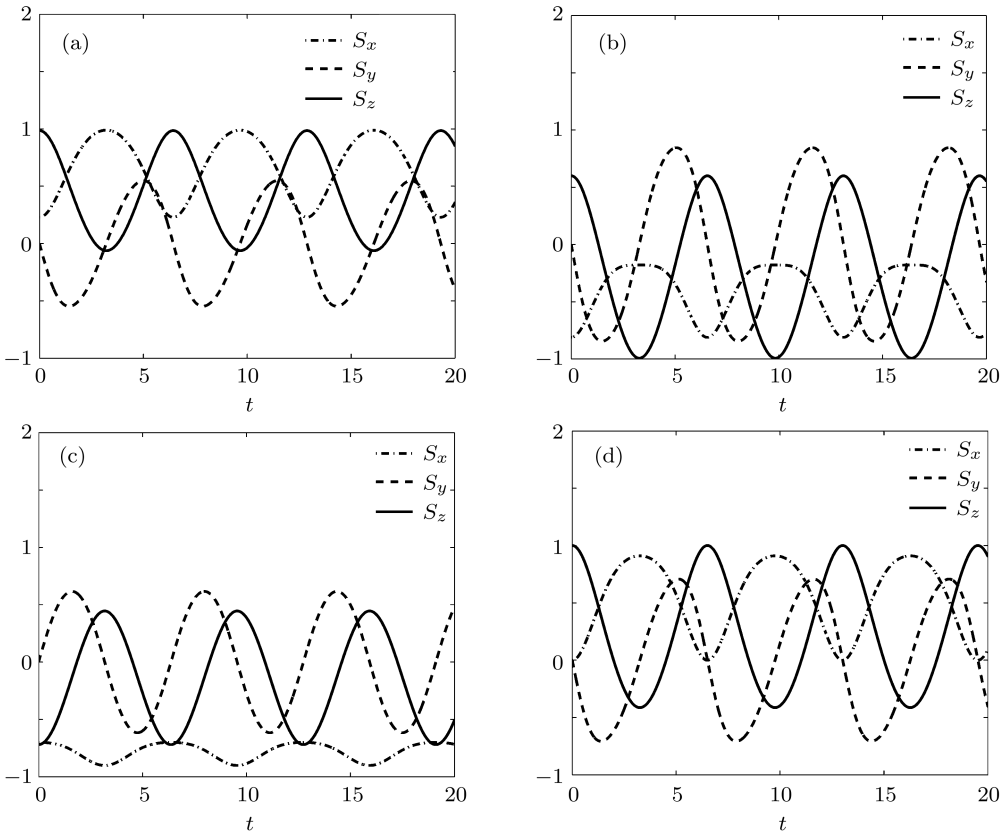


Fig. 1. The angular momentum components as a function of dimensionless time ζt. (a) $S_x(0) = 0.245, S_y(0) = 0, S_z(0) = 0.969,\ g = 100A,\ N = 100$. (b) $S_x(0) = -0.812, S_y(0) = 0, S_z(0) = 0.6,\ g = 100A,\ N = 100$. (c) $S_x(0) = -0.7, S_y(0) = 0, S_z(0) = -0.728,\ g = 500A,\ N = 200$. (d) $S_x(0) = 0, S_y(0) = 0, S_z(0) = 1,\ g = 200A,\ N = 200$. The dashed-dotted, dashed and solid lines in the figures are correspond to $S_x(t),\ S_y(t),\ S_z(t)$ respectively.

It is clear seen that $S_x(t), S_y(t), S_z(t)$ exhibit periodic oscillations between the two modes with a period T,

$$T = \frac{4K}{\zeta} = \frac{4}{\zeta}\int_0^{\pi/2} \frac{\mathrm{d}\theta}{\sqrt{1 - k^2 \sin^2\theta}}. \tag{18}$$

In order to compare our results with the numerical simulations, we plot the time evolution for the three angular momentums in Fig. 2(a) with the same system parameters and initial conditions. The solid, dotted and dashed-dotted lines represent our analytical results, and the markers of pentagram, ring and cross denote the results for numerical simulations, i.e., numerically solving Eqs. (8)–(10). It is clearly seen from the figure that our analytical results are consistent with the numerical results. Furthermore, we plot the phase space structure generated by the the angular momentums S_x, S_y, S_z in Fig. 2(b). Also, a periodic evolution behaviour has been revealed for the three angular momentums.

Next, we may return to the three real variables which have been defined ahead, giving the expressions as follows,

$$S_x = b^* a + a^* b, \tag{19}$$

$$S_y = -\mathrm{i}(b^* a - a^* b), \tag{20}$$

$$S_z = |b|^2 - |a|^2. \tag{21}$$

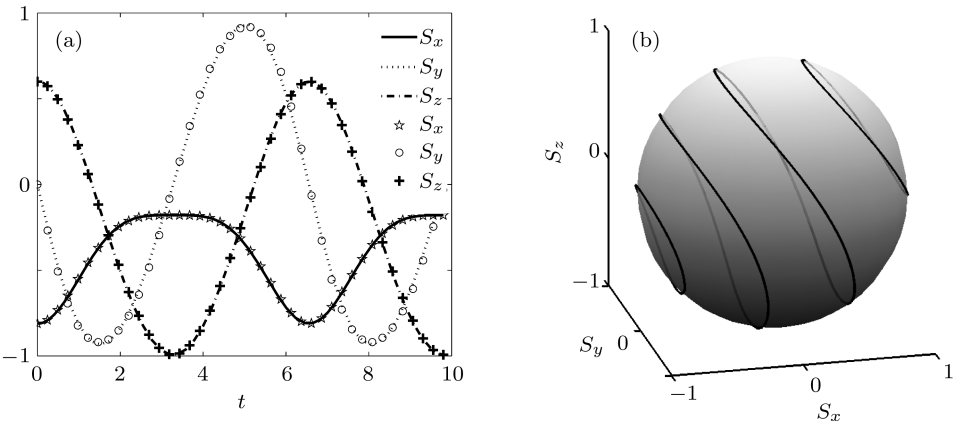


Fig. 2. (a) The angular momentum components as a function of dimensionless time ζt. $S_x(0) = -0.812, S_y(0) = 0, S_z(0) = 0.6,\ g = 100A,\ N = 100$. The dashed-dotted, dashed and solid lines in the figures are correspond to $S_x(t),\ S_y(t),\ S_z(t)$ respectively for the analytical results. The markers of pentagram, ring and cross denote the results of numerical simulations. (b) Mean-field dynamics for the angular momentum components in the phase space.

In Fig. 3(a), we show the mean-field solution for the quantity $S_z = b^\dagger b - a^\dagger a$ with different parameters. We set $g = 200A$, and the dashed-dotted line is for $N = 200$, the dashed line and solid line are corresponding to $N = 400$ and $N = 800$ respectively. It is clearly seen that when $N = 200, k = AN/g = 0.5$, almost complete exchange between the two modes can be observed which is analogous to Rabi-like oscillations.[37] With the increase of k, a rather uncompleted oscillations is shown in Fig. 3(a) which means for larger numbers of atoms, the macroscopic coherent tunneling is suppressed, and it is similar to the self-trapping transition in a double well BEC or a spit BEC.[38−44] Suppose the system is completely in the exciton mode at the initial time, few will invert into the photon mode during the evolution. In the dilute limit, excitons act as weakly interacting bosons and undergo BEC. Also, the excitonic insulator at high densities[45] have been studied experimentally.[46] We note that, from another viewpoint, when we set g, which stands for the coupling strength between the exciton mode and the photon mode, relatively large to NA, then in the Hamiltonian the term containing g will dominate, resulting in Rabi oscillation behaviour to the system. While the particle number N is larger, the term which includes NA shall dominate, and probably leads to a self-trapping like phenomenon. Also, for the case of self-trapping, we plot the phase space structure generated by the the angular momentums S_x, S_y, S_z in Fig. 3(b) for different initial conditions.

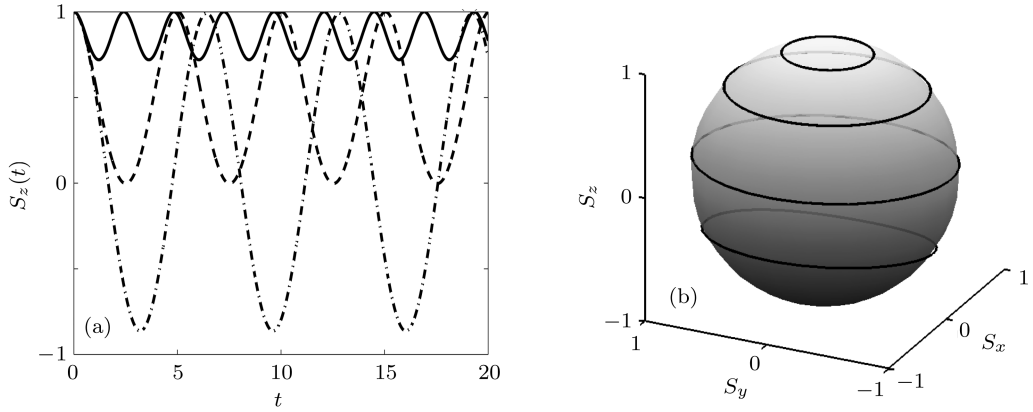


Fig. 3. (a) Mean-field solution for the occupation difference $S_z = b^\dagger b - a^\dagger a$ versus dimensionless time $945At$. The dashed-dotted, dashed and solid lines in the figure are for $k = 0.5, k = 1, k = 2$, $k = AN/g$ respectively. (b) Mean-field dynamics for the angular momentum components in the phase space.

See from the figure, S_z changes small during the evolution, and when the system is completely in the case of self-trapping, the curves will form the latitude lines of the Bloch ball. Before ending this section, we would like to point out that the system's dynamics can be surely solved in a straightforward way by numerically simulating first-order differential equations, however, by using our analytical method, we can give the solutions in the form of Jacobi elliptic functions and predict the character of the system's dynamic behaviour to some extent.

5. Conclusion

Stating from the two-mode mean-field model, we have derived the analytical solutions describing the dynamics of the exciton–cavity system. The analytical solutions satisfy relevant parameters including possible initial conditions. We have shown that the three components of the angular momentum manifest periodic oscillation behaviour. In particular, when the number of atom is relatively larger, the oscillating can be suppressed, which is analogous to the self-trapping transition.

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Appendix

In order to get the analytical solutions, let us do some manipulation to Eq. (14)

$$\left(\frac{\mathrm{d}S_z(t)}{\mathrm{d}t}\right)^2 = aS_z^4(t) + bS_z^3(t) + cS_z^2(t) + dS_z(t) + e.$$

Let

$$\begin{aligned} & P(x) \\ &= aS_z^4(t) + bS_z^3(t) + cS_z^2(t) + dS_z(t) + e \\ &= P_1(x)P_2(x) \\ &= (a_1x^2 + 2b_1x + c_1)(a_2x^2 + 2b_2x + c_2) \\ &= a_1a_2(x-\alpha_1)(x-\beta_1)(x-\alpha_2)(x-\beta_2), \end{aligned} \tag{A1}$$

where (α_1, β_1), (α_2, β_2) are the roots for $P_1(x) = 0$, $P_2(x) = 0$. Setting $S_z = (\lambda_1 s - \lambda_2)/(s-1)$, one can gain the expression as follows,

$$\begin{aligned} P_1(x) &= \frac{p_1 s^2 + q_1}{(s-1)^2}, \\ P_2(x) &= \frac{p_2 s^2 + q_2}{(s-1)^2}, \\ p_1 &= a_1\lambda_1^2 + 2b_1\lambda_1 + c_1, \\ q_1 &= a_1\lambda_2^2 + 2b_1\lambda_2 + c_1, \\ p_2 &= a_2\lambda_1^2 + 2b_2\lambda_1 + c_2, \\ q_2 &= a_1\lambda_2^2 + 2b_2\lambda_2 + c_2, \end{aligned} \tag{A2}$$

λ_1, λ_2 are the roots of the equation

$$\begin{aligned} &(a_1b_2 - a_2b_1)\lambda^2 + (a_1c_2 - a_2c_1)\lambda \\ &+ (b_1c_2 - b_2c_1) = 0. \end{aligned} \tag{A3}$$

One can easily prove λ_1, λ_2 are real whenever the roots for $P(x) = 0$ are real or conjugated. And $p_1p_2 < 0$ since the parameter $a < 0$. Then we find

$$\begin{aligned} \left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 &= \frac{|p_1p_2|}{(\lambda_2 - \lambda_1)^2}Q(s), \\ Q(s) &= \left(-s^2 - \frac{q_1}{p_1}\right)\left(s^2 + \frac{q_2}{p_2}\right). \end{aligned} \tag{A4}$$

By using the famous Jacobi elliptic functions and assuming that at the initial time, $t = 0$, the solution of Eq. (A4) can be given as follows, for $q_1/p_1 < 0$, $q_2/p_2 > 0$, $s < \sqrt{|q_1/p_1|}$,

$$s(t) = s(0)\mathrm{cn}(\zeta t, k), \tag{A5}$$

where $s(0) = (S_z(0) - \lambda_2)/(S_z(0) - \lambda_1)$,

$$k = \sqrt{\frac{\left|\frac{q_1}{p_1}\right|}{\left|\frac{q_1}{p_1}\right| + \frac{q_2}{p_2}}}, \quad \zeta = \sqrt{\frac{|p_1p_2|\left(\left|\frac{q_1}{p_1}\right| + \frac{q_2}{p_2}\right)}{(\lambda_2 - \lambda_1)^2}}, \tag{A6}$$

$$\begin{aligned} S_x(t) =& -\frac{AN}{4g}\left[\left(\frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1}\right)^2 - S_z^2(0)\right] \\ & - \frac{\Theta}{2g}\left[\frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1} - S_z(0)\right] + S_x(0), \\ S_y(t) =& \frac{1}{2g}\frac{\mathrm{d}S_z(t)}{\mathrm{d}t} \\ =& -\frac{(\lambda_2 - \lambda_1)s(0)}{2g}\frac{\mathrm{sn}(\zeta t,k)\mathrm{dn}(\zeta t,k)}{(s(0)\mathrm{cn}(\zeta t,k) - 1)^2}, \\ S_z(t) =& \frac{\lambda_1 s(0)\mathrm{cn}(\zeta t,k) - \lambda_2}{s(0)\mathrm{cn}(\zeta t,k) - 1}, \end{aligned} \tag{A7}$$

for $q_1/p_1 < 0$, $q_2/p_2 < 0$, $\sqrt{|q_2/p_2|} < s < \sqrt{|q_1/p_1|}$,

$$s(t) = s(0)\mathrm{dn}(\zeta t, k), \tag{A8}$$

where $s(0) = (S_z(0) - \lambda_2)/(S_z(0) - \lambda_1)$,

$$k = \sqrt{1 - \frac{\left|\frac{q_2}{p_2}\right|}{\left|\frac{q_1}{p_1}\right|}}, \quad \zeta = \sqrt{\frac{|p_1p_2|\left|\frac{q_1}{p_1}\right|}{(\lambda_2 - \lambda_1)^2}}. \tag{A9}$$

Then

$$S_x(t) = -\frac{AN}{4g}\left[\left(\frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k)-\lambda_2}{s(0)\mathrm{dn}(\zeta t,k)-1}\right)^2 - S_z^2(0)\right] - \frac{\Theta}{2g}\left[\frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k)-\lambda_2}{s(0)\mathrm{dn}(\zeta t,k)-1} - S_z(0)\right] + S_x(0),$$

$$S_y(t) = \frac{1}{2g}\frac{\mathrm{d}S_z(t)}{\mathrm{d}t} = -\frac{(\lambda_2-\lambda_1)k^2 s(0)}{2g}\frac{\mathrm{sn}(\zeta t,k)\mathrm{cn}(\zeta t,k)}{(s(0)\mathrm{dn}(\zeta t,k)-1)^2},$$

$$S_z(t) = \frac{\lambda_1 s(0)\mathrm{dn}(\zeta t,k)-\lambda_2}{s(0)\mathrm{dn}(\zeta t,k)-1}. \quad \text{(A10)}$$

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