Influence of spin–orbit coupling induced by in-plane external electric field on the intrinsic spin-Hall effect in a Rashba two-dimensional electron gas*

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We study theoretically the influence of spin–orbit coupling induced by in-plane external electric field on the intrinsic spin-Hall effect in a two-dimensional electron gas with Rashba spin–orbit coupling. We show that, after such an influence is taken into account, the static intrinsic spin-Hall effect can be stabilized in a disordered Rashba two-dimensional electron gas, and the static intrinsic spin-Hall conductivity shall exhibit some interesting characteristics as conceived in some original theoretical proposals.

Keywords: Rashba spin–orbit coupling, spin–orbit coupling induced by in-plane external electric field, spin-Hall effect

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The spin-Hall effect, which refers to the phenomenon that a spin current is generated in the direction perpendicular to an external electric field when it drives a charge current to circulate through a nonmagnetic semiconductor (or metal), has recently attracted considerable theoretical[1−4] and experimental[5−10] interest for possible applications in the emerging field of spintronics. This effect arises from the spin–orbit interaction of charge carriers, and two different types of the effect have been distinguished according to the origin of the spin–orbit interaction, namely the extrinsic spin-Hall effect (due to spin–orbit dependent impurity scattering) and the intrinsic spin-Hall effect (due to intrinsic spin–orbit splitting of the band structures), respectively. Since the intrinsic spin-Hall effect is a band structure effect, some theoretical proposals predicted that this effect would exhibit some unusual characteristics. For example, the spin-Hall conductivity due to this effect would be independent of disorder scattering and all states below the Fermi level could contribute substantially to it,[3,4] thus its magnitude could be several orders larger than that due to the extrinsic spin-Hall effect.[1,2] Such possibilities give explosive interest to the study of the intrinsic spin-Hall effect in various types of spin–orbit coupled semiconducting electronic systems with intrinsic spin–orbit split bands.[11−29] One of the most widely studied semiconducting electronic systems is two-dimensional electron gases in quantum wells that have substantial Rashba spin–orbit coupling. Such a system is very attractive both theoretically and experimentally because the strength of the spin–orbit coupling can be tuned effectively by applying a gate voltage. But unfortunately, after the original theoretical proposal of the effect,[4] it has been realized by many subsequent theoretical works that a static electric field cannot induce such an effect in a diffusive two-dimensional electron gas with Rashba spin–orbit coupling.[11−16] These works found that the static spin-Hall conductivity in a diffusive Rashba two-dimensional electron gas will be suppressed completely by disorder scattering (even in the limit of weak disorder scattering). Nevertheless, it should be noted that these conclusions do not mean that no static spin-Hall effect can survive in a real two-dimensional electron gas with Rashba spin–orbit coupling. This is due to the fact that, in addition to the Rashba spin–orbit coupling, other types of spin–orbit interaction may also exist in a realistic two-dimensional electron gas. From theoretical points of view, when different types of spin–orbit coupling

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are present simultaneously, the non-Abelian nature of spin may lead to significantly new effects, which will not just be a linear superposition of those encountered with only one type of spin–orbit coupling. Due to this fact, the study of the interplay of different types of spin–orbit coupling on spin-dependent transport will not only be experimentally relevant but also a subject of theoretical interest. In this paper we study theoretically the influence of an unusual type of spin–orbit coupling on the intrinsic spin-Hall effect in a Rashba two-dimensional electron gas. This type of spin–orbit interaction is induced by the in-plane external electric field that drives the charge current to circulate through a two-dimensional electron gas. According to the principle of relativistic quantum mechanics, like the perpendicular confining electric field normal to the interface of a two-dimensional electron gas, which induces the Rashba spin–orbit coupling, the in-plane external electric field that drives the charge current to circulate through a two-dimensional electron gas will also induce a spin–orbit interaction.[30] But unlike the usual intrinsic or the usual impurity-induced extrinsic spin–orbit coupling, this type of spin–orbit interaction will disappear completely when the external electric field is turned off. In this paper we will show that this type of spin–orbit interaction may have an important influence on the intrinsic spin-Hall effect in a Rashba two-dimensional electron gas, and, especially, it may play a determinant role in the stabilization of the static intrinsic spin-Hall effect in such systems.

For clarity, below we will iterate first some details of the usual two-dimensional Rashba model. For a two-dimensional electron gas confined in the xy plane and subjected to a confining electric field normal to the interface, the single-particle Hamiltonian of the system will read

$$\hat{H}_{\rm R} = \frac{\hbar^2 k^2}{2m} - \alpha_{\rm R}(\hat{\boldsymbol{u}}_z \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}, \tag{1}$$

where m is the effective mass and $\boldsymbol{k}$ is the wave vector of electrons, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the Pauli matrices of spin 1/2, $\hat{\boldsymbol{u}}_i$ $(i = x, y, z)$ denotes the unit vector along the i-th direction, and $\alpha_{\rm R}$ denotes the Rashba spin–orbit coupling constant. For a given wave vector $\boldsymbol{k} = (k\cos\theta, k\sin\theta)$, two spinor eigenstates can be obtained and expressed as

$$|\boldsymbol{k}\lambda\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -\mathrm{i}\lambda\, \mathrm{e}^{-\mathrm{i}\theta} \\ 1 \end{pmatrix}, \tag{2}$$

where $\lambda = \pm 1$ is the helicity of electrons. The corresponding eigenenergies will be given by

$$\epsilon_{\boldsymbol{k}\lambda} = \frac{\hbar^2 \boldsymbol{k}^2}{2m} - \lambda\alpha_{\rm R} k. \tag{3}$$

When an in-plane external electric field is applied and taking the spin–orbit coupling induced by the in-plane external electric field into account, the effective single-particle Hamiltonian of the system will read

$$\begin{aligned} \hat{H} = {} & \frac{\hbar^2 k^2}{2m} - \alpha_{\rm R}(\hat{\boldsymbol{u}}_z \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} + e\boldsymbol{E} \cdot \boldsymbol{r} \\ & + e\alpha_{\rm E}(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} + V_i(\boldsymbol{r}), \end{aligned} \tag{4}$$

where $\boldsymbol{E}$ is the in-plane external electric field, $V_i(\boldsymbol{r})$ denotes a spinless impurity potential, and the term proportional to $\alpha_{\rm E}$ is the spin–orbit coupling induced by the in-plane external electric field, with $\alpha_{\rm E}$ denoting the spin–orbit coupling constant. For simplicity and clarity, we will neglect the extrinsic spin–orbit coupling induced by impurity scattering. It also should be noted that the dimension of $\alpha_{\rm E}$ is different from that of $\alpha_{\rm R}$ when the Hamiltonian is written in the form of Eq. (4), and due to the enhancement by the crystal field in real semiconductors, the value of $\alpha_{\rm E}$ in some semiconductors can be several orders of magnitude larger than the value in vacuum. For example, for n-doped GaAs, one finds $\alpha_{\rm E} \approx 5.3$ Å^2 (1 Å = 0.1 nm),[30] which is about six orders of magnitude larger than the vacuum value.

From theoretical points of view, the intrinsic spin-Hall effect results from dynamic spin precession of electrons under the combined action of the external electric field and the spin–orbit interaction.[4] This mechanism can be described by the Heisenberg equations of motion governing the time evolution of the electrons' orbital and spin degrees of freedom, which read

$$\hbar\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{k} = -e\boldsymbol{E} - \nabla_{\boldsymbol{r}} V_i(\boldsymbol{r}), \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{S} = \boldsymbol{S} \times \boldsymbol{\Delta}(\boldsymbol{k}). \tag{6}$$

Equation (6) is just the usual Bloch equation, in which $\boldsymbol{\Delta}(\boldsymbol{k})$ represents a momentum-dependent effective magnetic field due to the spin–orbit interaction of charge carriers,

$$\boldsymbol{\Delta}(\boldsymbol{k}) = \frac{2\alpha_{\rm R}}{\hbar}(\hat{\boldsymbol{u}}_z \times \boldsymbol{k}) - \frac{2e\alpha_{\rm E}}{\hbar}(\boldsymbol{E} \times \boldsymbol{k}). \tag{7}$$

From Eqs. (5)–(7) one can see that, when an electron is accelerated by the external electric field, the effective magnetic field $\boldsymbol{\Delta}$ will become time-dependent and dynamic spin precession around the effective magnetic

field will hence be induced. For details of such a mechanism please refer to the illustration shown in Fig. 1 of Ref. [4]. But it should be noted that in the calculation performed in Ref. [4] the effects of disorder scattering were not considered. For a real system the effects of disorder scattering may be important and need to be considered in a proper way. In this paper we will treat the effects of disorder scattering within the usual Drude model, which has proven to be very powerful in describing both dc and ac transport phenomena, in spite of being classical in nature.[31] Recently it was shown that the influence of disorder scattering on the spin-Hall effect can also be properly described within this model.[25] In the spirit of the Drude model, the influence of isotropic disorder scattering on the motion of electrons can be appropriately included by replacing the drag force due to impurity scattering with a relaxation time as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{k} = -\frac{e\boldsymbol{E}}{\hbar} - \frac{\boldsymbol{k}-\boldsymbol{k}_0}{\tau}, \tag{8}$$

where τ is the relaxation time determined by impurity scattering and $\boldsymbol{k}_0 = (k_{0x}, k_{0y})$ is the initial wave vector of electrons. Similarly, the effects of weak spin dampings on the dynamic spin precession can also be properly included by adding a damping term in the Bloch equation as follows:

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{S}\times\boldsymbol{\Delta}(k_x,k_y) + \gamma\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t}\times\boldsymbol{S}, \tag{9}$$

where γ is a parameter characterizing the strength of spin dampings. Though the inclusion of such damping effects is very straightforward within the approach applied below, as was done in some previous studies, in this paper we will neglect such damping effects for both simplicity and clarity.

Now we consider the electrically-induced dynamic spin precession of electrons based on the above equations of motion. We will consider first the ac response and then take the dc limit. To this end, we assume that a weak ac electric field $\boldsymbol{E} = \boldsymbol{E}_x\,\mathrm{e}^{\mathrm{i}\omega t}$ is applied in the x direction. Then from Eq. (8) one obtains

$$k_y = k_{0y},\ k_x = k_{0x} - \frac{eE_x/\hbar}{\mathrm{i}\omega+\tau^{-1}}\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{10}$$

Substituting Eq. (10) into Eq. (7), a time-dependent effective magnetic field will be obtained. Within a semiclassical description, for an electron with the initial wave vector $\boldsymbol{k}_0$ and helicity λ, the dynamic precession of spin around the time-dependent effective magnetic field can be expressed as

$$\tilde{S}_{\lambda,i}(\boldsymbol{k}_0) = S^{(0)}_{\lambda,i}(\boldsymbol{k}_0) + \Omega_{\lambda,i}(\boldsymbol{k}_0)\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{11}$$

Here $S^{(0)}_{\lambda,i}(\boldsymbol{k}_0) = (\hbar/2)\langle\boldsymbol{k}_0\lambda|\hat{\sigma}_i|\boldsymbol{k}_0\lambda\rangle$ $(i = x,y,z)$ denotes the equilibrium expectation value of spin and the term $\Omega_{\lambda,i}(\boldsymbol{k}_0)\,\mathrm{e}^{\mathrm{i}\omega t}$ describes the dynamic precession of spin around the equilibrium orientation. Substituting Eqs. (10) and (11) into Eqs. (6) and (7) and keeping only the linear terms of $\Omega_{\lambda,i}$ and E_x, one can find that $\Omega_{\lambda,i}$ $(i = x,y,z)$ will satisfy the following equations,

$$\begin{aligned}&\mathrm{i}\omega\Omega_{\lambda,x}(\boldsymbol{k}_0)\\ &= -\frac{2\alpha_{\mathrm{R}}}{\hbar}k_{0x}\Omega_{\lambda,z}(\boldsymbol{k}_0) + \lambda\frac{e\alpha_{\mathrm{E}}E_xk_{0x}k_{0y}}{k_0},\end{aligned} \tag{12}$$

$$\begin{aligned}&\mathrm{i}\omega\Omega_{\lambda,y}(\boldsymbol{k}_0)\\ &= -\frac{2\alpha_{\mathrm{R}}}{\hbar}k_{0y}\Omega_{\lambda,z}(\boldsymbol{k}_0) + \lambda\frac{e\alpha_{\mathrm{E}}E_xk_{0y}^2}{k_0},\end{aligned} \tag{13}$$

$$\begin{aligned}&\mathrm{i}\omega\Omega_{\lambda,z}(\boldsymbol{k}_0)\\ &= \frac{2\alpha_{\mathrm{R}}}{\hbar}[k_{0x}\Omega_{\lambda,x}(\boldsymbol{k}_0) + k_{0y}\Omega_{\lambda,y}(\boldsymbol{k}_0)]\\ &\quad - \lambda\frac{e\alpha_{\mathrm{R}}E_x(k_{0y}/k_0)}{\mathrm{i}\omega\hbar+\hbar/\tau},\end{aligned} \tag{14}$$

and from which $\Omega_{\lambda,i}$ $(i = x,y,z)$ can be determined. For the calculation of the spin-Hall current with spin parallel to the normal of the interface, only the z component $\Omega_{\lambda,z}$ will be involved, whose solution reads

$$\begin{aligned}\Omega_{\lambda,z}(\boldsymbol{k}_0) &= \lambda\frac{e\hbar\alpha_{\mathrm{R}}E_x(k_{0y}/k_0)}{\omega^2\hbar^2 - 4\alpha_{\mathrm{R}}^2k_0^2}\frac{\mathrm{i}\omega}{\mathrm{i}\omega+\tau^{-1}}\\ &\quad - \lambda\frac{2e\hbar\alpha_{\mathrm{R}}\alpha_{\mathrm{E}}E_x(k_{0y}k_0)}{\omega^2\hbar^2 - 4\alpha_{\mathrm{R}}^2k_0^2}.\end{aligned} \tag{15}$$

After $\tilde{S}_{\lambda,z}(\boldsymbol{k}_0)$ is obtained according to Eqs. (11) and (15), the spin-Hall current flowing to the y direction with spin parallel to the z direction can be calculated through the following formula,[4,16]

$$J_y^z = \sum_\lambda\int\frac{\mathrm{d}^2\boldsymbol{k}_0}{(2\pi)^2}\tilde{S}_{\lambda,z}(\boldsymbol{k}_0)(\hbar k_{0y}/m)f_\lambda(\boldsymbol{k}_0), \tag{16}$$

where $f_\lambda(\boldsymbol{k})$ is the distribution function of electrons. Within the relaxation time approximation assumed above, the distribution function of electrons in the presence of a weak external electric field can be given by

$$f_\lambda(\boldsymbol{k}) = f_0(\epsilon_{\boldsymbol{k}\lambda}) - e\tau(\boldsymbol{E}\cdot\boldsymbol{v}_{\boldsymbol{k}})\frac{\partial f_0(\epsilon_{\boldsymbol{k}\lambda})}{\partial\epsilon_F}, \tag{17}$$

where $\boldsymbol{v}_{\boldsymbol{k}} = \hbar\boldsymbol{k}/m$ is the semiclassical velocity and $f_0(\epsilon_{\boldsymbol{k}\lambda})$ is the equilibrium Fermi–Dirac distribution function. Then the following ac spin-Hall conductivity (defined by $\sigma_s \equiv -J_y^z/E_x$) can be obtained from Eq. (16),

$$\sigma_s(\omega)$$

$$= \frac{\mathrm{i}\omega e\hbar^2\alpha_{\mathrm{R}}}{m(\mathrm{i}\omega+\tau^{-1})}\left[\sum_{\lambda}\int\frac{\mathrm{d}^2\boldsymbol{k}_0}{(2\pi)^2}\frac{\lambda f_0(\epsilon_{\boldsymbol{k}_0\lambda})(k_{0y}^2/k_0)}{4\alpha_{\mathrm{R}}^2k_0^2-\omega^2\hbar^2}\right] - \frac{2e\hbar^2\alpha_{\mathrm{R}}\alpha_{\mathrm{E}}}{m}\left[\sum_{\lambda}\int\frac{\mathrm{d}^2k_0}{(2\pi)^2}\frac{\lambda f_0(\epsilon_{k_0\lambda})(k_{0y}^2k_0)}{4\alpha_{\mathrm{R}}^2k_0^2-\omega^2\hbar^2}\right]. \tag{18}$$

Several interesting conclusions can be obtained from Eq. (18). Firstly, one can notice that, if the spin–orbit coupling induced by the in-plane external electric field is neglected (i.e, setting $\alpha_{\mathrm{E}}=0$), then from Eq. (18) one can recover some well established results obtained in some recent papers.[11−15] By setting $\alpha_{\mathrm{E}}=0$, the following ac spin-Hall conductivity will be obtained from Eq. (18),

$$\sigma_s(\omega) = \frac{\mathrm{i}\omega e\hbar^2\alpha_{\mathrm{R}}}{m(\mathrm{i}\omega+\tau^{-1})}\left[\sum_{\lambda}\int\frac{\mathrm{d}^2\boldsymbol{k}_0}{(2\pi)^2}\frac{\lambda f_0(\epsilon_{\boldsymbol{k}_0\lambda})(k_{0y}^2/k_0)}{4\alpha_{\mathrm{R}}^2k_0^2-\omega^2\hbar^2}\right]. \tag{19}$$

The spin-Hall conductivity given by this formula is nonvanishing if $\omega\neq 0$, indicating that an ac electric field can drive an ac spin-Hall current in the system. However, the spin-Hall conductivity given by this formula will vanish exactly in the static limit if the impurity scattering rate $\tau^{-1}\neq 0$, indicating that the static spin-Hall effect in a diffusive Rashba two-dimensional electron gas will be suppressed completely by disorder scattering (even in the limit of weak disorder scattering). This conclusion is in agreement with those obtained in Refs. [11]–[15]. On the other hand, in the ballistic transport limit where disorder scattering can be neglected (i.e., $\tau^{-1}=0$), a finite static spin-Hall conductivity can also be obtained from Eq. (19) by setting $\tau^{-1}=0$ first and then taking the static limit, and the result reads

$$\sigma_{\mathrm{s.dc}} = \frac{e\hbar^2}{4m\alpha_{\mathrm{R}}}\left[\sum_{\lambda}\int\frac{\mathrm{d}^2\boldsymbol{k}_0}{(2\pi)^2}\lambda f_0(\epsilon_{\boldsymbol{k}_0\lambda})(k_{0y}^2/k_0^3)\right] = \frac{e\hbar^2}{16\pi m\alpha_{\mathrm{R}}}[k_{\mathrm{F}+}-k_{\mathrm{F}-}], \tag{20}$$

where $k_{\mathrm{F}+}$ and $k_{\mathrm{F}-}$ are the Fermi wave vectors of the majority and minority spin Rashba bands. Equation (20) is just the main result obtained in Ref. [4]. Especially, in the usual cases where both spin–orbit split bands are occupied, one will have $k_{\mathrm{F}+}-k_{\mathrm{F}-}=2m\alpha_{\mathrm{R}}/\hbar^2$. Then a universal dc spin-Hall conductivity of $e/8\pi$ will be obtained in such cases, in agreement with the result obtained in Ref. [4].

Secondly, from Eq. (18) one can see that, if the spin–orbit coupling induced by the in-plane external electric field is taken into account, then not only is the ac spin-Hall conductivity nonvanishing but the static spin-Hall conductivity will also become nonvanishing, even in the presence of disorder scattering (i.e., $\tau^{-1}\neq 0$). The nonvanishing static spin-Hall conductivity obtained from Eq. (18) reads

$$\sigma_{\mathrm{s.dc}} = -\frac{e\hbar^2}{2m}\frac{\alpha_{\mathrm{E}}}{\alpha_{\mathrm{R}}}\left[\sum_{\lambda}\int\frac{\mathrm{d}^2\boldsymbol{k}_0}{(2\pi)^2}\lambda f_0(\epsilon_{\boldsymbol{k}_0\lambda})(k_{0y}^2/k_0)\right] = \frac{e\hbar^2}{24\pi m}\frac{\alpha_{\mathrm{E}}}{\alpha_{\mathrm{R}}}[k_{\mathrm{F}-}^3-k_{\mathrm{F}+}^3]. \tag{21}$$

It is interesting to note that the static spin-Hall conductivity given by Eq. (21) is independent of the impurity scattering rate (i.e., independent of the disorder scattering effect), similar to that as conceived in the original theoretical proposals of the intrinsic spin-Hall effect.[3,4] Moreover, as conceived in the original theoretical proposals, all states below the Fermi level will contribute to the spin-Hall current. This is significantly different from that encountered with usual dissipative charge and/or spin transports, where it is usually only the states in the vicinity of the Fermi level that can contribute substantially to a charge or spin current. Furthermore, since the spin-Hall conductivity given by Eq. (21) depends on both the Rashba spin–orbit coupling and the spin–orbit coupling induced by the in-plane external electric field, its magnitude can be tuned by applying the gate voltage, which implies that the controllability of the Rashba spin–orbit interaction combined with the influence of the spin–orbit coupling induced by the in-plane external electric field might offer an effective way to achieve electric control of spins based on the intrinsic spin-Hall effect. Of course, it should be pointed out that, though some unusual characteristics conceived in some original theoretical proposals of the intrinsic spin-Hall effect may survive after the influence of the spin–orbit coupling induced by in-plane external electric field is taken into account, some significant differences may also arise. For example, the nonvanishing static spin-Hall conductivity obtained above cannot take a universal value independent of material parameters as was conceived in Ref. [4], and its magnitude may be substantially smaller than the original theoretical prediction. To get a quantitative estimate, we take n-doped GaAs for an example. For n-doped GaAs, typical values of the material parameters involved are $\alpha_{\mathrm{E}}\sim 5.3$ Å^2,

$\alpha_{\rm R} \sim 10$–100 nm·meV, $m \sim 0.01 m_{\rm e}$, and the electron density $n \sim 10^{10}$–$10^{12}/{\rm cm}^2$.[30] Then the static spin-Hall conductivity given by Eq. (21) can be estimated to be around the order of $(10^{-2}$–$10^{-4}) \times e/8\pi$, which is substantially smaller than the universal value of $e/8\pi$ in the clean limit, but is still within the range of physical interest and experimentally detectable by current technology.[5−10]

In summary, we have presented a theoretical investigation for the influence of the spin–orbit coupling induced by in-plane external electric field on the intrinsic spin-Hall effect in a Rashba two-dimensional electron gas. The results obtained suggest that, after such an influence is taken into account, the static intrinsic spin-Hall effect could be stabilized in a disordered Rashba two-dimensional electron gas, and the nonvanishing static spin-Hall conductivity will exhibit some interesting characteristics as was conceived in some original theoretical proposals, while some significant differences may also arise.

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