Electronic transport through a periodic array of quantum-dot rings*

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Using the tight-binding approximation and the transfer matrix method, this paper studies the electronic transport properties through a periodic array of quantum-dot (QD) rings threaded by a magnetic flux. It demonstrates that the even–odd parity of the QD number in a single ring and the number of the QD rings in the array play a crucial role in the electron transmission. For a single QD ring, the resonance and antiresonance transmission depend not only on the applied magnetic flux but also on the difference between the number of QDs on the two arms of the ring. For an array of QD rings, the transmission properties are related not only to the even–odd parity of the number N_0 of QDs in the single ring but also to the even–odd parity of the ring number N in the array. When the incident electron energy is aligned with the site energy, for the array of N rings with $N_0 =$ odd the antiresonance transmission cannot occur but the resonance transmission may occur and the transmission spectrum has N resonance peaks ($N-1$ resonance peaks) in a period for $N =$ odd (for $N =$ even). For the array of N rings with $N_0 =$ even the transmission properties depend on the flux threading the ring and the QD number on one arm of the ring. These results may be helpful in designing QD devices.

Keywords: quantum dots, electronic transport, antiresonance transmission, resonance transmission

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1. Introduction

The quantum dot (QD) system, as a fundamental branch of mesoscopic physics systems, has been extensively investigated both experimentally and theoretically.[1−5] A single quantum dot (QD) is usually called an artificial atom due to its charge quantization and discrete energy spectrum,[6] while a QD array can be considered as an artificial molecule or artificial crystal.[7,8] In the last two decades, many authors have studied the transport properties of the single quantum dot connected to two electron reservoirs in order to probe its discrete electronic spectrum.[9−18] Furthermore, one has found that the on-site Coulomb interaction may lead to so-called Coulomb-blockade oscillations of the conductance[11,19] and the Kondo peak in differential conductance.[14,15] In recent years, progress in the nanofabrication technique and the potential application in electronic devices have stimulated the increasing interest in theoretical investigation of the electronic transport through the coupled QD systems.[20−26] It was found that the transport current and the differential conductance depend considerably on the ratio of the reservoir resonant width to the interdot tunnelling coupling.[25] On the other hand, the odd–even parity of the conductance is also an interesting problem in quantum transport through the mesoscopic system. In general, the conductance is higher for the atomic wire with an odd number of atoms than that with an even number of atoms.[27−29] When the number of sites in the atomic wire is odd, the conductance can reach the maximum possible value $2e^2/h$ (the conductance quantum),[27−29] and the conductance does not even depend on the details of the hopping matrices in the atomic wire, but is solely determined by the coupling strength of the two leads.[28] When the number is even, the conductance is sensitive to the values of the hopping matrices.[28] Similar even–odd conductance behaviour has been reproduced in the QD array.[22,23,30−33] For a noninteracting QD array side coupled to a quantum wire, if the number of QDs in the array is even, perfect transmission (the conductance is equal to $2e^2/h$) takes place, while perfect reflection (the conductance vanishes) oc-

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curs when this number is odd,[30,31] which is called antiresonance transmission. Recently, antiresonance has been observed experimentally in a structure consisting of a quantum wire laterally coupled to a QD.[34] Another similar system is a Kondo QD with a side-coupled QD array,[32] in which the same odd–even parity in the linear conductance is also found when the energy levels of quantum dots are pinned at the Fermi energy in the leads. This odd–even parity is opposed to that found in embedded QD arrays.[28,29,33] When two arbitrary QDs in one-dimensional QD array couple with the electron reservoirs of continuum states, the linear conductance also exhibits interesting odd–even parity if the chemical potential of the reservoir electrons aligns with the electron energy levels of all QDs, i.e. the conductance vanishes if the dangling QD number outside the leads is odd; otherwise, the conductance is equal to $2e^2/h$ due to perfect transmission for an odd-number QD array.[23] In addition, the odd–even behaviour of the conductance was also found in the transport through QD ring coupled to two electron reservoirs.[35,36] For example, when the QD level is aligned with the Fermi level in the electron reservoirs, an antiresonance zero point persists in the transmission spectrum for an even-numbered QD ring. When the QD number is a multiple of 4, a resonant peak occurs in the conductance spectrum at the zero-magnetic-field case. But such a peak is so fragile that a very small magnetic field perpendicular to the ring plane can change it into an antiresonance zero point.[35] All these results indicate that the features of the electron transport through coupled QD systems are tightly associated with the structural geometry and the odd–even parity of the total QD number.

In the present work, we will investigate the coherent electronic transport through a periodic array of QD rings threaded by a magnetic flux. Our main purpose is to calculate the linear conductance spectrum of such an array and determine the positions of resonant peaks and antiresonance zero points in the linear conductance spectrum. In particular, for an array of QD rings, we will demonstrate that the transmission properties through the array are related not only to the even–odd parity of the number N_0 of QDs in the single ring but also to the even–odd parity of the ring number N in the array. The matching between the number of QDs in the single ring and the ring number in the array may give rise to richer transmission properties.

2. Formalism

Consider a QD ring consisting of multiple coupled QDs with the same site energy. The QDs are assumed to be equally spaced on the ring. The two semi-infinite ordered QD leads with the site energy ε_0 are attached at two arbitrary sites denoted by L and R. The numbers of QDs on the upper and lower arms are n and m (the total number of QDs in a ring is $N_0 = n + m + 2$, see Fig. 1), respectively, and a magnetic flux ϕ penetrates the ring. The full system is described by a tight-binding Hamiltonian

$$H = H_{L,\mathrm{Lead}} + H_{R,\mathrm{Lead}} + H_{\mathrm{ring}} + H_{\mathrm{T}} \tag{1}$$

with

$$H_{L,\mathrm{Lead}} = \sum_{j=-\infty}^{-1} \left[\varepsilon_0 |j\rangle\langle j| + t_0 \left(|j+1\rangle\langle j| + \mathrm{h.c.}\right)\right],$$

$$H_{R,\mathrm{Lead}} = \sum_{j=1}^{\infty} \left[\varepsilon_0 |j\rangle\langle j| + t_0 \left(|j\rangle\langle j-1| + \mathrm{h.c.}\right)\right],$$

$$H_{\mathrm{ring}} = \sum_i \left[\varepsilon |i\rangle\langle i| + t\mathrm{e}^{\mathrm{i}\,\gamma} |i\rangle\langle i+1| + t\mathrm{e}^{-\mathrm{i}\,\gamma} |i+1\rangle\langle i|\right],$$

$$H_{\mathrm{T}} = \left[t_0 \left(|L\rangle\langle -1| + |R\rangle\langle +1|\right) + \mathrm{h.c.}\right],$$

where t and t_0 are the tunnel coupling between nearest-neighbouring QDs in the ring and in leads, respectively. The phase factor $\gamma = 2\pi\phi/\left(N_0\phi_0\right)$ with $\phi_0 = hc/e$ (the flux quantum). Using the procedure described by Chakrabarti *et al.*[37] we obtain the renormalized site energy $\tilde{\varepsilon}$ and effective tunnel coupling $\tilde{t}_f$ ($\tilde{t}_b$) at the sites L and R,

$$\tilde{\varepsilon} = \varepsilon - t\mathrm{e}^{\mathrm{i}\,\gamma}\frac{M_{12}^{n}}{M_{11}^{n}} - t\mathrm{e}^{-\mathrm{i}\,\gamma}\frac{M_{12}^{m*}}{M_{11}^{m*}}, \tag{2}$$

$$\tilde{t}_f = \frac{t\mathrm{e}^{\mathrm{i}\,\gamma}}{M_{11}^{n}} + \frac{t\mathrm{e}^{-\mathrm{i}\,\gamma}}{M_{11}^{m*}}, \quad \tilde{t}_b = \tilde{t}_f^{*} \tag{3}$$

with

$$M = \begin{pmatrix} \dfrac{E-\varepsilon}{t}\mathrm{e}^{-\mathrm{i}\,\gamma} & -\mathrm{e}^{-2\,\mathrm{i}\,\gamma} \\ 1 & 0 \end{pmatrix}.$$

The transmission probability through a single ring is given by[37]

$$T = \frac{4\sin^2 k}{|P_{12} - P_{21} + (P_{11} - P_{22})\cos k|^2 + |P_{11} + P_{22}|^2 \sin^2 k}, \tag{4}$$

where P_{ij} are the matrix elements of the transfer matrix

$$P = \begin{pmatrix} \dfrac{(E-\tilde{\varepsilon})^2}{t_0 \tilde{t}_f} - \dfrac{\tilde{t}_b}{t_0} & -\dfrac{E-\tilde{\varepsilon}}{\tilde{t}_f} \\ \dfrac{E-\tilde{\varepsilon}}{\tilde{t}_f} & -\dfrac{t_0}{\tilde{t}_f} \end{pmatrix}, \quad (5)$$

and the parametrization $E = \varepsilon_0 + 2t_0 \cos k$ is used (the "lattice" constant has been chosen to be unity). For the sake of simplicity, we restrict ourselves to $t_0 = t$ and $\varepsilon_0 = \varepsilon$ throughout the paper.

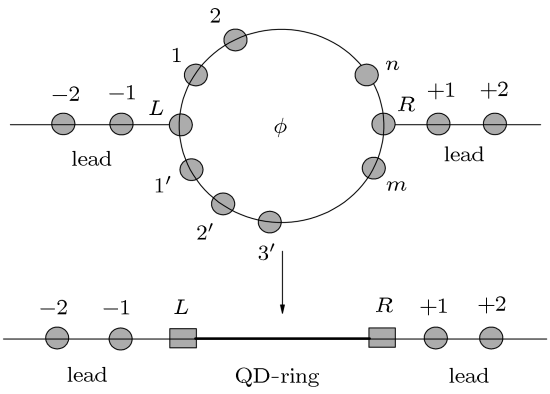


Fig. 1. Schematic representation of a QD ring and the effective dimers, penetrated by a magnetic flux ϕ, sandwiched symmetrically or asymmetrically between the two semi-infinite ordered QD leads.

3. Analytical results and discussion

The electron through a QD ring may obtain a phase because of the flux threading the ring, the transmission probability of the electron through the ring can be strengthened or weakened due to the quantum interference between the wave functions passing through the two arms of the ring. When the phase difference between two arms of the ring is equal to π, the destructive interference between two arms leads to the antiresonance transmission ($T = 0$), which has become a concern of recent theoretical[23,30,35] and experimental[34] works. We consider a QD ring with $m = n + p$ (p is an integer), the renormalized site energy $\tilde{\varepsilon}$ and effective tunnel coupling $\tilde{t}_f$ can be rewritten as

$$\tilde{\varepsilon} = \varepsilon + t \frac{\sin(nk)\sin[(n+p+1)k] + \sin[(n+p)k]\sin[(n+1)k]}{\sin[(n+1)k]\sin[(n+p+1)k]}, \quad (6)$$

$$\tilde{t}_f = t \sin k \frac{\sin[(n+p+1)k]\, \mathrm{e}^{\mathrm{i}(n+1)\gamma} + \sin[(n+1)k]\, \mathrm{e}^{-\mathrm{i}(n+p+1)\gamma}}{\sin[(n+1)k]\sin[(n+p+1)k]}, \quad (7)$$

where we have utilized the dispersion relation $\cos k = (E-\varepsilon)/2t$, $k \in (0, \pi)$. Physically, if the effective tunnel coupling $\tilde{t}_f = 0$ or the renormalized site energy $\tilde{\varepsilon} \to \infty$, the electron hopping from the site L to the site R is completely suppressed or the electron essentially "sees" an infinite potential barrier. As a consequence, the antiresonance phenomenon takes place and the ring becomes completely opaque for the incoming electron. Now, we study the antiresonance transmission through the system in terms of the renormalized site energy $\tilde{\varepsilon}$ and effective tunnel coupling $\tilde{t}_f$. In the case of $\sin[(n+1)k] \to 0$ ($\sin[(n+p+1)k] \neq 0$) or $\sin[(n+p+1)k] \to 0$ ($\sin[(n+1)k] \neq 0$), although $\tilde{\varepsilon} \to \infty$, $\lim \left| \tilde{\varepsilon}/\tilde{t}_f \right| \to 1$ and therefore the antiresonance transmission corresponding to $\tilde{\varepsilon} \to \infty$ does not occur. For the case of $\sin[(n+1)k] \to 0$ and $\sin[(n+p+1)k] \to 0$, we have

$$\lim \left| \tilde{\varepsilon}/\tilde{t}_f \right| = \frac{N_0}{\sqrt{N_0^2 - \sin^2(\pi\phi/\phi_0)(N_0^2 - p^2)}}, \quad \text{for} \quad \cos(pk) = 1, \quad (8)$$

and

$$\lim \left| \tilde{\varepsilon}/\tilde{t}_f \right| = \frac{N_0}{\sqrt{N_0^2 - \cos^2(\pi\phi/\phi_0)(N_0^2 - p^2)}}, \quad \text{for} \quad \cos(pk) = -1, \quad (9)$$

where $N_0 = 2n + p + 2$ is the total number of QDs in the ring. Obviously, the left-hand side of Eq. (8) (Eq. (9)) equals unity at the integer flux (at the half-integer flux), which means that the antiresonance transmission does not occur. But for the other flux values, i.e., $\phi/\phi_0 \neq l$ ($\cos(pk) = 1$) and $\phi/\phi_0 \neq (2l+1)/2$ ($\cos(pk) = -1$) with l being an integer, $\lim \left| \tilde{\varepsilon}/\tilde{t}_f \right| > 1$ which implies that the divergent speed of $\tilde{\varepsilon}$ is faster than $\tilde{t}_f$, thus the electron actually "sees" an infinite potential barrier or well at the site L and leads to the transmission probability $T = 0$. The antiresonance positions are given by $k \in \{l_1\pi/(n+1)\} \cap \{l_2\pi/(n+p+1)\}$ with $l_1 = 1, \ldots, n$, and $l_2 = 1, \ldots, n+p$. For the case of $\sin[(n+1)k] \neq 0$ and $\sin[(n+p+1)k] \neq 0$, the antiresonance transmission condition is determined by

$$\sin\left[(n+p+1)\,k\right]+\mathrm{e}^{-\,\mathrm{i}\,2\pi\phi/\phi_0}\sin\left[(n+1)\,k\right]=0, \tag{10}$$

which corresponds to $\tilde{t}_f = 0$. The condition can be rewritten as

$$\sin\frac{N_0k}{2}\cos\frac{pk}{2}=0 \quad \text{for} \quad \phi=l\phi_0, \tag{11}$$

$$\cos\frac{N_0k}{2}\sin\frac{pk}{2}=0 \quad \text{for} \quad \phi=\frac{2l+1}{2}\phi_0, \tag{12}$$

where l is an integer. For the integer flux and half-integer flux, from Eqs. (11) and (12) one can find that the antiresonance occurs at $k = 2q\pi/N_0$ ($q \in [1, N_0/2)$ $\cup\, k = (2r+1)\,\pi/p$ with $r \in [0, (p-1)/2]$) for $\phi = l\phi_0$ and $k = (2q+1)\pi/N_0$ ($q \in [1, (N_0-1)/2) \cup k = 2r\pi/p$ with $r \in [0, p/2)$) for $\phi = (2l+1)\,\phi_0/2$, and these wave number values must satisfy the conditions of $k \notin \{k = l_1\pi/(n+1)\} \cup \{k = l_2\pi/(n+p+1)\}$. These analytical results show that for a QD ring the number of the antiresonance points and the antiresonance transmission positions depend not only on the applied magnetic flux but also on the difference between the number of QDs on the two arms of the ring. Furthermore, one can find that the antiresonance transmission depends on the even–odd parity of the total number of QDs in the ring at $k = \pi/2$, namely, perfect reflection takes place in the even-numbered ring (due to $\tilde{\varepsilon} \to \infty$ with $\lim\left|\tilde{\varepsilon}/\tilde{t}_f\right| > 1$ or $\tilde{t}_f = 0$ for $k = \pi/2$), which is consistent with Ref. [35], but that cannot occur in the odd-numbered ring (due to $\tilde{\varepsilon} \to \infty$ with $\lim\left|\tilde{\varepsilon}/\tilde{t}_f\right| = 1$ for $k = \pi/2$).

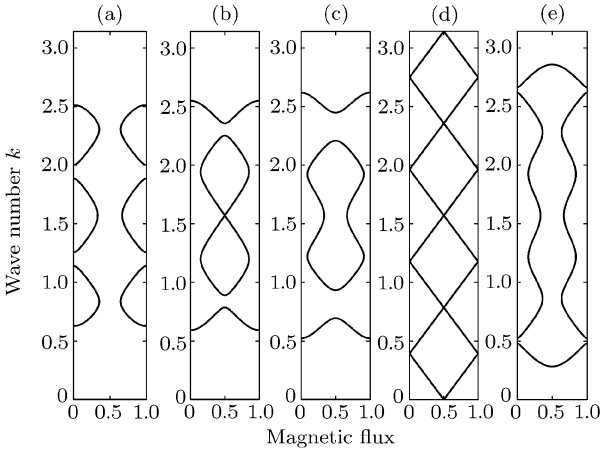


Fig. 2. The resonance points as a function of the magnetic flux ϕ/ϕ_0 and the wave number k for an even-numbered ring with $N_0 = 10$. (a) $p = 0$, (b) $p = 2$, (c) $p = 4$, (d) $p = 6$, (e) $p = 8$.

As for the resonance transmission case, the elements of the transfer matrix P through a single QD ring can be obtained from Eq. (5) as

$$P_{11}=\frac{2\sin^2 k\left[\cos\left(N_0k\right)-\cos\left(2\pi\phi/\phi_0\right)\right]}{\alpha}, \tag{13}$$

$$P_{12}=-P_{21}=-\frac{\sin\left(N_0k\right)\sin k}{\alpha}, \tag{14}$$

$$P_{22}=\frac{\cos\left(N_0k\right)-\cos\left(pk\right)}{2\alpha} \tag{15}$$

with

$$\alpha=\mathrm{e}^{\mathrm{i}(n+1)\gamma}\sin k\{\sin\left[(n+p+1)\,k\right] +\sin\left[(n+1)\,k\right]\mathrm{e}^{-\,\mathrm{i}\,2\pi\phi/\phi_0}\}.$$

After straightforward algebra, the relation between the number N_0 of QDs in the ring, the difference p between the number of QDs on the two arms of the ring, the incident electron energy E and the magnetic flux ϕ at the resonance points is given by

$$4\sin^2 k\cos\left(2\pi\phi/\phi_0\right)+4\sin k\sin\left[(N_0-1)\,k\right] +\cos\left(N_0k\right)-\cos\left(pk\right)=0. \tag{16}$$

We now discuss the relation between the number of the resonance points and the number of QDs in the ring. Since the flux ϕ has the period ϕ_0, we only need to discuss $\phi/\phi_0 \in [0, 1]$. In Fig. 2 we plot the resonance point as a function of the flux ϕ and the wave number k for $N_0 = 10$ and $p = 0, 2, 4, 6, 8$, respectively. For a fixed flux, the the maximum number of resonance points is 8 for $p = 6, 8$ (see Figs. 2(d) and 2(e)) and 6 for $p = 0, 2, 4$ (see Figs. 2(a), 2(b) and 2(c)). It is interesting to note that for a symmetric QD ring with $p = 0$ the flux has a special region where resonance never occurs as displayed in Fig. 2(a). This novel behaviour can be understood by the effective tunnel coupling. If the flux takes the half-integer,

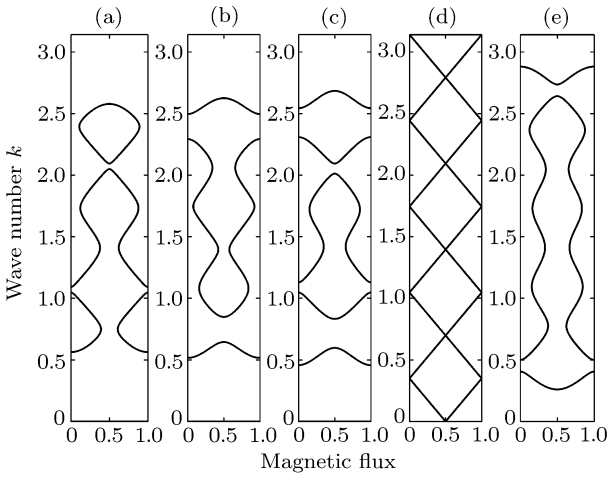


Fig. 3. The resonance points as a function of the magnetic flux ϕ/ϕ_0 and the wave number k for an odd-numbered ring with $N_0 = 11$. (a) $p = 1$, (b) $p = 3$, (c) $p = 5$, (d) $p = 7$, (e) $p = 9$.

from Eq. (10) one can find that $\tilde{t}_f = 0$ for $p = 0$, i.e., the electron tunnelling transition from the site L to the site R is completely prohibited, therefore the transmission probability $T = 0$. This result is consistent with that of Ref. [35]. In Fig. 3 we plot the resonance point as a function of the flux ϕ and wave number k for $N_0 = 11$ and $p = 1, 3, 5, 7, 9$, respectively. The maximum number of resonance points is 9 for $p = 7, 9$ (see Figs. 3(d) and 3(e)) and 7 for $p = 1, 3, 5$ (see Figs. 3(a), 3(b) and 3(c)) at a fixed value of the flux. Comparing with the symmetric case, we find that the asymmetric number p plays a crucial role in the resonance transmission[38] and there is at least one resonance transmission point in the asymmetric QD ring. For the general case, further analysis shows that the maximum number of resonance points is $N_0 - 2$ for $p = N_0 - 2, N_0 - 4$ and $N_0 - 4$ for the other possible p values at $N_0 > 2$, i.e., for a fixed flux value the maximum number of resonance points depends only on the number of QDs in the ring.

In the above analysis the behaviour of the transmission for $k = 0$ (or π) is not considered. When $k \to 0$, the resonance condition is given by

$$16 \sin^2 (\pi\phi/\phi_0) = N_0 (8 - N_0) + p^2. \tag{17}$$

For the case of $k \to \pi$, the resonance conditions are related to the even–odd parity of N_0 (or p) by

$$\begin{cases} 16 \cos^2 (\pi\phi/\phi_0) = N_0 (8 - N_0) + p^2, & \text{for } N_0 \text{ being odd}, \\ 16 \sin^2 (\pi\phi/\phi_0) = N_0 (8 - N_0) + p^2, & \text{for } N_0 \text{ being even}. \end{cases} \tag{18}$$

These conditions require that N_0 and p satisfy $0 \leq N_0 (8 - N_0) + p^2 \leq 16$. Furthermore, further calculations show that if the above conditions cannot be satisfied antiresonance occurs, i.e., $T = 0$.

The double-QD ring is an ideal model for investigating the Fano effect,[39,40] Coulomb blockade[7] and the influence of the intradot and interdot interaction.[41,42] On the other hand, the multi-QD ring and the array of the QD rings may be used to study the influence of the even–odd parity of the QD number in a single ring and the number of the QD rings in the array on the electron transmission. In order to obtain precise analytical results, we restrict ourselves to the $k = \pi/2$ case, i.e., the incident electron energy E is aligned with the site energy ε. The transfer matrix for $N_0 =$ odd can be written as

$$P = -\frac{1}{\alpha} S \begin{pmatrix} \mathrm{e}^{\mathrm{i}\, 2\pi\phi/\phi_0} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\, 2\pi\phi/\phi_0} \end{pmatrix} S^{-1}, \tag{19}$$

where

$$S = \begin{pmatrix} 1 & 1 \\ (-1)^{(N_0+1)/2}\, \mathrm{e}^{-\mathrm{i}\, 2\pi\phi/\phi_0} & (-1)^{(N_0+1)/2}\, \mathrm{e}^{\mathrm{i}\, 2\pi\phi/\phi_0} \end{pmatrix}.$$

Then, the transfer matrix across a periodic array of N QD rings has a compact form

$$P^N = \frac{(-1)^N}{\alpha^N} \begin{pmatrix} \dfrac{\sin 2\pi\,(N+1)\,\phi/\phi_0}{\sin (2\pi\phi/\phi_0)} & \dfrac{(-1)^{(N_0-1)/2} \sin (2N\pi\phi/\phi_0)}{\sin (2\pi\phi/\phi_0)} \\ \dfrac{-(-1)^{(N_0-1)/2} \sin (2N\pi\phi/\phi_0)}{\sin (2\pi\phi/\phi_0)} & \dfrac{-\sin [2\pi\,(N-1)\,\phi/\phi_0]}{\sin (2\pi\phi/\phi_0)} \end{pmatrix}, \tag{20}$$

and the transmission probability is given by

$$T^N = \frac{\sin^2 (2\pi\phi/\phi_0)}{1 - \cos^2 (2N\pi\phi/\phi_0) \cos^2 (2\pi\phi/\phi_0)}. \tag{21}$$

It is evident from Eq. (21) that the transmission probability oscillates with the flux for a given N. The resonance condition is determined by

$$\cos (2\pi\phi/\phi_0) \sin (2N\pi\phi/\phi_0) = 0, \tag{22}$$

which gives rise to the flux at resonance points. The number of resonance points in a period is related to the even–odd parity of the ring number N in the array, i.e., there are N resonance peaks in a period for $N =$ odd and

$N-1$ resonance peaks for $N=$ even. When the flux ϕ/ϕ_0 takes the integer or half-integer, $T^N = 1/\left(1+N^2\right)$, which means that antiresonance transmission does not occur in the array and is consistent with Ref. [37].

Compared to the odd-numbered QDs case, when N_0 is an even number, the even–odd parity of n plays an important role in the electron transmission. The transfer matrix of the array of N QD rings has a simple form

$$P^N = \begin{pmatrix} \dfrac{2\left[(-1)^{N_0/2} - \cos\left(2\pi\phi/\phi_0\right)\right]}{\alpha} & 0 \\ 0 & \dfrac{(-1)^{N_0/2} - (-1)^{p/2}}{2\alpha} \end{pmatrix}^N . \tag{23}$$

If n is an odd number and $\phi/\phi_0 \neq N_0/4$, the transmission probability $T^N \to 0$ due to $P_{11}^N \to \infty$ and $P_{22}^N = 0$, i.e., antiresonance occurs, which corresponds to the case of $\tilde{\varepsilon} \to \infty$ and $\lim_{k=\pi/2} \left|\tilde{\varepsilon}/\tilde{t}_f\right| > 1$. As for the $\phi/\phi_0 = N_0/4$ case, one can check that the transmission probability across an arbitrarily long array of such rings is unity. When n is an even number, $|\alpha|^2 = 2[1+(-1)^{p/2}\cos\left(2\pi\phi/\phi_0\right)]$, the corresponding transmission probabilities are given by

$$T^N = \frac{4\left[2\sin\left(\pi\phi/\phi_0\right)\right]^{2N}}{\left\{1+\left[2\sin\left(\pi\phi/\phi_0\right)\right]^{2N}\right\}^2} \tag{24}$$

for $N_0 = 4l$ (l is an integer, corresponding to $p = 2, 6, 10, \ldots$) and

$$T^N = \frac{4\left[2\cos\left(\pi\phi/\phi_0\right)\right]^{2N}}{\left\{1+\left[2\cos\left(\pi\phi/\phi_0\right)\right]^{2N}\right\}^2} \tag{25}$$

for $N_0 = 4l+2$ (l is an integer, corresponding to $p = 0, 4, 8, \ldots$). When $N_0 = 4l$, the resonance transmission occurs at $\phi/\phi_0 = 1/6, 5/6$ in a period, and when $N_0 = 4l+2$, the resonance transmission occurs at $\phi/\phi_0 = 1/3, 2/3$ in a period. The antiresonance transmission occurs at integer flux for $N_0 = 4l$ and half-integer flux for $N_0 = 4l+2$, which corresponds to the $\tilde{t}_f = 0$ case. When N gets large, these oscillations turn into sharp peaks. Furthermore, one can find from Eqs. (24) and (25) that electron transmission is suppressed near $\phi/\phi_0 = j$ ($j = 0, 1, 2, \ldots$) for $N_0 = 4l$, which is consistent with Ref. [35], and the suppression occurs near $\phi/\phi_0 = (2j+1)/2$ ($j = 0, 1, 2, \ldots$) for $N_0 = 4l+2$. In fact, the eigenstate of the QD ring is modulated by an applied magnetic flux; when the electron passes the system along different paths, the phase factor induced by the magnetic field can be different, and thus the constructive and destructive interference of the partial electron wave functions through different paths may lead to the flux dependence of the resonance or antiresonance transmission.

4. Summary

We have studied the resonance and antiresonance transmission properties through a periodic array of the QD rings threaded by a magnetic flux. For a single QD ring, the resonance and antiresonance transmission depend not only on the applied magnetic flux but also on the difference between the number of QDs on the two arms of the ring. In addition, the resonance and antiresonance transmission depend on the even–odd parity of the total number of QDs in the ring. When the incident electron energy is aligned with the site energy ($k = \pi/2$), perfect reflection may take place in the even-numbered ring but cannot occur in the odd-numbered ring. For an array of QD rings, the transmission property is related not only to the even–odd parity of the number N_0 of QDs in the single ring but also to the even–odd parity of the ring number N in the array. For the array of N rings, if each ring contains odd-numbered QDs ($N_0 =$ odd) the antiresonance transmission cannot occur but the resonance transmission may take place at $k = \pi/2$. The transmission spectrum has N resonance peaks in a period for $N =$ odd and $N-1$ resonance peaks for $N =$ even. If each ring contains even-numbered QDs ($N_0 =$ even), the transmission property is related not only to the number N_0 of QDs in the single ring and the flux threading the ring but also to the even–odd parity of the QD number n on the upper arm of the ring. When $n =$ odd, the resonance transmission occurs at $\phi/\phi_0 = N_0/4$ and the antiresonance transmission occurs at $\phi/\phi_0 \neq N_0/4$. When $n =$ even, the resonance transmission occurs at $\phi/\phi_0 = 1/6, 5/6$ in a period for $N_0 = 4l$ and $\phi/\phi_0 = 1/3, 2/3$ in a period for $N_0 = 4l+2$, while the antiresonance transmission occurs at integer flux for $N_0 = 4l$ and half-integer flux for $N_0 = 4l+2$. These results may be helpful in designing QD devices.

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