Entanglement evolution in an anisotropic two-qubit Heisenberg XYZ model with Dzyaloshinskii–Moriya interaction*

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(Received 3 August 2009; revised manuscript received 13 October 2009)

This paper investigates the entanglement evolution of a two-qubit anisotropic Heisenberg XYZ chain in the presence of Dzyaloshinskii–Moriya interaction. The time evolution of the concurrence is studied for the initial pure entangled states $\cos\theta\,|00\rangle + \sin\theta\,|11\rangle$ and $\cos\phi\,|01\rangle + \sin\phi\,|10\rangle$ at zero temperature. The influences of Dzyaloshinskii–Moriya interaction D, anisotropic parameter Δ and environment coupling strength γ on entanglement evolution are analysed in detail. It is found that the effect of noisy environment obviously suppresses the entanglement evolution, and the Dzyaloshinskii–Moriya interaction D acts on the time evolution of entanglement only when the initial state is $\cos\phi\,|01\rangle + \sin\phi\,|10\rangle$. Finally, a formula of steady state concurrence is obtained, and it is shown that the stable concurrence, which is independent of different initial states and Dzyaloshinskii–Moriya interaction D, depends on the anisotropic parameter Δ and the environment coupling strength γ.

Keywords: Heisenberg model, entanglement evolution, decoherence, Dzyaloshinskii–Moriya interaction

PACC: 0365, 7510H

1. Introduction

Quantum entanglement, as a nonlocal quantum correlation without any classical analog, has attracted much attention mainly due to its key role in quantum communication and information processing, such as quantum dense coding,[1,2] quantum teleportation,[3−6] quantum key distribution and quantum computation.[7−9] In recent years, there has been an outgoing effort to characterise the entanglement properties of condensed matter systems and apply them in quantum information. The quantum entanglement in solid state systems such as spin chains is an important emerging field.[10−16] Spin chains can be used as qubits in many real physical system, and Heisenberg spin chain is the simplest spin chain in condensed matter physics, which is regarded as one of the natural candidates to construct a quantum computer and quantum dots.[17] Heisenberg exchange interaction between two quantum dots can be used to implement universal one and two-qubit quantum gates.[18] In real quantum systems, inevitable interactions with the surrounding environment may lead to decoherence, resulting in degradation of entanglement. Taking the decoherence effect into account, Yu and Eberly[19,20] have shown that the entanglement of a bipartite qubit system may decrease abruptly to zero in a finite time due to the influence of quantum noise, this phenomenon was termed as entanglement sudden death (ESD). Recently, a great deal of the theoretical investigations of ESD have been reported.[21−25] Opposite to the current extensively discussed ESD, entanglement sudden birth (ESB) is the creation or rebirth of entanglement where the initial unentangled qubits can be entangled after a finite evolution time.[26−28]

In the presence of decoherence, Wang *et al.*[29] studied the entanglement evolution of a two-qubit Heisenberg XY spin chain, and the entanglement dynamic of the Heisenberg XY spin chain with magnetic field had been discussed in Refs. [30] and [31]. A stable entanglement, in which once qubits become entangled they will never be disentangled, was also demonstrated in Refs. [29]–[31]. In those studies the influence of spin–orbit coupling interaction on entanglement dynamics was rarely considered. Taking the

*Project supported by the National Natural Science Foundation of China (Grant No. 10904033), Educational Commission of Hubei Province (Grant No. D20092204) and Natural Science Foundation of Hubei Normal University (Grant No. 2007D21).

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spin-orbit coupling effect into account, Dzyaloshinskii and Moriya found the anisotropic antisymmetric Dzyaloshinskii–Moriya (DM) interaction.[32,33] In a variety of quasi-one-dimensional materials, DM interaction is mainly along the z-axis.[34] In this paper, we present an exact calculation of the entanglement dynamics between two qubits coupling with a common environment at zero temperature. The two qubits interact via a Heisenberg XYZ spin chain and with z-axis DM interaction. We examine the entanglement evolution of different initial states, including the unentangled states and the maximally entangled states, find that the effect of DM interaction on the time evolution of concurrence is due to the initial state, and the stable concurrence depends on the anisotropic parameter and the coupling strength with the environment which is independent of different initial states and DM coupling interaction.

2. Heisenberg XYZ model with DM interaction

We note that the Hamiltonian of an anisotropic two-qubit Heisenberg XYZ chain with z-axis DM interaction is

$$H = \frac{1}{2}\Big[J_x\sigma_{1x}\sigma_{2x} + J_y\sigma_{1y}\sigma_{2y} + J_z\sigma_{1z}\sigma_{2z} + \boldsymbol{D}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\Big], \quad (1)$$

where $J_i(i=x,y,z)$ are the spin–spin coupling coefficients, $J_i>0$ corresponds to the antiferromagnetic case, and $J_i<0$ corresponds to the ferromagnetic case. $\boldsymbol{D}$ is called DM vector coupling, here we choose $\boldsymbol{D}=D\boldsymbol{z}$, and $\boldsymbol{\sigma}_i(i=x,y,z)$ are the Pauli matrices. The Hamiltonian can be expressed as

$$H = (J+\mathrm{i}D)\sigma_{1+}\sigma_{2-} + (J-\mathrm{i}D)\sigma_{1-}\sigma_{2+} + \Delta(\sigma_{1+}\sigma_{2+}+\sigma_{1-}\sigma_{2-}) + \frac{J_z}{2}\sigma_{1z}\sigma_{2z}, \quad (2)$$

where $J=(J_x+J_y)/2$, $\Delta=(J_x-J_y)/2$, and $\sigma_\pm=(\sigma_x\pm\mathrm{i}\sigma_y)/2$ are the spin raising and lowering operators, the parameter Δ describes the spatial anisotropy of the spin–spin interaction.

In the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, the eigenvalues and eigenvectors can be derived as

$$H|\Psi_\pm\rangle = \left(-\frac{J_z}{2}\pm u\right)|\Psi_\pm\rangle,$$
$$H|\Phi_\pm\rangle = \left(\frac{J_z}{2}\pm\Delta\right)|\Phi_\pm\rangle, \quad (3)$$

where the eigenvectors are

$$|\Psi_\pm\rangle = \frac{\sqrt{2}}{2}(|01\rangle \pm \mathrm{e}^{\mathrm{i}\theta}|10\rangle),$$
$$|\Phi_\pm\rangle = \frac{\sqrt{2}}{2}(|00\rangle \pm |11\rangle), \quad (4)$$

and $u=\sqrt{D^2+J^2}$, $\theta=\arctan(D/J)$.

3. Entanglement evolution at zero temperature

In order to investigate the entanglement dynamics of two qubits, we use the concurrence as a measure of entanglement.[35] For a system described by the density matrix of the system ρ, the concurrence is defined by

$$C = \max(\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4},0), \quad (5)$$

where λ_1, λ_2, λ_3, and λ_4 are the eigenvalues in a decreasing order of the density operator R which is defined as $R=\rho(\sigma_{1y}\otimes\sigma_{2y})\rho^*(\sigma_{1y}\otimes\sigma_{2y})$, $*$ denotes the complex conjugate, σ_y is the usual Pauli matrix. The concurrence is available, no matter whether ρ is pure or mixed.

The description of time evolution of an open system at zero temperature is provided by the master equation, which can be written most generally in the Lindblad form with the assumption of weak system-reservoir coupling and Born–Markov approximation.[36] The Lindblad equation for the density matrix dynamics of the particles can be written as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H,\rho] + \sum_{j=1,2}\gamma_j \times\left[\sigma_{j-}\rho\sigma_{j+} - \frac{1}{2}\{\sigma_{j+}\sigma_{j-},\rho\}\right], \quad (6)$$

where the term $\gamma_1(\gamma_2)$ is the decay rate of qubit 1(2) to the environment, and here we assume $\gamma_1=\gamma_2=\gamma$; $\{\}$ means anticommutator. For our Hamiltonian, the density matrix can be represented by

$$\boldsymbol{\rho} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \quad (7)$$

Then the concurrence can be expressed as

$$C = \max[2(\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}), 2(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}), 0]. \tag{8}$$

The solution of Eq. (6) depends on the initial state of the qubits, and we consider the system initial state is in a pure state $\cos\theta\,|00\rangle + \sin\theta\,|11\rangle$. On solving the master equation in Eq. (6), we gain the time evolution of the reduced density matrix elements in the standard basis as

$$\begin{aligned}
\rho_{11}(t) &= \frac{1}{2(\gamma^2+4\Delta^2)}[2\Delta^2 + \mathrm{e}^{-2\gamma t}(\gamma^2+2\Delta^2) - \mathrm{e}^{-2\gamma t}\gamma^2\cos(2\theta) \\
&\quad -4\,\mathrm{e}^{-\gamma t}(\Delta^2\cos(2\Delta t)\cos(2\theta) + \Delta\gamma\sin(2\Delta t)\sin^2\theta)], \\
\rho_{14}(t) &= \frac{1}{4(\gamma^2+4\Delta^2)}[\gamma(2\mathrm{i}\,\mathrm{e}^{-\gamma t}\gamma\sin(2\Delta t) + 4\mathrm{i}\Delta(\mathrm{e}^{-\gamma t}\cos(2\Delta t) - 1)) \\
&\quad -2\mathrm{i}\,\mathrm{e}^{-\gamma t}(\gamma^2+4\Delta^2)\sin(2\Delta t)\cos(2\theta) + 2\,\mathrm{e}^{-\gamma t}(\gamma^2+4\Delta^2)\sin(2\theta), \\
\rho_{22}(t) &= \frac{1}{4(\gamma^2+4\Delta^2)}[4\Delta^2 - 2\,\mathrm{e}^{-2\gamma t}\gamma^2 + 2\,\mathrm{e}^{-\gamma t}\gamma^2\cos(2\Delta t) - 4\,\mathrm{e}^{-2\gamma t}\Delta^2 \\
&\quad -\gamma\cos(2\theta)\cdot(-2\,\mathrm{e}^{-2\gamma t}\gamma + 2\,\mathrm{e}^{-\gamma t}\cos(2\Delta t) + 4\,\mathrm{e}^{-\gamma t}\Delta\sin(2\Delta t))], \\
\rho_{44}(t) &= \frac{1}{2(\gamma^2+4\Delta^2)}[2(\gamma^2+\Delta^2) + \mathrm{e}^{-2\gamma t}(\gamma^2+2\Delta^2) - 2\,\mathrm{e}^{-\gamma t}\cos(2\Delta t)\cdot(\gamma^2 - 2\Delta^2\cos(2\theta)) \\
&\quad +\gamma^2\cos(2\theta)\cdot(2\,\mathrm{e}^{-\gamma t}\cos(2\Delta t) - \mathrm{e}^{-2\gamma t}) + 4\,\mathrm{e}^{-\gamma t}\gamma\Delta\sin(2\Delta t)\cos^2\theta],
\end{aligned} \tag{9}$$

where $\rho_{41}(t) = \rho_{14}^*(t), \rho_{33}(t) = \rho_{22}(t)$, and all other matrix elements are zero. From Eq. (9), we know that the coupling coefficient J_z, coefficient J and DM interaction D have nothing to do with the time evolution of concurrence for this initial state.

In the following, we investigate the entanglement dynamics under different system parameters for several different initial cases: unentangled state ($\theta = 0, \theta = \pi/2$), not maximal entangled state ($\theta = \pi/8$), maximal entangled state ($\theta = \pi/4$). In Fig. 1, the time evolution of the concurrence is plotted for various values of the anisotropic parameter Δ without the noisy environment when the qubits are initially in the different states. One can find that the entanglement evolves periodically for $\theta = 0$, $\theta = \pi/8$ and $\theta = \pi/2$. The amplitude of these oscillations is the same for different Δ, but the frequency increases with increasing Δ. However, the concurrence is invariant with different Δ when the initial state is maximal entangled state ($\theta = \pi/4$).

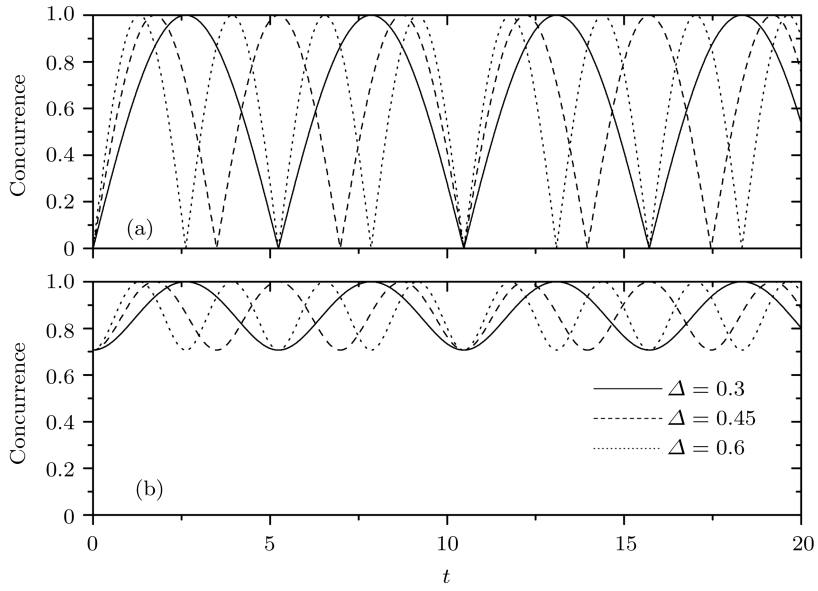


Fig. 1. Time evolution of the concurrence for various values of the anisotropic parameter Δ with $\gamma = 0$ for different initial states: (a) $\theta = 0$ or $\theta = \pi/2$, (b) $\theta = \pi/8$.

Figure 2 illustrates the time evolution of the concurrence for different Δ with the decoherence. From Fig. 2(a), we can find that the concurrence of the initial unentangled state increases from zero to a certain value with increasing time, and then fall abruptly to zero when the Δ is big enough, it occurs more rapidly with

increasing Δ. Figures 2(b), 2(c), and 2(d) show the ESD clearly when the $\Delta = 0.6$. Figure 2(d) reveals that the concurrence suddenly birth, ESB appears, and the ESD can occur twice with the increase of the system anisotropy. From Fig. 2, we see that the stable concurrences are different with different values of the anisotropic parameter Δ, and this has no relation to the initial states.

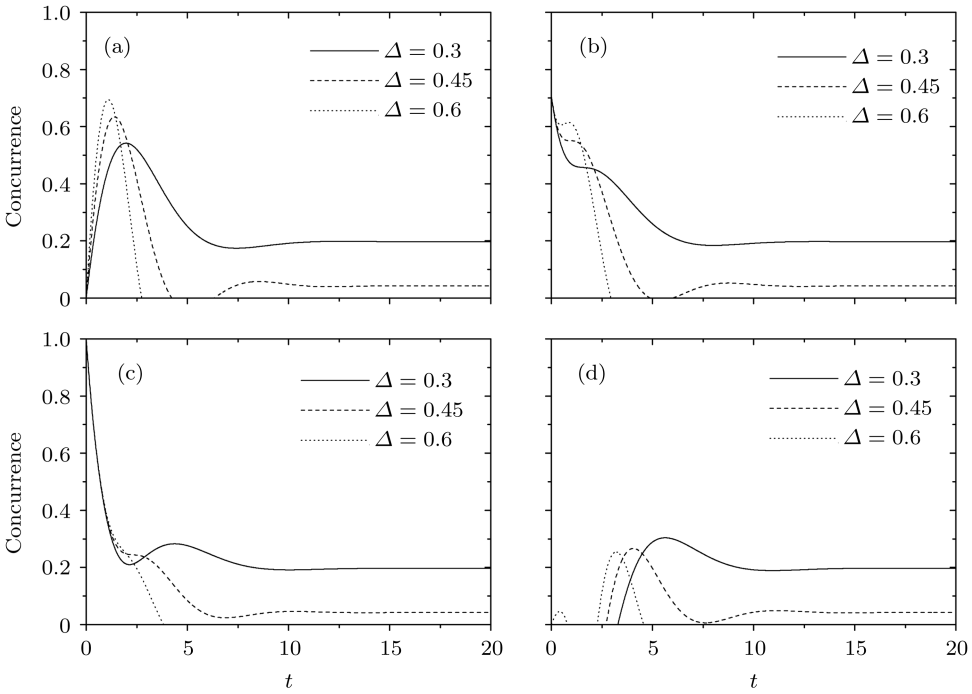


Fig. 2. Time evolution of the concurrence for various values of the anisotropic parameter Δ with $\gamma = 0.5$ for different initial states: (a) $\theta = 0$, (b) $\theta = \pi/8$, (c) $\theta = \pi/4$, (d) $\theta = \pi/2$.

We study the two qubits to be initially in another form of pure state $\cos\phi\,|01\rangle + \sin\phi\,|10\rangle$. By calculation from Eq. (6), we obtain the following reduced density matrix elements

$$
\begin{aligned}
\rho_{11}(t) &= \frac{\Delta}{(\gamma^2+4\Delta^2)}[\,\mathrm{e}^{-\gamma t}(-\gamma\sin(2\Delta t)+2\Delta\sinh(\gamma t))],\\
\rho_{14}(t) &= \frac{1}{4(\gamma^2+4\Delta^2)}[2\,\mathrm{e}^{-\gamma t}\gamma^2\sin(2\Delta t)+4\mathrm{i}\gamma\Delta(\,\mathrm{e}^{-\gamma t}\cos(2\Delta t)-1)],\\
\rho_{22}(t) &= \frac{1}{4u(\gamma^2+4\Delta^2)}[u(4\Delta^2+2\,\mathrm{e}^{-\gamma t}\gamma^2\cos(2\Delta t)+4\Delta^2\,\mathrm{e}^{-2\gamma t})\\
&\quad -2\,\mathrm{e}^{-\gamma t}(\gamma^2+4\Delta^2)(u\cosh(2ut)\cos(2\phi)-D\sinh(2ut)\sin(2\phi))],\\
\rho_{33}(t) &= \frac{1}{4u(\gamma^2+4\Delta^2)}[u(4\Delta^2+2\,\mathrm{e}^{-\gamma t}\gamma^2\cos(2\Delta t)+4\Delta^2\,\mathrm{e}^{-2\gamma t})\\
&\quad +2\,\mathrm{e}^{-\gamma t}(\gamma^2+4\Delta^2)(u\cosh(2ut)\cos(2\phi)-D\sinh(2ut)\sin(2\phi))],\\
\rho_{23}(t) &= \frac{1}{4u(d+\mathrm{i}J)}[2\,\mathrm{e}^{-\gamma t}u^2\sinh(2ut)\cos(2\phi)+2u\sin(2\phi)\cdot(\,\mathrm{e}^{-\gamma t}D\cosh(2ut)+\mathrm{i}\,\mathrm{e}^{-\gamma t}J)],\\
\rho_{44}(t) &= \frac{1}{(\gamma^2+4\Delta^2)}[\,\mathrm{e}^{-\gamma t}(-\gamma^2\cos(2\Delta t)+\gamma^2\cosh(\gamma t)+\gamma\Delta\sin(2\Delta t)+(\gamma^2+2\Delta^2)\sinh(\gamma t))],
\end{aligned}
\tag{10}
$$

where $\rho_{41}(t) = \rho_{14}^*(t), \rho_{32}(t) = \rho_{23}^*(t)$, and all other matrix elements are zero. From Eq. (10), we know that ϕ should range from 0 to $\pi/2$, and the coupling coefficient J_z have nothing to do with the time evolution of the concurrence.

The influence of the anisotropic parameter Δ for this initial state is similar to that for the initial state $\cos\theta\,|00\rangle + \sin\theta\,|11\rangle$, so we assume $\Delta = 0.3$ and the coefficient $J = 1$. We investigate the entanglement dynamics under DM interaction for several different

initial cases: unentangled state ($\phi = 0$), not maximal entangled state ($\phi = \pi/8$), maximal entangled state ($\phi = \pi/4$). If there is no decoherence, the entanglement evolves periodically with the D. The amplitude and the frequency of the oscillations increase with the increase of D in Fig. 3(c) when the initial state is the maximal entangled state ($\phi = \pi/4$), the DM interaction makes slow oscillation around the maximal value of concurrence. Figure 3(a) shows the DM interaction can change the frequency of the oscillations when the initial state is the unentangled state ($\phi = 0$). With the decoherence, from Figs. 3(b) and 3(d), we can see that the two-qubit state can evolve into a stationary entanglement state, the different values of the DM coupling parameter D just generate the same stable concurrences, and this also has no relation to the initial states.

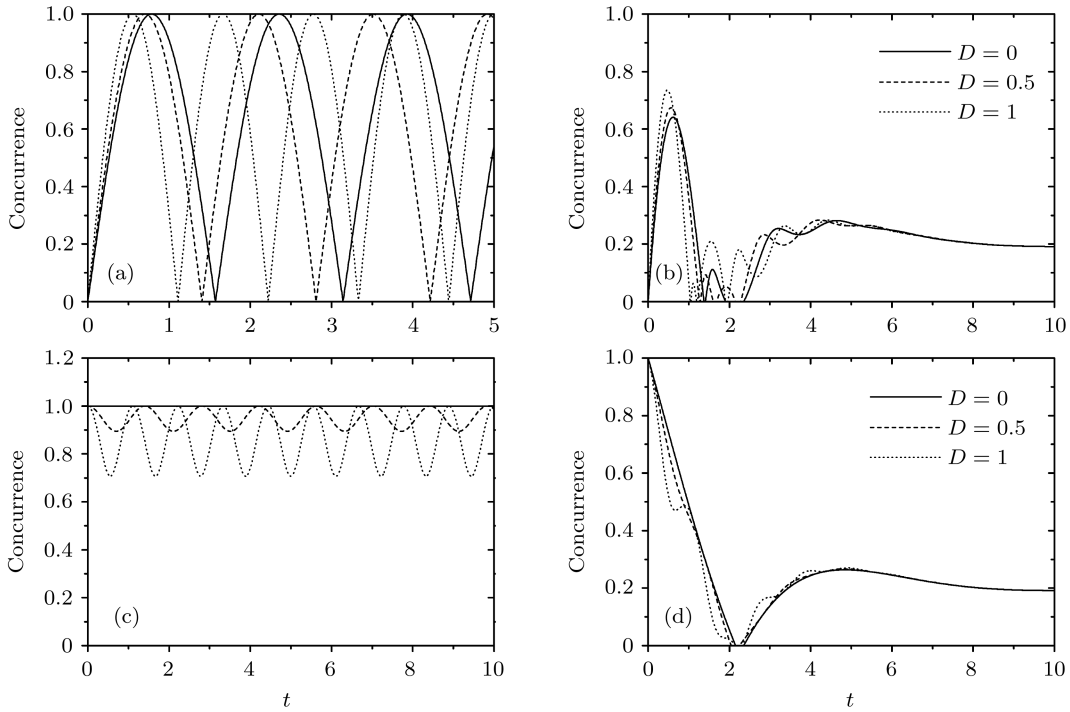


Fig. 3. Time evolution of the concurrence for various values of DM coupling parameter D with $\Delta = 0.3$ and $J = 1$ for different initial states: (a) $\gamma = 0, \phi = 0$; (b) $\gamma = 0.5, \phi = 0$; (c) $\gamma = 0, \phi = \pi/4$; (d) $\gamma = 0.5, \phi = \pi/4$.

Calculating the steady entanglement from Eqs. (9) and (10) with the decoherence, a uniform concurrence can be obtained as follows:

$$C_{\text{steady}} = \max\left(\frac{2\Delta\gamma - 2\Delta^2}{\gamma^2 + 4\Delta^2}, 0\right). \tag{11}$$

The stable concurrence depends on the anisotropic parameter Δ and the decay rate of qubits γ, but not on the DM coupling parameter D. And if $\Delta = 0$, the stable concurrence is always zero. Figure 4 shows the time evolution of the concurrence with $\gamma = 0.5$, $\Delta = 0.3$, $J = 1$ and $D = 0.5$ for different initial states ($\theta = 0, \theta = \pi/4, \phi = 0, \phi = \pi/4$), it is observed that the steady concurrences are the same.

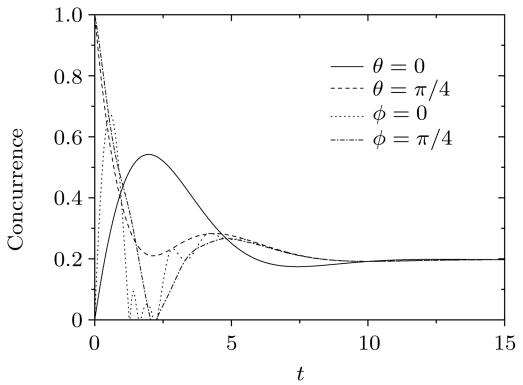


Fig. 4. Time evolution of the concurrence with $J = 1$, $\Delta = 0.3$, $\gamma = 0.5$ and $D = 0.5$ for different initial states: $\theta = 0, \theta = \pi/4, \phi = 0, \phi = \pi/4$.

4. Conclusions

In this paper, we investigate the entanglement evolution of a two-qubit anisotropic Heisenberg XYZ chain in the presence of DM interaction for different initial states of the system. When the initial state is $\cos\theta\,|00\rangle + \sin\theta\,|11\rangle$, the entanglement evolution is independent of the DM interaction. However, the DM interaction has a remarkable influence on the entanglement dynamics when the initial state is $\cos\phi\,|01\rangle + \sin\phi\,|10\rangle$. Finally, we obtain a formula of the steady concurrence, and by controlling the values of the anisotropic parameter we can achieve a certain stable concurrence when DM interaction coupling parameter D has no effect on it.

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