Approximate symmetry reduction for perturbed nonlinear Schrödinger equation*

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We investigate the one-dimensional nonlinear Schrödinger equation with a perturbation of polynomial type. The approximate symmetries and approximate symmetry reduction equations are obtained with the approximate symmetry perturbation theory.

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1. Introduction

It is well known that there are several integrable nonlinear evolution equations which play an important role in many branches of physics, such as Korteweg-de Vries (KdV), nonlinear Schrödinger (NLS), and Kadomtsev-Petviashvili (KP) equations, etc. The solutions of these equations can be obtained by various methods such as the inverse scattering transformation (IST), the Hirota bilinear method, and the Darboux transformations, etc.[1−6] However, the exact integrable nonlinear equations tend to be highly idealized. As a matter of fact, in more realistic situations, one has to study the effects of the perturbations due to the external drive, the fibre loss, the dissipative and dispersion, etc. Nonlinear systems with perturbation generally exist and their integrability could easily be broken. Therefore, it is more difficult to analyse these equations with perturbation terms than the standard equations. So far, many powerful perturbation methods have been proposed such as the inverse scattering perturbation method,[7] the direct perturbation approach, etc.[8−10]

Symmetry study is considered as one of the most important and powerful methods to solve the partial differential equation (PDE). Especially in integrable systems, exact solutions of a nonlinear equations can be obtained with an infinite number of symmetries.[11] For the perturbed equations, Lou and his colleagues proposed the approximate symmetry approach to study. It is significant to take into account symmetry when treating with perturbed partial differential equations.[12−14] In this paper, we apply the approximate symmetry reduction method to the following perturbed NLS equation[15]

$$\mathrm{i}\,u_t + \alpha u_{xx} + u^2 v - \epsilon u_{xxxx} = 0, \tag{1a}$$

$$-\mathrm{i}\,v_t + \alpha v_{xx} + v^2 u - \epsilon v_{xxxx} = 0, \tag{1b}$$

where α is an arbitrary constant with ϵ being a small parameter.

This paper is organised as follows. In Section 2, the approximate symmetry is obtained in the infinite perturbation series. In Section 3, we find out the reduced equations by the symmetry reduction. The last section is a simple summary and discussion.

2. The approximate symmetry for perturbed NLS equation

According to the perturbation theory, the solution can be expressed as a series containing a small parameter

$$u = \sum_{j=0}^{\infty} \epsilon^j u_j, \quad v = \sum_{n=0}^{\infty} \epsilon^j v_j, \tag{2}$$

where u_j and v_j are functions of (x, t). Substituting Eq. (2) into Eqs. (1a) and (1b) and the proper balance the coefficients of all different powers of ϵ, we obtain

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$$O(\epsilon^0): \quad \mathrm{i}\,u_{0,t}+\alpha u_{0,xx}+u_0^2v_0=0, \tag{3a}$$

$$-\mathrm{i}\,v_{0,t}+\alpha v_{0,xx}+v_0^2u_0=0, \tag{3b}$$

$$O(\epsilon^1): \quad \mathrm{i}\,u_{1,t}+\alpha u_{1,xx}+u_0^2v_1+2u_0u_1v_0-u_{0,xxxx}=0, \tag{4a}$$

$$-\mathrm{i}\,v_{1,t}+\alpha v_{1,xx}+v_0^2u_1+2v_0v_1u_0-v_{0,xxxx}=0, \tag{4b}$$

$$O(\epsilon^2): \quad \mathrm{i}\,u_{2,t}+\alpha u_{2,xx}+u_0^2v_2+u_1^2v_0+2u_0u_1v_1+2u_0u_2v_0-u_{1,xxxx}=0, \tag{5a}$$

$$-\mathrm{i}\,v_{2,t}+\alpha v_{2,xx}+v_0^2u_2+v_1^2u_0+2v_0v_1u_1+2v_0v_2u_0-v_{1,xxxx}=0, \tag{5b}$$

$$O(\epsilon^3): \quad \mathrm{i}\,u_{3,t}+\alpha u_{3,xx}+u_0^2v_3+u_1^2v_1+2u_0u_1v_2+2u_0u_2v_1+2u_0u_3v_0+2u_1u_2v_0-u_{2,xxxx}=0, \tag{6a}$$

$$-\mathrm{i}\,v_{3,t}+\alpha v_{3,xx}+v_0^2u_3+v_1^2u_1+2v_0v_1u_2+2v_0v_2u_1+2v_0v_3u_02v_1v_2u_0-v_{2,xxxx}=0, \tag{6b}$$

$$O(\epsilon^j): \quad \mathrm{i}\,u_{j,t}+\alpha u_{j,xx}+\sum_{k=0}^{j}\sum_{l=0}^{k}v_lu_{k-l}u_{j-k}-u_{j-1,xxxx}=0, \tag{7a}$$

$$-\mathrm{i}\,v_{j,t}+\alpha v_{j,xx}+\sum_{k=0}^{j}\sum_{l=0}^{k}u_lv_{k-l}v_{j-k}-v_{j-1,xxxx}=0 \tag{7b}$$

with $j=1,2,\ldots,n$, and $u_{-1}=v_{-1}=0$.

Equation (7a) and (7b) are the series expansion for arbitrary order of ϵ. In the following, we will use the Lie point symmetry to study Lie-symmetry reductions of Eqs. (7a) and (7b). The symmetry can be supposed to have the following form

$$\begin{aligned}\sigma_{u_j}&=Xu_{j,x}+Tu_{j,t}-U_j,\\ \sigma_{v_j}&=Xv_{j,x}+Tv_{j,t}-V_j,\quad (j=0,\ldots,n),\end{aligned} \tag{8}$$

where X, T, U_n and V_n are functions of x, t, u_j and v_j $(j=0,\ldots,n)$, which means that Eqs. (7a) and (7b) are formal invariant under transformations

$$u_j\to u_j+\epsilon'\sigma_{u_j},\qquad v_j\to v_j+\epsilon'\sigma_{v_j},\qquad (j=0,\ldots,n) \tag{9}$$

with an infinitesimal parameter ϵ'. The symmetry equations for Eqs. (3)–(7) read

$$\mathrm{i}\,\sigma_{u0,t}+\alpha\sigma_{u0,xx}+u_0^2\sigma_{v0}+2u_0v_0\sigma_{u0}=0, \tag{10a}$$

$$-\mathrm{i}\,\sigma_{v0,t}+\alpha\sigma_{v0,xx}+v_0^2\sigma_{u0}+2v_0u_0\sigma_{v0}=0, \tag{10b}$$

$$\mathrm{i}\,\sigma_{u1,t}+\alpha\sigma_{u1,xx}+u_0^2\sigma_{v1}+2u_0v_1\sigma_{u0}+2u_0u_1\sigma_{v0}+2u_0v_0\sigma_{u1}+2u_1v_0\sigma_{u0}-\sigma_{u0,xxxx}=0, \tag{11a}$$

$$-\mathrm{i}\,\sigma_{v1,t}+\alpha\sigma_{v1,xx}+\beta v_0^2\sigma_{u1}+2v_0u_1\sigma_{v0}+2v_0v_1\sigma_{u0}+2u_0v_0\sigma_{v1}+2v_1u_0\sigma_{v0}-\sigma_{v0,xxxx}=0, \tag{11b}$$

$$\begin{aligned}&\mathrm{i}\,\sigma_{u2,t}+\alpha\sigma_{u2,xx}+u_1^2\sigma_{v0}+u_0^2\sigma_{v2}+2u_1v_0\sigma_{u1}+2u_2v_0\sigma_{u0}+2u_0v_2\sigma_{u0}\\&+2u_0u_1\sigma_{v1}+2\beta u_0v_1\sigma_{u1}+2u_1v_1\sigma_{u0}+2\beta u_0u_2\sigma_{v0}+2\beta u_0v_0\sigma_{u2}-\sigma_{u1,xxxx}=0,\end{aligned} \tag{12a}$$

$$\begin{aligned}&-\mathrm{i}\,\sigma_{v2,t}+\alpha\sigma_{v2,xx}+v_1^2\sigma_{u0}+v_0^2\sigma_{u2}+2v_0u_1\sigma_{v1}+2v_1u_0\sigma_{v1}+2u_2v_0\sigma_{v0}\\&+2v_0v_1\sigma_{u1}+2v_1u_1\sigma_{v0}+2v_0v_2\sigma_{u0}+2v_0u_0\sigma_{v2}+2u_0v_2\sigma_{v0}-\sigma_{v1,xxxx}=0,\end{aligned} \tag{12b}$$

$$\begin{aligned}&\mathrm{i}\,\sigma_{u3,t}+\alpha\sigma_{u3,xx}+u_0^2\sigma_{v3}+u_1^2\sigma_{v1}+2u_1u_2\sigma_{v0}+2\beta u_1v_1\sigma_{u1}+2u_0u_1\sigma_{v2}\\&+2u_0u_2\sigma_{v1}+2u_0v_3\sigma_{u0}+2u_0v_2\sigma_{u1}+2u_1v_2\sigma_{u0}+2u_0v_1\sigma_{u2}+2u_2v_1\sigma_{u0}\\&+2u_0u_3\sigma_{v0}+2u_0v_0\sigma_{u3}+2u_3v_0\sigma_{u0}+2u_1v_0\sigma_{u2}+2u_2v_0\sigma_{u1}-\sigma_{u2,xxxx}=0,\end{aligned} \tag{13a}$$

$$\begin{aligned}&-\mathrm{i}\,\sigma_{v3,t}+\alpha\sigma_{v3,xx}+v_0^2\sigma_{u3}+v_1^2\sigma_{u1}+2v_0u_3\sigma_{u0}+2v_0u_1\sigma_{v1}+2v_0v_1\sigma_{u2}\\&+2v_0u_2\sigma_{v1}+2v_1u_2\sigma_{v0}+2v_0v_2\sigma_{u1}+2v_0u_1\sigma_{v2}+2v_2u_1\sigma_{v0}+2v_0v_3\sigma_{u0}\\&+2v_0u_0\sigma_{v3}+2v_3u_0\sigma_{v0}+2v_1v_2\sigma_{u0}+2v_1u_0\sigma_{v2}+2v_2u_0\sigma_{v1}-\sigma_{v2,xxxx}=0,\end{aligned} \tag{13b}$$

$$\mathrm{i}\,\sigma_{uj,t}+\alpha\sigma_{uj,xx}+\sum_{k=0}^{j}\sum_{l=0}^{k}(\sigma_{vl}u_{k-l}u_{j-k}+v_l\sigma_{k-l}u_{j-k}+v_lu_{k-l}\sigma_{j-k})-\sigma_{u\,j-1,xxxx}=0, \tag{14a}$$

$$-\mathrm{i}\,\sigma_{vj,t}+\alpha\sigma_{vj,xx}+\sum_{k=0}^{j}\sum_{l=0}^{k}(\sigma_{ul}v_{k-l}v_{j-k}+u_l\sigma_{k-l}v_{j-k}+u_lv_{k-l}\sigma_{j-k})-\sigma_{v\,j-1,xxxx}=0. \tag{14b}$$

Substituting Eq. (8) into the symmetry equations with $u_{j,t}$ and $v_{j,t}$ satisfying Eqs. (3)–(7), we have the determining equations by identifying all coefficients of derivatives of u and v. The solutions of the functions X, T, U_j and V_j ($j = 0, 1, \ldots$) can be concluded using the determining equations

$$
\begin{aligned}
&X = \frac{c_1}{2}t + c_3, \quad T = c_1 t + c_2,\\
&U_0 = c_4 u_0, \quad U_1 = (c_4 - c_1)u_1 + c_5 u_0,\\
&U_2 = (c_4 - 2c_1)u_2 + c_6 u_1 + c_5 u_0,\\
&U_3 = (c_4 - 3c_1)u_3 + c_5 u_2 + c_6 u_1 + c_7 u_0,\\
&V_0 = -(c_4 + c_1)v_0,\\
&V_1 = -(c_4 + 2c_1)v_1 - c_5 v_0,\\
&V_2 = -(c_4 + 3c_1)v_2 - c_5 v_1 - c_6 v_0,\\
&V_3 = -(c_4 + 4c_1)v_3 - c_6 v_2 - c_5 v_1 - c_7 v_0,\\
&\ldots\\
&U_j = -jc_1 u_j + \sum_{k=0}^{j} c_{k+4} u_{j-k},\\
&V_j = -c_1(j+1)v_j - \sum_{k=0}^{j} c_{k+4} v_{j-k}. \qquad (15)
\end{aligned}
$$

The specific forms of symmetries can be written as

$$
\begin{aligned}
\sigma_{u_j} &= \left(\frac{c_1}{2}t + c_3\right)u_{j,x} + (c_1 t + c_2)u_{j,t} + jc_1 u_j\\
&\quad - \sum_{k=0}^{j} c_{k+4} u_{j-k}, \qquad (16a)\\
\sigma_{v_j} &= \left(\frac{c_1}{2}t + c_3\right)v_{j,x} + (c_1 t + c_2)v_{j,t} + c_1(j+1)v_j\\
&\quad + \sum_{k=0}^{j} c_{k+4} v_{j-k}, \qquad (16b)
\end{aligned}
$$

where c_1, c_2, c_3, …, c_j are arbitrary constants.

3. The symmetry reduction for Eqs. (3)–(7)

A similarity reduction of a differential equation is closely connected with the invariance of the equation. In the following, we will reduce Eqs. (3)–(7) into the ordinary differential equations in the invariant conditions.

To obtain similarity reductions related to Eqs. (16a) and (16b), one can solve the characteristic equations

$$
\begin{aligned}
\frac{\mathrm{d}x}{X} &= \frac{\mathrm{d}t}{T} = \frac{\mathrm{d}u_0}{U_0} = \frac{\mathrm{d}v_0}{V_0} = \frac{\mathrm{d}u_1}{U_1} = \frac{\mathrm{d}v_1}{V_1} = \ldots\\
&= \frac{\mathrm{d}u_j}{U_j} = \frac{\mathrm{d}v_j}{V_j} = \ldots, \qquad (17)
\end{aligned}
$$

where X, T, U_0, V_0, …, U_j, V_j are given by Eq. (15).

(A) Case 1: $c_1 \neq 0$.

Here, we take $c_1 = 1$ and make transformations $c_2 \to n_0$, $c_3 \to c_1 n_0$, $c_4 \to c_2 n_0$, …, $c_j \to c_{j-2} n_0$. In this case, the reduction equations are too lengthy for $n \geq 2$ so we only write out the reduction equations for the first two equations. After solving the characteristic equations, we can conclude that

$$
\begin{aligned}
u_0 &= U_0(\xi)(x + 2n_0 c_1)^{2c_2 n_0}, \qquad (18a)\\
v_0 &= V_0(\xi)(x + 2n_0 c_1)^{-2-2c_2 n_0}, \qquad (18b)\\
u_1 &= (n_0 c_3 x^2 U_0(\xi) + 4n_0^2 c_3 c_1 x U_0(\xi)\\
&\quad + U_1(\xi))(x + 2n_0 c_1)^{-2+2c_2 n_0}, \qquad (19a)\\
v_1 &= (-n_0 c_3 x^2 V_0(\xi) - 4n_0^2 c_3 c_1 x V_0(\xi)\\
&\quad + V_1(\xi))(x + 2n_0 c_1)^{-4-2c_2 n_0}, \qquad (19b)
\end{aligned}
$$

with the group invariant $\xi = \dfrac{x + 2n_0 c_1}{\sqrt{t + n_0}}$. Substitution of Eqs. (18) and (19) into Eqs. (3) and (4) leads to the following reduction equations

$$
\begin{aligned}
4\alpha\xi^2 U_{0,\xi\xi} &= -\mathrm{i}\,U_{0,\xi} - 6\alpha\xi U_{0,\xi} + 8\alpha c_2 n_0 \xi U_{0,\xi} - 4c_2^2 n_0^2 \alpha U_0 + 2n_0 c_2 \alpha U_0 - U_0^2 V_0, \qquad (20a)\\
4\alpha\xi^2 V_{0,\xi\xi} &= \mathrm{i}\,V_{0,\xi} - 14\alpha\xi V_{0,\xi} - 8\alpha c_2 n_0 \xi V_{0,\xi} - 4\alpha c_2^2 n_0^2 V_0 - 6\alpha V_0 - 10\alpha c_2 n_0 V_0 - U_0 V_0^2, \qquad (20b)\\
16\alpha^4\xi^4 U_{1,\xi\xi} &= (-4\,\mathrm{i}\,\alpha^3\xi^2 - 56\alpha^4\xi^3 U_{1,\xi} + 32c_2 n_0 \alpha^4 \xi^3) U_{1,\xi}\\
&\quad + (\mathrm{i} - 8c_2 n_0 \alpha\xi + 6\alpha\xi + 128 n_0^3 c_1^2 c_3 \alpha^4 \xi^3) U_{0,\xi} - 32\alpha^2\xi^4 U_{0,\xi}^2 V_0\\
&\quad - 64\alpha^2\xi^3 U_0 V_0 U_{0,\xi} + 64c_2 n_0 \xi^3 \alpha^2 U_0^2 V_{0,\xi} - 64\alpha^2\xi^4 U_0 V_{0,\xi} U_{0,\xi}\\
&\quad + 16\,\mathrm{i}\,\alpha\xi^2 U_0 V_0 U_{0,\xi} + 32c_2^2 n_0^2 \alpha^2 \xi^2 U_0^2 V_0 + 64c_2 n_0 \alpha^2 \xi^2 U_0^2 V_0 + U_0^2 V_0\\
&\quad + 12\alpha\xi^2 U_0^3 V_0^2 - 24\alpha^4 \xi^2 U_1 - 4\alpha^3 \xi^2 U_0^2 V_1 - 8\alpha^3 \xi^2 U_0 V_0 U_1 + 4c_2^2 n_0^2 \alpha U_0 - 2c_2 n_0 \alpha U_0\\
&\quad - 128 c_3 c_2 c_1^2 n_0^4 \alpha^4 \xi^2 U_0 + 96 c_3 c_1^2 n_0^3 \alpha^4 \xi^2 U_0 + 40 c_2 n_0 \alpha^4 \xi^2 U_1 - 16 c_2^2 n_0^2 \alpha^4 \xi^2 U_1, \qquad (21a)
\end{aligned}
$$

$$
\begin{aligned}
16\alpha^4\xi^4 V_{1,\xi\xi} = {} & (4\,\mathrm{i}\,\alpha^3\xi^2 - 32c_2n_0\alpha^4\xi^3 - 88\alpha^4\xi^3 - 16\alpha^4\xi^4)V_{1,\xi} \\
& + (-\mathrm{i} + 14\alpha\xi + 8c_2n_0\alpha\xi - 128c_3c_1^2n_0^3\alpha^4\xi^3)V_{0,\xi} - 16\,\mathrm{i}\,\alpha\xi^2V_0U_0V_{0,\xi} \\
& - 64n_0c_2\alpha^2\xi^3V_0^2U_{0,\xi} - 32\alpha^2\xi^4V_{0,\xi}^2U_0 - 64\alpha^2\xi^3V_0^2U_{0,\xi} - 64\alpha^2\xi^4V_0U_{0,\xi}V_{0,\xi} \\
& - 64\alpha^2\xi^3V_0U_0V_{0,\xi} + V_0^2U_0 - 4\alpha^3\xi^2V_0^2U_1 + 32n_0^2c_2^2\alpha^2\xi^2U_0V_0^2 - 8\alpha^3\xi^2V_0U_0V_1 \\
& - 32\alpha^2\xi^2V_0^2U_0 + 12\alpha\xi^2U_0^2V_0 - 128c_3c_2c_1^2n_0^4\alpha^4\xi^2V_0 - 224c_3c_1^2n_0^3\alpha^4\xi^2V_0 \\
& + 4c_2^2n_0^2\alpha V_0 + 10c_2n_0\alpha V_0 + 6\alpha V_0 - 80\alpha^4\xi^2V_1 - 16n_0^2c_2^2\alpha^4\xi^2V_1 - 72c_2n_0\alpha^4\xi^2V_1. \qquad (21b)
\end{aligned}
$$

(B) Case 2: $c_1 = 0$, $c_j = 0 (j \geq 4)$.

In this case, we can find the general similarity equations after solving the characteristic equations

$$u_0 = U_0(\xi)\exp(c_2c_1x), \qquad (22a)$$

$$v_0 = V_0(\xi)\exp\left(-\frac{c_2}{c_1}x\right); \qquad (22b)$$

$$u_1 = U_1(\xi)\exp(c_2c_1x), \qquad (23a)$$

$$v_2 = V_2(\xi)\exp\left(-\frac{c_2}{c_1}x\right); \qquad (23b)$$

$$u_j = U_j(\xi)\exp(c_2c_1x), \qquad (24a)$$

$$v_j = V_j(\xi)\exp\left(-\frac{c_2}{c_1}x\right), \qquad (24b)$$

where group invariant ξ is given by $\xi = x - c_1t$. Substituting Eqs. (22)–(24) into Eqs. (3)–(7), the invariant functions U_0, V_0, U_1, V_1, …, U_j, V_j satisfy the following reduction systems

$$\alpha U_{0,\xi\xi} + \mathrm{i}\,c_1^2U_{0,\xi} - 2\alpha c_2U_{0,\xi} + \alpha c_2^2U_0 + c_1^2U_0^2V_0 = 0, \qquad (25a)$$

$$\alpha V_{0,\xi\xi} - \mathrm{i}\,c_1^2V_{0,\xi} + 2c_2\alpha V_{0,\xi} + \alpha c_2^2V_0 + c_1^2V_0^2U_0 = 0, \qquad (25b)$$

$$
\begin{aligned}
& c_1^2(\alpha U_{1,\xi\xi} + \mathrm{i}\,c_1^2U_{1,\xi} - 2\alpha c_2U_{1,\xi} + \alpha c_2^2U_1 + 2c_1^2U_0V_0U_1 + c_1^2U_0^2V_1) - U_{0,\xi\xi\xi\xi\xi} \\
& + 4c_2U_{0,\xi\xi\xi} - 6c_2^2U_{0,\xi\xi} + 4c_2^3U_{0,\xi} - c_2^4U_0 = 0, \qquad (26a) \\
& c_1^2(\alpha V_{1,\xi\xi} - \mathrm{i}\,c_1^2V_{1,\xi} + 2\alpha c_2V_{1,\xi} + \alpha c_2^2V_1 + 2c_1^2V_0U_0V_1 + c_1^2V_0^2U_1) - V_{0,\xi\xi\xi\xi\xi} \\
& - 4c_2V_{0,\xi\xi\xi} - 6c_2^2V_{0,\xi\xi} - 4c_2^3V_{0,\xi} - c_2^4V_0 = 0; \qquad (26b) \\
& c_1^2(\alpha U_{2,\xi\xi} + \mathrm{i}\,c_1^2U_{2,\xi} - 2\alpha c_2U_{2,\xi} + c_2^2\alpha U_2 + 2c_1^2U_0V_1U_1 + 2c_1^2U_0V_0U_2 + c_1^2U_0^2V_2 + c_1^2V_0U_1^2) \\
& - U_{1,\xi\xi\xi\xi} + 4c_2U_{1,\xi\xi\xi} - 6c_2^2U_{1,\xi\xi} + 4c_2^3U_{1,\xi} - c_2^4U_1 = 0, \qquad (27a) \\
& c_1^2(\alpha V_{2,\xi\xi} - \mathrm{i}\,c_1^2V_{2,\xi} + 2\alpha c_2V_{2,\xi} + \alpha c_2^2V_2 + 2c_1^2V_0U_1V_1 + 2c_1^2V_0U_0V_2 + c_1^2V_0^2U_2 + c_1^2U_0V_1^2) \\
& - V_{1,\xi\xi\xi\xi} + 4c_2V_{1,\xi\xi\xi} - 6c_2^2V_{1,\xi\xi} - 4c_2^3V_{1,\xi} - c_2^4V_1 = 0; \qquad (27b) \\
& c_1^2(\alpha U_{j,\xi\xi} + \mathrm{i}\,c_1^2U_{j,\xi} - 2\alpha c_2U_{j,\xi} + \alpha c_2^2U_j + c_1^2\sum_{k=0}^{j}\sum_{l=0}^{k}V_lU_{k-l}U_{j-k}) \\
& - U_{j-1,\xi\xi\xi\xi} + 4c_2U_{j-1,\xi\xi\xi} - 6c_2^2U_{j-1,\xi\xi} + 4c_2^3U_{j-1,\xi} - c_2^4U_{j-1} = 0, \qquad (28a) \\
& c_1^2(\alpha V_{j,\xi\xi} - \mathrm{i}\,c_1^2V_{j,\xi} + 2\alpha c_2V_{j,\xi} + \alpha c_2^2V_j + \sum_{k=0}^{j}\sum_{l=0}^{k}U_lV_{k-l}V_{j-k}) \\
& - V_{j-1,\xi\xi\xi\xi} + 4c_2V_{j-1,\xi\xi\xi} - 6c_2^2V_{j-1,\xi\xi} - 4c_2^3V_{j-1,\xi} - c_2^4V_{j-1} = 0, \qquad (28b)
\end{aligned}
$$

where $U_{-1} = V_{-1} = 0$, $(j = 0, 1, \ldots, n)$. In this case, we can easily solve the first reduced Eq. (25) which has bright soliton solution. Then, in principle, Eq. (26) can be solved because is linear ordinary differential equation in U_1 or V_1. In the same procedures, Eqs. (27)–(28) can be solved one by one.

4. Conclusion

In summary, we have demonstrated by the above calculation how we reduced the perturbed NLS equation into the ordinary differential equation by applying the approximate symmetry theory. The approximate similarity reduction method is much easier calculation than other methods for treating the nonlinear equations with perturbations. It is very appropriate for solving these problems and future work on these aspects is worthy of studying.

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