Unconditional preparation of multipartite continuous-variable entangled states among remote parties*

Hu Xue-Yuan(胡雪元)[a)], Gu Ying(古 英)[a)†], Gong Qi-Huang(龚旗煌)[a)], and Guo Guang-Can(郭光灿)[a)b)]

[a)] *State Key Laboratory for Mesoscopic Physics, School of Physics, Peking University, Beijing* 100871, *China*
[b)] *Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei* 230026, *China*

(Received 8 July 2009; revised manuscript received 9 November 2009)

We demonstrate that the n-partite continuous-variable entanglement can be unconditionally prepared among n parties that share no common past, from n two-mode squeezed states. Both GHZ-like and cluster-like states can be generated for any nonzero squeezing in the entangled sources. An application of the resulting multipartite entangled state to a teleportation network is illustrated.

Keywords: entanglement redistribution, multipartite continuous-variable entanglement, a teleportation network

PACC: 0367

1. Introduction

Entanglement is an essential resource in quantum communications. Multipartite entangled states embody interesting properties such as monogamy property,[1] and enable various quantum information tasks realised, such as quantum teleportation networks,[2] quantum secret sharing of classical messages,[3,4] quantum state sharing,[5,6] quantum computation,[7] etc. Recently, the generation of multipartite entangled states have drawn much attention in both theoretical and experimental fields. Multipartite entangled qubits can be generated from Bell pairs using beam splitters (BSs) and single-photon detectors,[8,9] and entangled state of up to six qubits has been experimentally realised.[10] The generation of continuous-variable (CV) entangled states is simpler and more efficient as compared with its discrete-variable counterpart. It was shown that, by applying a sequence of $n-1$ BSs (which give $1:(n-1)$, $1:(n-2),\ldots,$ $1:1$, respectively) to n single-mode squeezed vacuum states (including one phase-squeezed and $n-1$ amplitude-squeezed vacuum modes), n-partite Greenberger–Horne–Zeilinger (GHZ)-like Gaussian entangled states could be theoretically generated for n being arbitrary.[2] Using the same BS array but replacing one amplitude-squeezed input mode by a phase-squeezed mode, we can also prepare quadripartite cluster-like states.[11] Employing these schemes, tripartite GHZ-like Gaussian entangled state was experimentally prepared and applied to a teleportation network;[12] furthermore, quadripartite GHZ-like and cluster-like Gaussian entangled states were also experimentally prepared.[13] Another interesting scheme for tripartite Gaussian entangled state generation and controlled dense coding was proposed[14] and experimentally realised,[15] where only a two-mode squeezed state and linear optics were required. In all of the above schemes, multipartite entangled states are directly generated and then distributed to the agents.

Entanglement redistribution offers a flexible method for manipulating entangled states. For example, entanglement swapping[16−18] and cascading[19,20] can establish entanglement between two remote parties that do not share any common past, from a sequence of entangled pairs. As a multipartite generalisation of entanglement swapping, entangled states of many qubits can be established from singlets by a local measurement.[21] This method of preparing multipartite entangled states embodies many practical ad-

*Project supported by the National Natural Science Foundation of China (Grants Nos. 10674009, 10874004 and 10821062) and the National Key Basic Research Program of China (Grant No. 2006CB921601).
†Corresponding author. E-mail: ygu@pku.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

vantages: first, it allows a certain degree of freedom to entangle any set of the parties on demand; second, when the system is exposed to decoherence or eavesdropping, one only needs to modify entangled pairs instead of multipartite entangled states. Since entanglement redistribution protocols are believed to be more efficient and easier to manipulate in the CV scenario, establishing multipartite CV entangled states by entanglement redistribution would be significant.

In this paper, we propose an unconditional protocol for preparing multipartite Gaussian entangled states among remote parties that do not share any common past, and then illustrate a practical use of the resulted entangled states for a teleportation network. Consider n agent nodes in a quantum communication network, each of whom initially shares a two-mode squeezed state with a central node. These initial two-mode squeezed states can be experimentally generated by using one beam splitter and two single-mode squeezed states from optical parameter oscillators in analogy with that in Ref. [12], or by injecting two orthogonal polarised coherent states with the same frequency into a nondegenerate optical parametric amplifier as in Refs. [14] and [15]. In order to obtain the agent nodes entangled, the central node transmits his/her modes through a BS array composed of $n-1$ BSs, measures the amplitude or phase quadratures of the outputs using balanced homodyne detectors, and communicates the classical results to $n-1$ agents. The agents then optimally displace their modes according to the measurement results and finally share an n-partite Gaussian entangled state. The prepared entangled state can be used as a source in quantum teleportation networks, where the teleportation fidelity is higher than 1/2 and also serves as a practical witness of the multipartite entanglement. The multipartite entangled state among any set of agents can be prepared on demand for any nonzero squeezing in all of the entangled sources. Compared with the previous protocol as in Refs. [2], [12], and [15], the teleportation fidelity achieved in our scheme is a little lower, which is a certain consequence of the nonincreasing property of entanglement under local quantum operations and classical communications; however, our method is advantaged in its flexibility, i.e., Hebe need not know in advance which set of agents will be involved in the teleportation network and where exactly the agents are. Besides, with the same experimental arrangement, our protocol can prepare different types of multipartite Gaussian entangled states, such as GHZ-like states and cluster-like states,[11] depending on which quadratures are measured.

2. The protocol for unconditionally preparing multipartite entangled states by entanglement redistribution

Consider N independent agents (named Alice, Bob, *et al.*) of a workshop, each of whom shares a two-mode squeezed state with a central node named Hebe. Now Hebe wants n of the agents to collaborate for some practical missions, so she tries to build an n-mode Gaussian entangled state shared among them. The aim of this section is to help Hebe solve this problem. Notice that for the $n=2$ case, the unconditional CV entanglement swapping protocol[17] would accomplish this task. For $n \geqslant 3$, we here propose an unconditional protocol for preparing multipartite Gaussian entangled states among remote nodes. We first describe the protocol for $n=3$ in the Wigner picture, and then generalise it to the cases for a generic n. It is worth mentioning that, when Hebe chooses different quadratures to measure, our protocol can prepare Gaussian GHZ-like states or cluster-like states on demand with the same experimental arrangement.

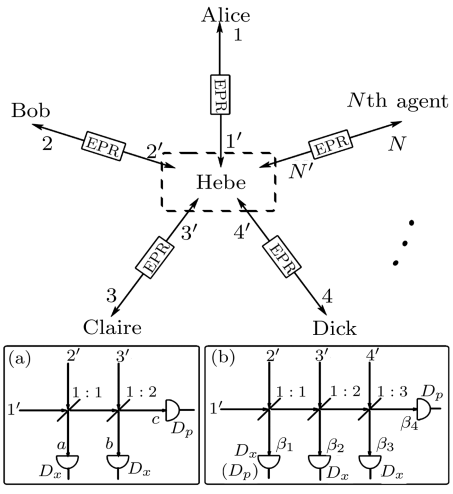


Fig. 1. Scheme for unconditional generation of multipartite Gaussian entangled states among a set of agents that share no common past. Top: Each of the agents initially share with Hebe Einstein–Podolsky–Rosen (EPR) pairs. Bottom: Hebe's operations for generation of (a) tripartite GHZ-like states and (b) quadripartite GHZ-like (cluster-like) states. Here D_x, D_p present the balanced homodyne detections (BHDs) for detecting the corresponding quadratures, and (D_p) in (b) means using D_p instead of D_x for beam β_1 for preparing quadripartite cluster-like states.

In Fig. 1(a) is shown the scheme for the unconditional preparation of tripartite Gaussian entangled states among Alice, Bob and Claire. Initially, each agent shares with Hebe a two-mode squeezed state of mode j and mode j' ($j = 1, 2, 3$), whose Wigner function reads

$$\begin{aligned} & W_{j,j'}^{TMSV}(\alpha_j, \alpha_{j'}) \\ = & \frac{1}{\pi^2} \exp\left\{ -\frac{\mathrm{e}^{2r}}{2}[(x_j - x_{j'})^2 + (p_j + p_{j'})^2] \right. \\ & \left. -\frac{\mathrm{e}^{-2r}}{2}[(x_j + x_{j'})^2 + (p_j - p_{j'})^2] \right\}, \end{aligned} \tag{1}$$

where $\alpha_{j(j')} = (x_{j(j')} + \mathrm{i}p_{j(j')})/\sqrt{2}$, and r is the squeezing parameter. Here $x_{j(j')}$ and $p_{j(j')}$ are respectively amplitude and phase quadratures of mode $j(j')$. For the first step, Hebe combines her three modes on a BS array (BS1 and BS2 give $1:1$ and $1:2$ respectively). The quadratures of the three output beams are

$$\begin{aligned} \beta_a &= \frac{1}{\sqrt{2}}(\alpha_{1'} - \alpha_{2'}), \\ \beta_b &= \frac{1}{\sqrt{6}}(\alpha_{1'} + \alpha_{2'} - 2\alpha_{3'}), \\ \beta_c &= \frac{1}{\sqrt{3}}(\alpha_{1'} + \alpha_{2'} + \alpha_{3'}), \end{aligned} \tag{2}$$

where $\beta_k = (x_k + \mathrm{i}p_k)/\sqrt{2}$, $k = a, b, c$. Now Hebe measures the amplitude quadratures of mode a and mode b as well as the phase quadrature of mode c, yielding classical results x_a, x_b and p_c. This measurement is equivalent to Hebe's projecting her three modes on an ideal tripartite Gaussian GHZ-like state. Now the normalised Wigner function of Alice's mode 1, Bob's mode 2 and Claire's mode 3 becomes

$$\begin{aligned} W_p(\alpha_1, \alpha_2, \alpha_3 | x_a, x_b, p_c) = & \frac{1}{\mathcal{P}(x_a, x_b, p_c)} \int \mathrm{d}\alpha_{1'}^2 \int \mathrm{d}\alpha_{2'}^2 \int \mathrm{d}\alpha_{3'}^2 \prod_{j=1}^{3} W_{j,j'}^{TMSV}(\alpha_j, \alpha_{j'}) \cdot \delta(x_{1'} - x_{2'} - \sqrt{2}x_a) \\ & \times \delta(x_{1'} + x_{2'} - 2x_{3'} - \sqrt{6}x_b)\delta(p_{1'} + p_{2'} + p_{3'} - \sqrt{3}p_c) \\ = & \frac{1}{\pi^3} \exp\left[-\cosh(2r)\left(\frac{x_1 - x_2 + g\sqrt{2}x_a}{\sqrt{2}}\right)^2 - \cosh(2r)\left(\frac{x_1 + x_2 - 2x_3 + g\sqrt{6}x_b}{\sqrt{6}}\right)^2 \right. \\ & -\cosh(2r)\left(\frac{p_1 + p_2 + p_3 + g\sqrt{3}p_c}{\sqrt{3}}\right)^2 - \frac{1}{\cosh(2r)}\left(\frac{x_1 + x_2 + x_3}{\sqrt{3}}\right)^2 \\ & \left. -\frac{1}{\cosh(2r)}\left(\frac{p_1 - p_2}{\sqrt{2}}\right)^2 - \frac{1}{\cosh(2r)}\left(\frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)^2 \right], \end{aligned} \tag{3}$$

where $g = \tanh(2r)$, and $\mathcal{P}(x_a, x_b, p_c) = \int \mathrm{d}\alpha_{1'}^2 \int \mathrm{d}\alpha_{2'}^2 \int \mathrm{d}\alpha_{3'}^2 \prod_{j=1}^{3} W_{j,j'}^{TMSV}(\alpha_j, \alpha_{j'})$ is the probability of obtaining the measurement results (x_a, x_b, p_c). A direct calculation leads to $\langle \Delta(x_i - x_j/\sqrt{2})^2 \rangle = 1/2\cosh(2r) < 1/2$ for $i, j = 1, 2, 3$, $i \neq j$ and $\langle \Delta((p_1 + p_2 + p_3)/\sqrt{2})^2 \rangle = 1/2\cosh(2r) < 1/2$, which indicates that the resulted state of mode 1, mode 2, and mode 3 is a tripartite GHZ-like entangled state.[11] However, because of the randomness of the measurement results, Alice, Bob and Claire cannot have entangled states without the information of Hebe's measurement. At least two of them should receive the results from Hebe through classical channels and displace their modes accordingly. Here Alice and Bob receive the results and displace mode 1 and mode 2 respectively as

$$\begin{aligned} & x_1 \to X_1 = x_1 + \frac{g}{2}\sqrt{2}x_a + \frac{g}{2}\sqrt{6}x_b, \quad p_1 \to P_1 = p_1, \\ & x_2 \to X_2 = x_2 - \frac{g}{2}\sqrt{2}x_a + \frac{g}{2}\sqrt{6}x_b, \quad p_2 \to P_2 = p_2 + g\sqrt{3}p_c. \end{aligned} \tag{4}$$

The state of the three modes then becomes

$$\begin{aligned} & W_d(\alpha_1^d, \alpha_2^d, \alpha_3 | x_a, x_b, p_c) \\ = & \frac{1}{\pi^3} \exp\left[-\cosh(2r)\left(\frac{X_1 - X_2}{\sqrt{2}}\right)^2 - \cosh(2r)\left(\frac{X_1 + X_2 - 2x_3}{\sqrt{6}}\right)^2 \right. \\ & \left. -\cosh(2r)\left(\frac{p_1 + P_2 + p_3}{\sqrt{3}}\right)^2 - \frac{1}{\cosh(2r)}\left(\frac{X_1 + X_2 + x_3 - g\sqrt{6}x_b}{\sqrt{3}}\right)^2 \right. \end{aligned}$$

$$-\frac{1}{\cosh(2r)}\left(\frac{p_1-P_2+g\sqrt{3}p_c}{\sqrt{2}}\right)^2-\frac{1}{\cosh(2r)}\left(\frac{p_1+P_2-2p_3-g\sqrt{3}p_c}{\sqrt{6}}\right)^2\Bigg], \tag{5}$$

where $\alpha^d_{1,2}=(X_{1,2}+\mathrm{i}P_{1,2})/\sqrt{2}$. Now we define the new variables as

$$\begin{aligned} X_a&=\frac{X_1-X_2}{\sqrt{2}},\quad X_b=\frac{X_1+X_2-2x_3}{\sqrt{6}},\quad X_c=\frac{X_1+X_2+x_3}{\sqrt{3}},\\ P_a&=\frac{p_1-P_2}{\sqrt{2}},\quad P_b=\frac{p_1+P_2-2p_3}{\sqrt{6}},\quad P_c=\frac{p_1+P_2+p_3}{\sqrt{3}}. \end{aligned} \tag{6}$$

For a tripartite Gaussian GHZ-like entangled state, the variances of variables $X_a,\ X_b,\ P_c$ should be smaller than $1/2$.[11] This condition is satisfied by the state in Eq. (5), where $\langle\Delta X_a^2\rangle=\langle\Delta X_b^2\rangle=\langle\Delta P_c^2\rangle=1/2\cosh(2r)<1/2$ for all $r>0$. The final three-mode state shared among the agents is a mixed state obtained by integrating Eq. (5) over all possible measurement results as

$$\begin{aligned} &W_f(\alpha_1^d,\alpha_2^d,\alpha_3)\\ &=\int \mathrm{d}x_a\int \mathrm{d}x_b\int \mathrm{d}p_c\mathcal{P}(x_a,x_b,p_c)\\ &\quad\times W_d(\alpha_1^d,\alpha_2^d,\alpha_3|x_a,x_b,p_c). \end{aligned} \tag{7}$$

Nevertheless, the mixing does not affect the variances of $X_a,\ X_b,\ P_c$, because from Eq. (5), the distributions of these variables are independent of the measurement results after the displacements in Eq. (4). In turn, Alice's mode 1, Bob's mode 2 and Claire's mode 3 are now entangled in a GHZ-like way.

From the above discussion, the essentials of our protocol are twofold: firstly, Hebe's measurement projects the modes shared among Alice, Bob and Claire on a tripartite GHZ-like state; secondly, Alice and Bob optimally displace their modes such that the distributions of variables $X_a,\ X_b,\ P_c$ are independent of the measurement results, and consequently, the entanglement is preserved after the mixing as in Eq. (7). Bearing these in mind, we naturally generalise the protocol to produce GHZ-like states of larger size. Now Hebe decides to make n agents ($n\geqslant 3$) involved in a mission. Still, the j-th agent ($j=1,2,\ldots,n$) initially shares with Hebe a two-mode squeezed state of mode j and mode j' described in Eq. (1). As the first step, Hebe transmits her n modes through a sequence of $n-1$ BSs where the k-th BS ($k=1,2,\ldots,n-1$) gives $1:k$ (for the $n=4$ case, see Fig. 1(b)). She then measures the amplitude quadratures of the $n-1$ output beams $\beta_1,\ \beta_2,\ldots,\ \beta_{n-1}$ as well as the phase quadrature of the output beam β_n, yielding classical results $x_{\beta_1}=(x_{1'}-x_{2'})/\sqrt{2},\ \ldots,\ x_{\beta_{n-1}}=\left[\sum_{j=1}^{n-1}x_{j'}-(n-1)x_{n'}\right]/\sqrt{n(n-1)}$, $p_{\beta_n}=\sum_{j=1}^{n}p_{j'}/\sqrt{n}$, which are then classically sent to the first $n-1$ agents. After receiving the results, the agents locally displace their modes as

$$\begin{aligned} p_1&\to P_1=p_1+\sqrt{n}gp_{\beta_n},\\ x_k&\to X_k=x_k-g\sqrt{\frac{k-1}{k}}x_{\beta_{k-1}}\\ &\quad+g\sum_{j=k}^{n-2}\frac{1}{\sqrt{j(j+1)}}x_{\beta_j}+g\sqrt{\frac{n}{n-1}}x_{\beta_{n-1}},\\ &\quad(k=1,2,\ldots,n-2),\\ x_{n-1}&\to X_{n-1}=x_{n-1}-g\sqrt{\frac{n-1}{n}}x_{\beta_{n-2}}\\ &\quad+g\sqrt{\frac{n}{n-1}}x_{\beta_{n-1}}, \end{aligned} \tag{8}$$

such that the distributions of the variables defined as $X_{\beta_1}=(X_1-X_2)/\sqrt{2},\ldots,\ X_{\beta_{n-1}}=\left[\sum_{j=1}^{n-1}X_j-(n-1)x_n\right]/\sqrt{n(n-1)}$, $P_{\beta_n}=(P_1+\sum_{j=2}^{n}p_j)/\sqrt{n}$ are independent of the measurement results. As a result, the n modes shared among the n agents are in an n-partite GHZ-like state.

Multipartite cluster-like states can also be prepared by using the same arrangement but detecting different quadratures. As discussed in Ref. [11], n-partite CV cluster-like states and GHZ-like states are equivalent for $n=3$ but not so for $n>3$. For $n=4$, the quadratures of an ideal zero-displaced CV cluster-like state satisfy $p_1-p_2=0,\ 2p_2+p_3+p_4=0,\ x_1+x_2-2x_3=0$ and $x_3-x_4=0$; and equivalently, $p_1-p_2=0,\ p_1+p_2+p_3+p_4=0,\ x_1+x_2-2x_3=0$ and $x_1+x_2+x_3-3x_4=0$. Therefore, when Hebe detects the phase quadrature $p_{\beta_1}=(p_{1'}-p_{2'})/\sqrt{2}$, instead of the amplitude quadrature x_{β_1}, of the output beam β_1 and keeps the rest of the arrangement the same as in preparation of GHZ-like states (as shown in Fig. 1(b)), her four modes are projected on an ideal CV cluster-like state, resulting in a cluster-like quadripartite state of the modes shared among Alice, Bob, Claire and Dick, described as

$$
\begin{aligned}
& W_p(\alpha_1,\alpha_2,\alpha_3,\alpha_4|p_{\beta_1},x_{\beta_2},x_{\beta_3},p_{\beta_4}) \\
\propto & \int \mathrm{d}\alpha_{1'}^2 \int \mathrm{d}\alpha_{2'}^2 \int \mathrm{d}\alpha_{3'}^2 \int \mathrm{d}\alpha_{4'}^2 \prod_{j=1}^{4} W_{j,j'}^{TMSV}(\alpha_j,\alpha_{j'}) \\
& \times \delta(p_{1'}-p_{2'}-\sqrt{2}p_{\beta_1}) \cdot \delta(x_{1'}+x_{2'}-2x_{3'}-\sqrt{6}x_{\beta_2}) \\
& \times \delta(x_{1'}+x_{2'}+x_{3'}-3x_{4'}-\sqrt{12}x_{\beta_3}) \cdot \delta(p_{1'}+p_{2'}+p_{3'}+p_{4'}-2p_{\beta_4}) \\
\propto & \exp\Bigg[-\cosh(2r)\left(\frac{p_1-p_2+g\sqrt{2}p_{\beta_1}}{\sqrt{2}}\right)^2-\cosh(2r)\left(\frac{x_1+x_2-2x_3+g\sqrt{6}x_{\beta_2}}{\sqrt{6}}\right)^2 \\
& -\cosh(2r)\left(\frac{x_1+x_2+x_3-3x_4+g\sqrt{12}x_{\beta_3}}{\sqrt{12}}\right)^2-\cosh(2r)\left(\frac{p_1+p_2+p_3+p_4+2gp_{\beta_4}}{2}\right)^2\Bigg]. \qquad (9)
\end{aligned}
$$

Here we ignore the terms that are irresponsible for the quadripartite entanglement. Hebe then sends the classical results (p_{β_1}, x_{β_2}, x_{β_3} and p_{β_4}) to Alice, Bob and Claire, who locally displace their modes as

$$
\begin{aligned}
x_1 &\to X_1 = x_1 + \frac{g}{3}\sqrt{6}x_{\beta_2} + \frac{2g}{3}\sqrt{12}x_{\beta_3}, \\
p_1 &\to P_1 = p_1 + \frac{g}{2}\sqrt{2}p_{\beta_1} + gp_{\beta_4}, \\
p_2 &\to P_2 = p_2 - \frac{g}{2}\sqrt{2}p_{\beta_1} + gp_{\beta_4}, \\
x_3 &\to X_3 = x_3 - \frac{g}{3}\sqrt{6}x_{\beta_2} + \frac{g}{3}\sqrt{12}x_{\beta_3}. \qquad (10)
\end{aligned}
$$

After the displacement, the distributions of the variables $P_{\beta_1} = (P_1 - P_2)/\sqrt{2}$, $X_{\beta_2} = (X_1 + x_2 - 2X_3)/\sqrt{6}$, $X_{\beta_3} = (X_1 + x_2 + X_3 - 3x_4)/\sqrt{12}$ and $P_{\beta_4} = (P_1 + P_2 + p_3 + p_4)/2$ are independent of the measurement results. Consequently, after the mixing caused by the randomness of the measurement results, the covariances of the above variables all remain as $1/2\cosh(2r)$, which is smaller than $1/2$ for any $r > 0$ and approaches zero for $r \to \infty$. This means that CV cluster-like state has been prepared.

We have illustrated the protocol for unconditionally preparing tripartite GHZ-like states among remote parties that share no common past, and then generalised it to the cases of n-partite ($n \geqslant 3$) GHZ-like states and cluster-like states. The advantages of our method lie in its flexibility and high-efficiency. Specifically, Hebe is able to build any types (GHZ-like or cluster-like) of entangled states among any set of the agents in case of necessity. Notice that the first BS of Hebe's BS array always gives $1:1$, so Hebe just need to add proper BSs at the end of her BS array when she wants more agents involved; however, if the multipartite entangled state is prepared as in Refs. [2] and [11] (where the first BS gives $1:(n-1)$, depending on how many agents are involved), the whole BS array has to be rearranged for this task. Moreover, multipartite entangled states can be produced by using only linear optics, as long as the squeezing in the initial entangled sources is nonzero; on the other hand, preparation of multipartite entangled qubits in an 'entanglement swapping' fashion[21] requires a sequence of logic gates, and the efficiency is limited by the photon counting rates and the indistinguishability of all four Bell states.

3. An application to a teleportation network

The output n-partite Gaussian entangled states can be used as sources in a teleportation network. Here we illustrate this in the $n = 3$ case, and it is almost the same for the generic n. In a teleportation network,[2] quantum information is teleported from Claire to Bob, assisted by Alice. Here, Alice would not gain any information of the teleported states, but the teleportation would fail without Alice's assistance.

The scheme for third-man-assistant teleportation using a tripartite entangled state generated as in the last section is shown in Fig. 2. As a preparation step, Hebe transmits her three modes through a BS array and measures the quadratures of the output beams in order to build tripartite entanglement among the three agents, as discussed in the last section. Alice and Bob displace their modes as in Eq. (4), except for $p_2 \to P_2 = p_2 + g'\sqrt{3}p_c$, where $g' = 2g\cosh^2(2r)/(2\cosh^2(2r)+1)$. Here the gain g' is chosen for maximising the anticorrelation between phase quadratures of Bob's mode 2 and Claire's mode 3 and consequently maximising the teleportation fidelity.

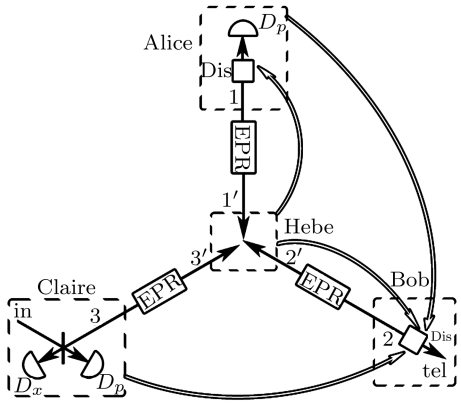


Fig. 2. Scheme for third-man-assistant teleportation using tripartite entangled state prepared by our protocol. Here, Hebe's local operations are the same as in Fig. 1(a) and empty arrows represent classical communications.

Now the three agents are ready to implement the teleportation network. First, Claire combines her mode 3 and an unknown coherent state $|\alpha_{\rm in}\rangle$ on a 1 : 1 BS and measures the quadratures of the outputs $x_{c_1} = (x_{\rm in} - x_3)/\sqrt{2}$, $p_{c_2} = (p_{\rm in} + p_3)/\sqrt{2}$. Here, $\alpha_{\rm in} = (X_{\rm in} + {\rm i}P_{\rm in})/\sqrt{2}$, and the Wigner function for the input state $|\alpha_{\rm in}\rangle$ is $W_{\rm in}(x_{\rm in}, p_{\rm in}) = \exp[-(x_{\rm in} - X_{\rm in})^2 - (p_{\rm in} - P_{\rm in})^2]/\pi$. Claire then sends her measurement results to Bob, who continues to displace his new mode 2 as

$$\begin{aligned} X_2 &\to X_2' = x_1 - \frac{g}{2}\sqrt{2}x_a + \frac{g}{2}\sqrt{6}x_b + g_c\sqrt{2}x_{c_1}, \\ P_2 &\to P_2' = p_2 + g'\sqrt{3}p_c + g_c\sqrt{2}p_{c_2}. \end{aligned} \tag{11}$$

The parameter g_c describes a normalised gain, and we set $g_c = 1$ for the optimal teleportation fidelity. However, Bob will gain no information encoded in the phase quadrature of $|\alpha_{\rm in}\rangle$ without Alice's help. Hence, Alice detects the phase quadrature of her mode 1 and sends the classical result p_1 to Bob. For the third time, Bob displaces the phase qudrature of his mode as $P_2' \to P_2^{\rm tel} = p_2 + g'\sqrt{3}p_c + \sqrt{2}p_{c_2} + g_a p_1$, and keeps the amplitude quadrature unchanged $X_2^{\rm tel} = X_2'$. Here, the optimal gain is $g_a = 2\sinh^2(2r)/(2\cosh^2(2r) + 1)$. The Wigner function of the teleported state reads

$$W_{\rm tel}(X_{\rm tel}, P_{\rm tel}) \propto \exp\left(-\frac{1}{2}\frac{(X_{\rm tel} - X_{\rm in})^2}{\dfrac{1}{\cosh(2r)} + \dfrac{1}{2}} - \frac{1}{2}\frac{(P_{\rm tel} - P_{\rm in})^2}{\dfrac{3}{2\cosh(2r) + 1/\cosh(2r)} + \dfrac{1}{2}}\right). \tag{12}$$

Clearly, for infinite squeezing $r \to \infty$, the teleported state is just the input state $W_{\rm tel} \to W_{\rm in}$. For finite squeezing, in order to estimate the ability for the teleportation channel, we calculate the teleportation fidelity defined as the overlap between the input and the teleported states $\rho_{\rm tel} : F = \langle\alpha_{\rm in}|\rho_{\rm tel}|\alpha_{\rm in}\rangle$. This fidelity is related to the Q-function of the teleported mode as $F = \pi Q_{\rm tel}(\alpha_{\rm in}) = 1/\sqrt{\sigma_{X_{\rm tel}}\sigma_{P_{\rm tel}}}$ ($g_c = 1$), where $\sigma_{X_{\rm tel}}$ and $\sigma_{P_{\rm tel}}$ are the variances of the Q-function of the teleported mode for the corresponding quadratures. According to the property of Q-function, $\sigma_{X_{\rm tel}}$ and $\sigma_{P_{\rm tel}}$ are related to the fluctuations of the corresponding quadratures as $\sigma_{X_{\rm tel}} = \langle\Delta X_{\rm tel}^2\rangle + 1/2$, $\sigma_{P_{\rm tel}} = \langle\Delta P_{\rm tel}^2\rangle + 1/2$. Directly from Eq. (12), $\langle\Delta X_{\rm tel}^2\rangle = 1/\cosh(2r) + 1/2$, $\langle\Delta P_{\rm tel}^2\rangle = 3/(2\cosh(2r) + 1/\cosh(2r)) + 1/2$. Consequently, the optimal teleportation fidelity reads

$$F = \left[\left(1 + \frac{1}{\cosh(2r)}\right) \times \left(1 + \frac{3}{2\cosh(2r) + 1/\cosh(2r)}\right)\right]^{-1/2} \tag{13}$$

which is plotted in Fig. 3 (solid line). For initially no squeezing in the entangled sources ($r = 0$), no entanglement is produced among Alice, Bob and Claire, and the fidelity $F_{\rm classical} = 1/2$, giving the limit for 'classical teleportation'. For infinite squeezing $r \to \infty$, the fidelity approaches unity. For any $r > 1/2$, the fidelity can exceed the classical limit $F > F_{\rm classical}$. Since the teleportation fidelity $F > 1/2$, only when entanglement exists, will this result give an experimental evidence that multipartite entanglement is actually established using our protocol. Compared with the fidelity achieved when the tripartite entangled state is prepared as in Ref. [2] (plotted as dashed line in Fig. 3), our fidelity is always lower, due to the non-increasing property of entanglement under local operations and classical communications. However, the difference is so tiny as to be ignored when the squeezing is large enough. Therefore, it is feasible to use the multipartite entangled state prepared by our protocol as an entangled source for third-man-assistant teleportation. Because of its flexibility, our protocol for

generating multi-mode entanglement is especially appropriate when Hebe does not know in advance which agents will be involved for the teleportation task.

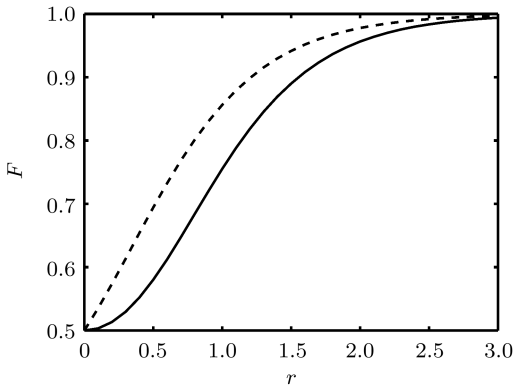


Fig. 3. The teleportation fidelity achieved when the tripartite entangled source shared among Alice, Bob and Claire is prepared directly as in Ref. [2] (dashed line), or in an 'entanglement redistribution' fashion using our protocol (solid line), as a function of squeezing parameter r of the entangled source for preparing tripartite entangled states.

4. Conclusion

We have proposed a scheme for preparing multipartite continuous-variable entangled states among remote parties that share no common past. Using only squeezed light and linear optics, this scheme can unconditionally produce on demand any type (GHZ-like or cluster-like) of multipartite Gaussian entangled state among any set of agents. Besides, an application of the produced multipartite entangled states to teleport networks is also illustrated, where the higher-than-1/2 teleportation fidelity indicates that multipartite entangled states are actually prepared by using our protocol. It is worth noting that, if each of the agents initially shares with Hebe a two-mode Gaussian entangled state of two atomic ensembles as prepared in Ref. [22], our protocol can also offer a method for preparing multipartite entangled states among atomic ensembles, which can serve as memories of entanglement.

References

[1] Osborne T J and Verstraete F 2006 *Phys. Rev. Lett.* **96** 220503
[2] van Loock P and Braunstein S L 2000 *Phys. Rev. Lett.* **84** 3482
[3] Hillery M, Bužek V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829
[4] Chen Y, Zhang A, Zhao Z, Zhou X, Lu C, Peng C, Yang T and Pan J 2005 *Phys. Rev. Lett.* **95** 200502
[5] Cleve R, Gottesman D and Lo H 1999 *Phys. Rev. Lett.* **83** 648
[6] Lance A M, Symul T, Bowen W P, Sanders B C and Lam P K 2004 *Phys. Rev. Lett.* **92** 177903
[7] Raussendorf R and Hans Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188
[8] Zeilinger A, Horne M A, Weinfurter H and Żukowski M 1997 *Phys. Rev. Lett.* **78** 3031
[9] Bodiya T P and Duan L M 2006 *Phys. Rev. Lett.* **97** 143601
[10] Lu C, Zhou X, Gühne O, Gao W, Zhang J, Yuan Z, Goebel A, Yang T and Pan J 1997 *Nature Phys.* **3** 91
[11] Zhang J and Braunstein S L 2006 *Phys. Rev.* A **73** 032318
[12] Yonezawa H, Aoki T and Furusawa A 2004 *Nature* **431** 430
[13] Su X, Tan A, Jia X, Zhang J, Xie C and Peng K 2007 *Phys. Rev. Lett.* **98** 070502
[14] Zhang J, Xie C and Peng K 2002 *Phys. Rev.* A **66** 032318
[15] Jing J, Zhang J, Yan Y, Zhao F, Xie C and Peng K 2003 *Phys. Rev. Lett.* **90** 167903
[16] Zukowski M, Zeilinger A, Horne M A and Ekert A K 1993 *Phys. Rev. Lett.* **78** 3031
[17] van Loock P and Braunstein S L 1999 *Phys. Rev.* A **61** 010302(R)
[18] Jia X, Su X, Pan Q, Gao J, Xie C and Peng K 2004 *Phys. Rev. Lett.* **93** 250503
[19] Ban M 2004 *J. Phys.* A: *Math. Gen.* **37** L385-90
[20] Hu X, Gu Y, Gong Q and Guo G 2009 *Opt. Commun.* **282** 1681-7
[21] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822
[22] Dantan A, Treps N, Bramati A and Pinard M 2005 *Phys. Rev. Lett.* **94** 050502